

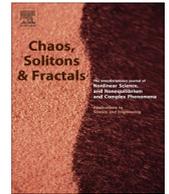


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Order to chaos transition studies in a DC glow discharge plasma by using recurrence quantification analysis



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ABSTRACT

Recurrence quantification analysis (RQA) is used to study dynamical systems and to identify the underlying physics when a system exhibits a transition due to changes in some control parameter. The tendency of reoccurrence of different states after certain interval reflects and reveals the hidden patterns of a complex time series data. The present work involves the study of the floating potential fluctuations of a glow discharge plasma obtained by using a Langmuir probe. Determinism, entropy and L_{\max} are important measures of RQA that show an increasing and decreasing trend with variation in the values of discharge voltages and indicate an order-chaos transition in the dynamics of the fluctuations. Statistical analysis techniques represented by skewness and kurtosis are also supportive of a similar phenomenon occurring in the system.

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1. Introduction

Experimental investigations on plasma systems have revealed many interesting features interspersed over regular and chaotic dynamics such as self-excited oscillations, period doubling, intermittency, strange attractor and other routes to chaos [1–5]. Like in many other dynamical systems, the transition from one state to another can be triggered by altering a specific control parameter of the system [6–8]. In a DC plasma discharge produced in a glow discharge device, neutral gas pressure and breakdown voltage serve as important control parameters, by varying which, signatures of various phenomena can be studied. Further, a DC glow discharge plasma not driven by any external periodic force exhibits a kind of self-generated oscillation whose frequency varies with the control parameters of the discharge. Experimental observations of deterministic chaos have been reported by Qin et al. [9] in an undriven

steady-state plasma wherein period-doubling sequence and intermittency have been found as two routes to chaotic behaviour. Jaman et al. [10] have used nonlinear techniques like correlation dimension, largest Lyapunov exponent etc. to investigate the inverse bifurcation behaviour in the floating potential fluctuations at different discharge voltages in a cylindrical electrode system with a hollow cathode. All the existing and so far extensively used nonlinear techniques to study various phenomena in a plasma assume that a stationary time series of sufficient length and free from noise is available. In many of the plasma experiments, such requirements are hard to fulfil, particularly those related to fusion plasmas that are pulsed in behaviour. These limitations prompt us to use statistical methods based on recurrent behaviour of turbulent phenomena where stationarity of time series or availability of long data does not pose restrictions. The recurrence plot (RP) is a relatively new technique for the qualitative analysis of time series signals and was introduced by Eckmann et al. [11]. Applying RPs we can graphically detect different

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patterns and structural changes in time series data. RPs are graphical, two dimensional representations showing the instants of time at which a phase space trajectory returns approximately to the same regions of phase space. The number and duration of recurrences of a dynamical system presented by its phase space trajectory can be quantified by recurrence based diagnostics [12–14], like determinism, Shannon entropy, laminarity, trapping time etc. Over the last two decades, RPs have been extensively used in diverse fields such as life science, earth science, astrophysics, chemical reactions and space plasmas to gain understanding about the nonlinear dynamics of complex systems [15]. In fusion plasmas, recurrence quantification analysis (RQA) techniques are emerging as important tools to understand plasma turbulence and associated cross field transport. RQA of electrostatic turbulent data from the plasma edge of the Tokamak Chauffage Alfvén Bresilien [16] tokamak reveal that biasing improves confinement through destruction of highly recurrent regions [17] within the plasma column that enhance particle and heat transport. The influence of external biasing on Texas Helimak turbulence reveals two kinds of perturbed turbulence that have been classified according to recurrence properties [18] and consistently validated by spectral and statistical properties. RQA has also been utilized to analyze simulation data of ion temperature gradient turbulence [19] and dissipative trapped electron mode turbulence [20], and to characterize transport dynamics. A method based on spatial recurrence property is also used to investigate the reconstruction and identification of complex dynamical regimes of a complex spatio-temporal system [21]. Intermittency type dynamics occurring in a low temperature discharge plasma [22] have been analyzed to show that the determinism as measured by the RQA variable DET depends on the control parameter as a decreasing exponential.

In this report an attempt has been made to understand the periodic, quasi periodic and chaotic behaviour of floating potential fluctuations in a glow discharge plasma under different experimental conditions using the RP technique. Typical time series obtained from floating potential fluctuations are not purely stochastic, but rather quasi-periodic or chaotic in nature. Visual changes in RPs are good approach to detect the transition in potential fluctuations easily. Skewness and kurtosis of potential fluctuations are estimated and it is observed that at 381v and 396v there is a transition in the dynamics of the system. In Section 2, we discuss the basics of RP and RQA quantification analysis and brief description about skewness and kurtosis. Section 3 describes the experimental setup while Section 4 discusses the results obtained. Conclusions are presented in Section 5.

2. Theory

2.1. Recurrence plot

Recurrence plot (RP) analysis of nonlinear time series is a relatively new and advanced technique based on phase space reconstruction which besides giving visual informa-

tion also has several measures to quantify various complexities associated with the different small scale recurrent structures. According to Taken's embedding theorem [23] using a time series data X_i an embedding can be made using the vector $\vec{Y}_i = \bar{X}_i; \bar{X}_{i+\tau}; \bar{X}_{i+(d-1)\tau}$ where d is the embedding dimension and τ is the time delay. The correct embedding parameters preserving the topological property of phase space are estimated by false nearest neighbour and mutual information method [24]. The original time series is now embedded into a d -dimensional reconstructed phase space. A recurrence is said to occur whenever a trajectory visits approximately the same region of phase space. The RP is a graphical representation of the square matrix

$$R_{ij} = H(\epsilon - \|\vec{Y}_i - \vec{Y}_j\|), \quad i, j = 1, 2, \dots, N$$

where ϵ is a predetermined threshold, and H is the Heaviside unit step function and N is the number of data points of the signal. Both the axes of the graph represent the temporal extent to which the signal spans. Diagonal lines in the plots are indicative of deterministic behaviour and indicate similar evolution of states at different times [25]. States that change slowly, like those occurring during laminar phases cause horizontal and vertical lines. Dynamical states that are rare cause isolated points.

2.2. Recurrence quantification analysis

Several statistical measures are there to quantify the characteristics of the different structures appearing in a RP form a diagnostic tool known as recurrence quantification analysis (RQA) [26]. For example, the RQA measure determinism (DET) gives the ratio of the number of recurrence points in the diagonal lines to all the recurrence points. For deterministic dynamics, determinism (DET) is close to unity and approaches zero when the behaviour is random. Other parameters related to diagonal lines such as average diagonal line length or Shannon entropy of the probability distribution of the diagonal line lengths also reflect the complexity of the deterministic structures in the system. The longest diagonal line ($line_{max}$) in the RP is related to the exponential divergence of phase space trajectory.

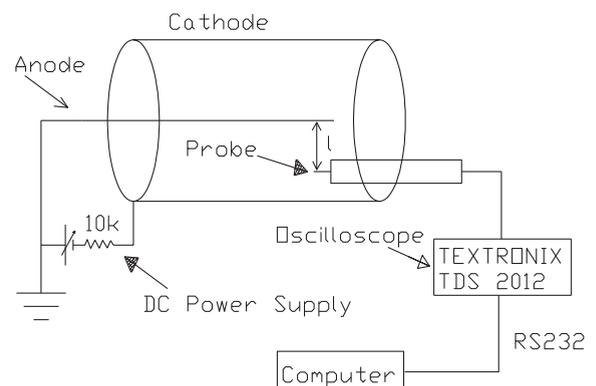


Fig. 1. Schematic diagram of the whole experimental setup.

2.3. Skewness and kurtosis

The skewness characterizes the degree of asymmetry of a distribution around the data mean. A distribution is symmetric if it looks same to both left and right to the centre point. If skewness is negative, the data are concentrated more to the left of the mean than to the right. If skewness is positive the data are concentrated more to the right.

Skewness of the normal distribution(Gaussian distribution) is zero [27]. Skewness of a distribution is given by

$$S = E[x - \mu]^3 / \sigma^3$$

where x is the data, μ is the mean of x and σ is the standard deviation of x and E represents the expected value of the quantity.

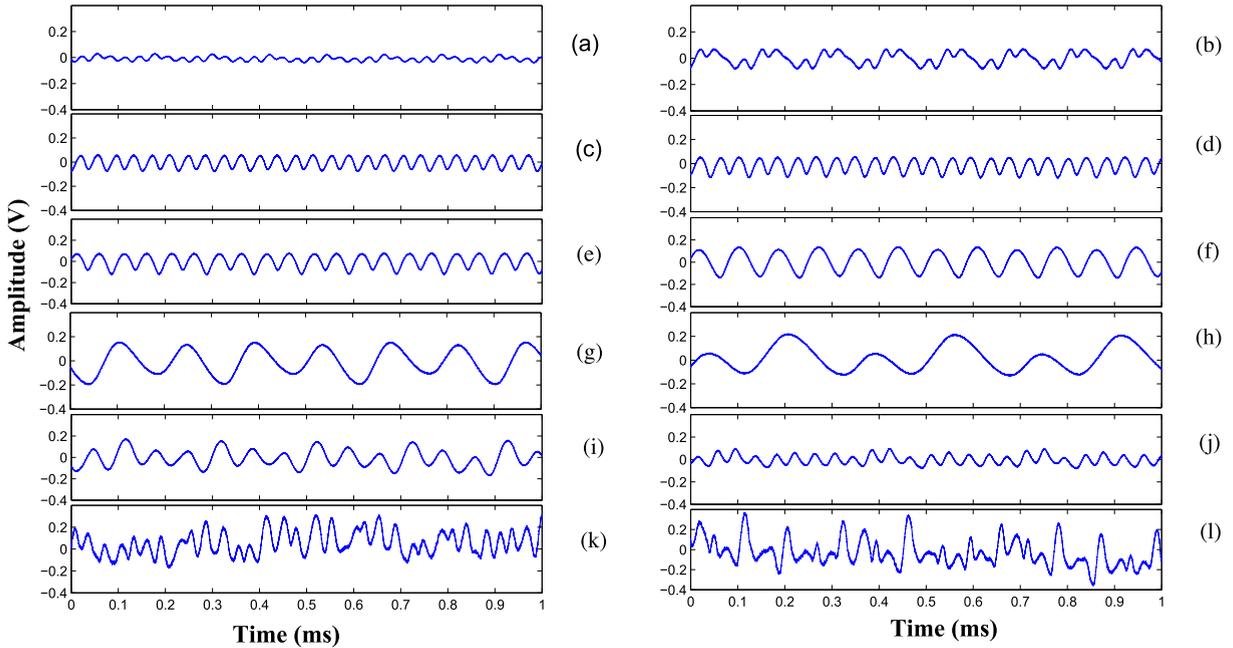


Fig. 2. Time series for floating potential fluctuations at different values of discharge voltages: (a) 295 V (b) 297 V (c) 300 V (d) 306 V (e) 312 V (f) 321 V (g) 336 V (h) 357 V (i) 371 V (j) 381 V (k) 387 V (l) 396 V.

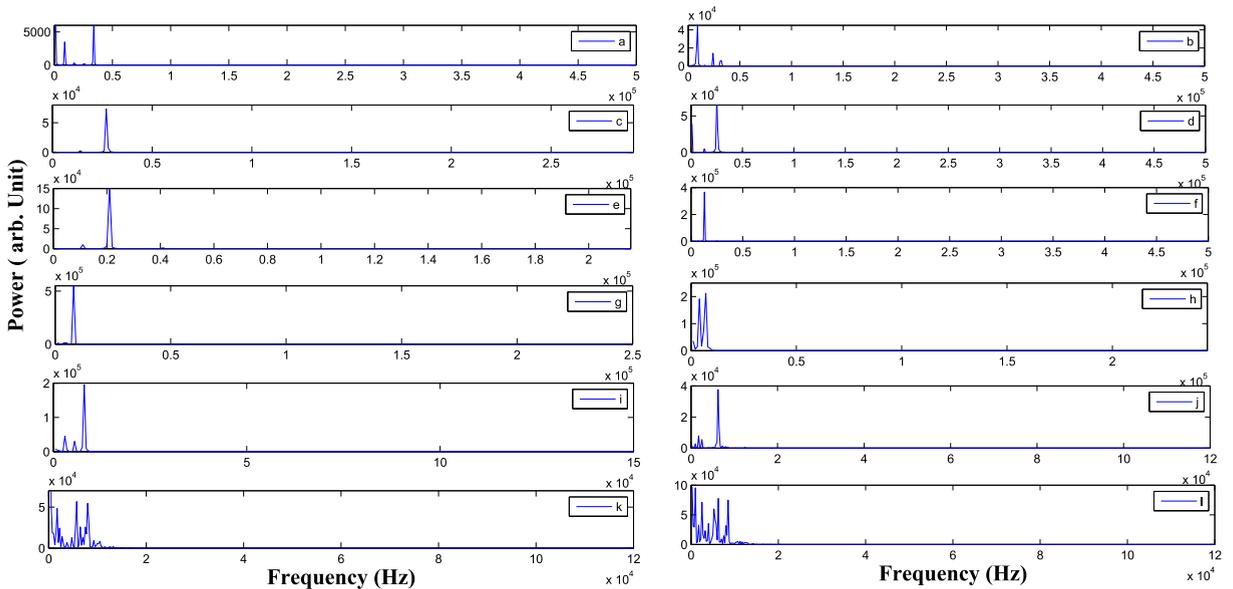


Fig. 3. Power spectra of floating potential fluctuations at different values of discharge voltages: (a) 295 V (b) 297 V (c) 300 V (d) 306 V (e) 312 V (f) 321 V (g) 336 V (h) 357 V (i) 371 V (j) 381 V (k) 387 V (l) 396 V.

Kurtosis evaluates the peakedness or flatness of a distribution of fluctuations around the data mean. A higher and lower value of kurtosis reflects a sharper peak and flatter peak than normal distribution, with values concentrated around the mean of the distribution. The kurtosis of the normal distribution is 3. Kurtosis of a distribution is given by

$$K = E[x - \mu]^4 / \sigma^4$$

where x is the data, μ is the mean of x and σ is the standard deviation of x and E is the expected value of the quantity.

3. Experimental setup

The complete set of experiment has been performed in glow discharge plasma system having stainless steel chamber of length 50 cm and diameter 20 cm acts as a cathode. A central rod (stainless steel) anode of length 7 cm and diameter 1.6 mm placed inside the system. The schematic diagram of experimental set up is shown in Fig. 1. A rotary pump is used to evacuate the system and the pressure inside the chamber has been controlled by a needle valve. The pressure could be varied from 0.001 mbar to 1 bar. Argon(Ar) plasma is produced inside the system at required working pressure by applying a voltage between anode and cathode as shown in Fig. 1.

A high voltage DC power supply of maximum output power 1 kW has been used for discharge process. Floating potential fluctuations (FPF) were measured by Langmuir

probe of diameter 2 mm. The electron density and temperature measured by Langmuir probe are of the order of 10^7 cm^{-3} and 2–6 eV, respectively. The Langmuir probe is placed in the middle of the device to measure the potential fluctuations. The detail of the system can be found in Jaman et al. [10]. The time series corresponding to the FPF data are collected in the oscilloscope, and further, various nonlinear analyses are carried out with the help of recurrence plot and recurrence quantification analysis for studying their behaviour.

4. Results and analysis

Pressure and discharge voltages are control parameters that have been considered for the experiment. Nature of plasma discharge depends on the control parameter. For a particular pressure gaseous discharge takes place in a particular voltage (Threshold voltage). In this experiment pressure is kept at 0.08 mbar and for this pressure plasma discharge is obtained at 295 V (threshold voltage). The dynamics of plasma fluctuations have been observed varying the DV (discharge voltage) from 295 V to 396 V. It has been observed that with the change of the discharge voltage plasma oscillations are also altering and with the increase of discharge voltage (V_d) the fluctuations gradually change from periodic to chaotic state. The transition of the dynamics has been visually studied by using the recurrence plot. In Fig. 2 the time series of floating potentials fluctuations have been shown from 295 V to 396 V. It

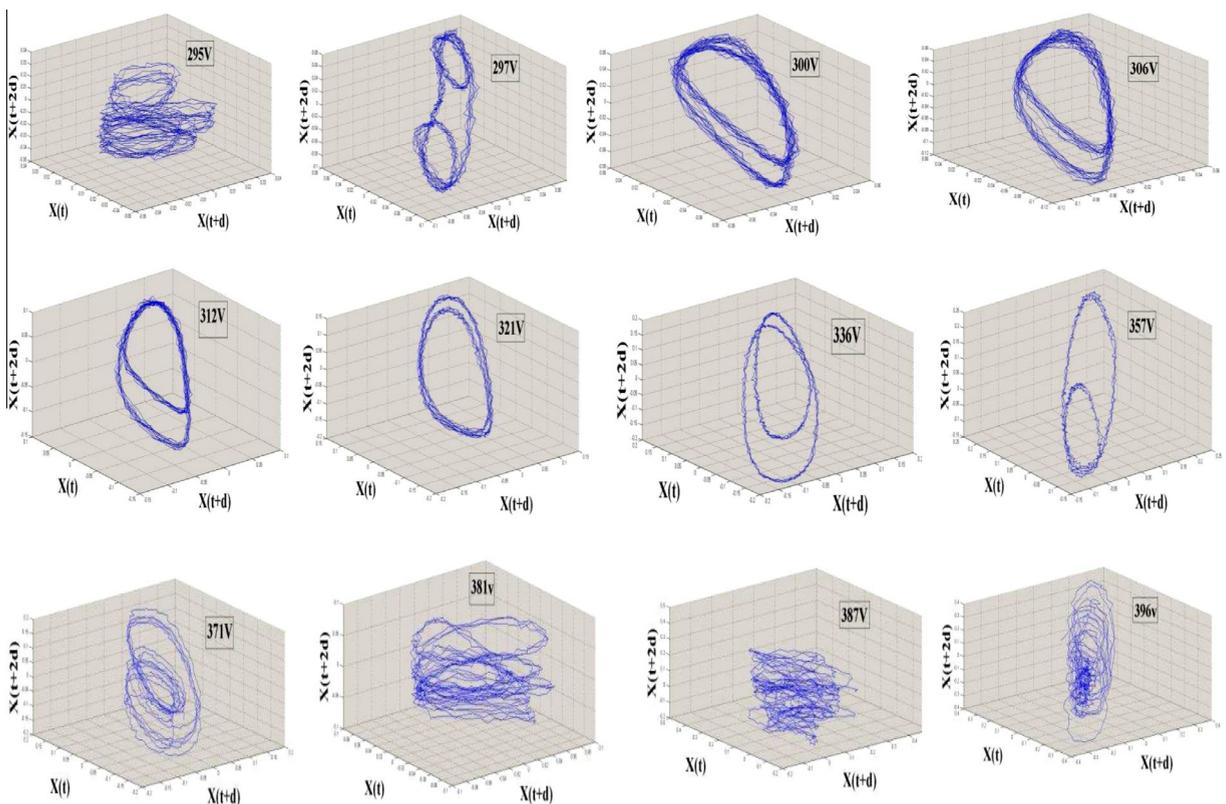


Fig. 4. State space plots of floating potential fluctuation of the time series shown in Fig. 2.

has been noticed that the amplitude of the FPF is increasing with the increase of discharge voltage.

In Fig. 3 power spectrum of corresponding discharge voltages have been shown. Fig. 3(a) indicates two periods of frequency 9 kHz and 34 kHz and Fig. 3(b) depicts three periods having frequency 9 kHz, 24 kHz and 32 kHz. Fig. 3(c) and (d) shows two periods of frequency 14 kHz, 27 kHz and 13 kHz, 25 kHz. Fig. 3(e) indicates two periods of frequency 11 kHz and 21 kHz. Fig. 3(f) reflects two periods of frequency 7 kHz and 13 kHz, Fig. 3(g) shows two periods of frequency 4 kHz and 8 kHz. Fig. 3(h) reflects two periods of frequency 4 kHz and 7 kHz. Fig. 3(i) reflects three periods of frequency 3 kHz, 5.5 kHz and 8 kHz. Fig. 3(j) shows quasi periodicity and Fig. 3(k) and (l) gives an impression of burst of frequency which indicates chaos. The pattern of dynamics of plasma oscillation is changing from periodic to chaos via quasi periodic behaviour with the increase of discharge voltage. It has been observed that the frequency is decreasing with the increase of discharge voltage.

State space reconstructions of the raw data of FPF were performed based on embedding technique [28]. For each twelve time series an appropriate state space was reconstructed from the original time series. Embedding dimensions (m) were computed for each time series by using false nearest neighbour method (FNN) [29]. The FNN compares the distance between neighbouring trajectories with that in higher dimension. Embedding dimension has been selected when the false nearest neighbour becomes zero. Time delays τ were calculated from the first minimum of the average mutual information function. It has been noticed that m and d for all the time series lie in the range of 3–5 and 4–20.

Fig. 4 shows the reconstructed state space plot of FPFs with the variation of discharge voltage. At 295 V several loops are observable. At 297 V three loops are noticed which reflects 3 period. Two loops are seen from 300 V to 357 V which represent two period that is observed also in Fig. 3. The complex nature of the state space plot at 396 V represents chaotic oscillation. Fig. 4 reflects that the nature of the loops are changing with respect to the

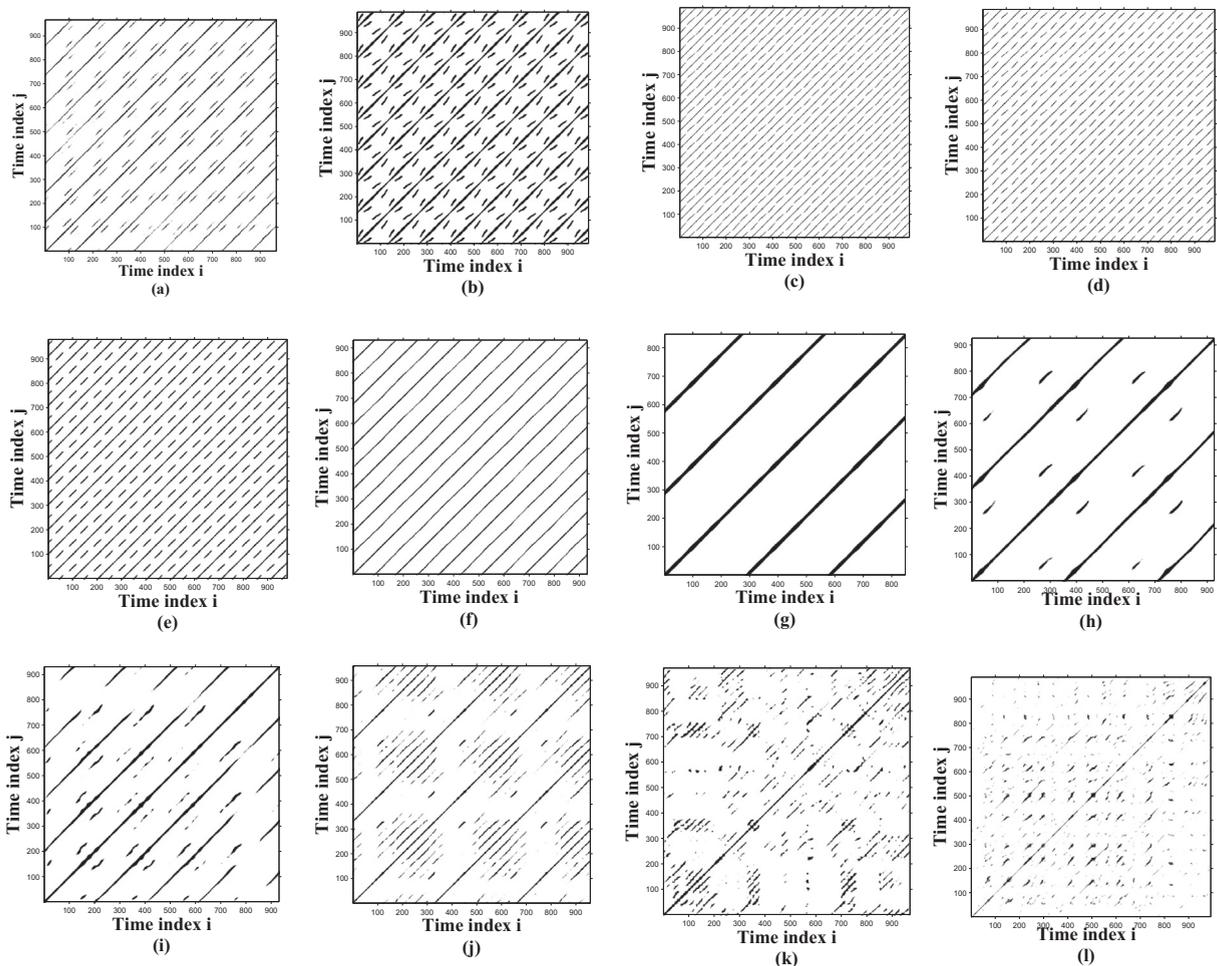


Fig. 5. Recurrence plots for floating potential fluctuation at different discharge voltages: (a) 295 V (b) 297 V (c) 300 V (d) 306 V (e) 312 V (f) 321 V (g) 336 V (h) 357 V (i) 371 V (j) 381 V (k) 387 V (l) 396 V.

discharge voltage which indicates the transition from order to chaos through quasi periodicity.

Fig. 5 depicts twelve RPs of FPF at different discharge voltages. The data length has been considered as thousand data points. Threshold has been selected in such a way that

the point density is approximately 1%. For $V_d = 295$ V, the RP of the floating potential shows non interrupted long diagonal lines and within it small diagonal lines are observable. Vertical distance between the two consecutive diagonal lines gives the information about the time period

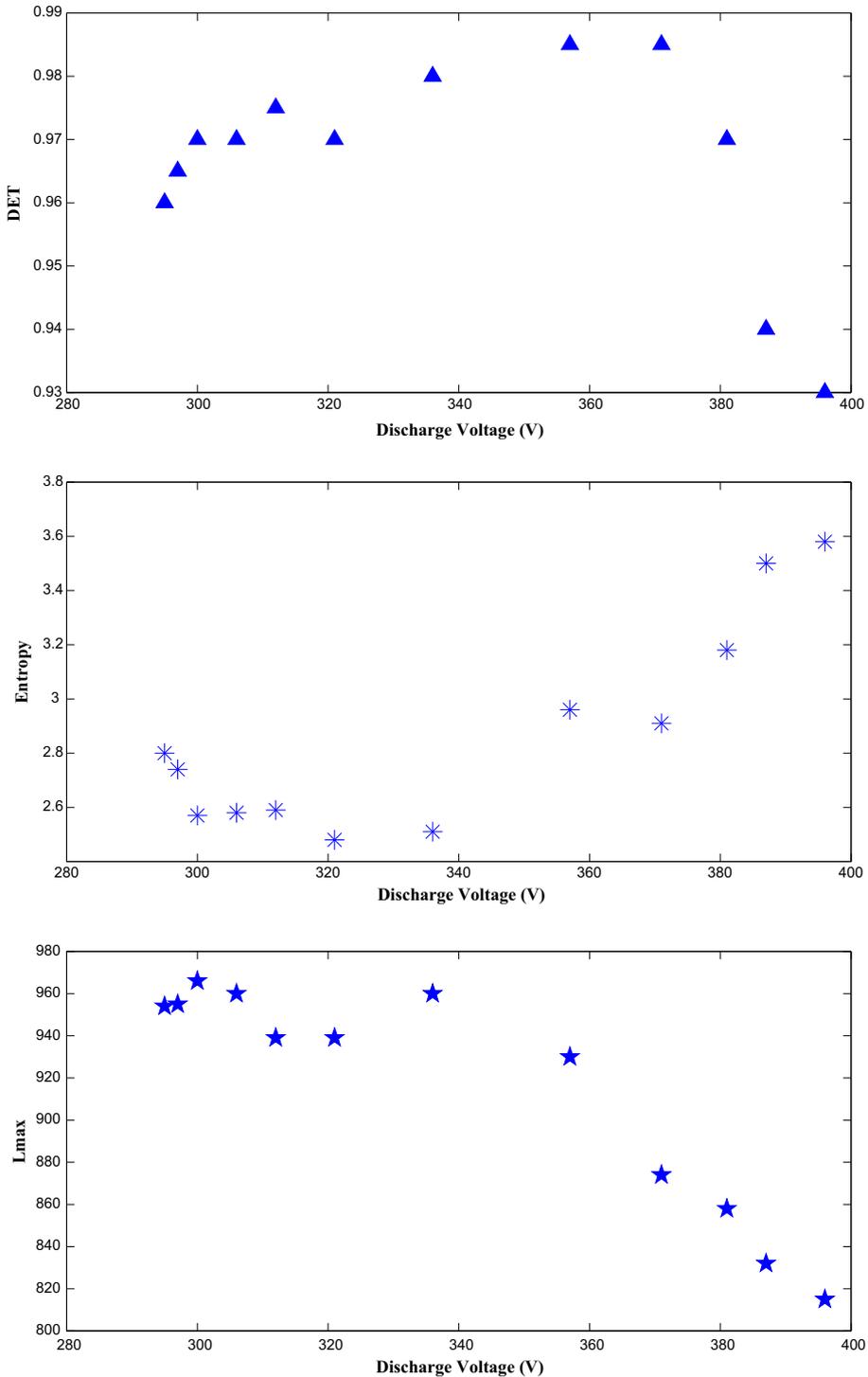


Fig. 6. Variations of recurrence quantification variables (a) DET (b) L_{max} (c) entropy at different discharge voltages.

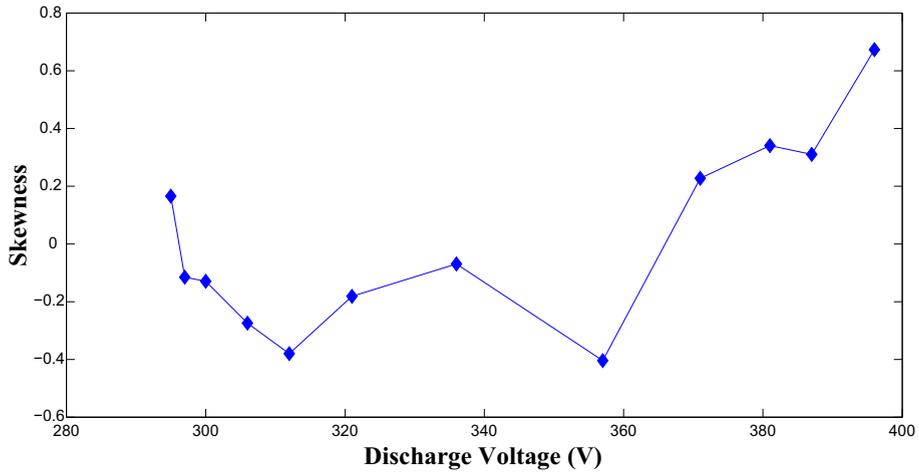


Fig. 7. Skewness of floating potential fluctuations with variation in discharge voltages.

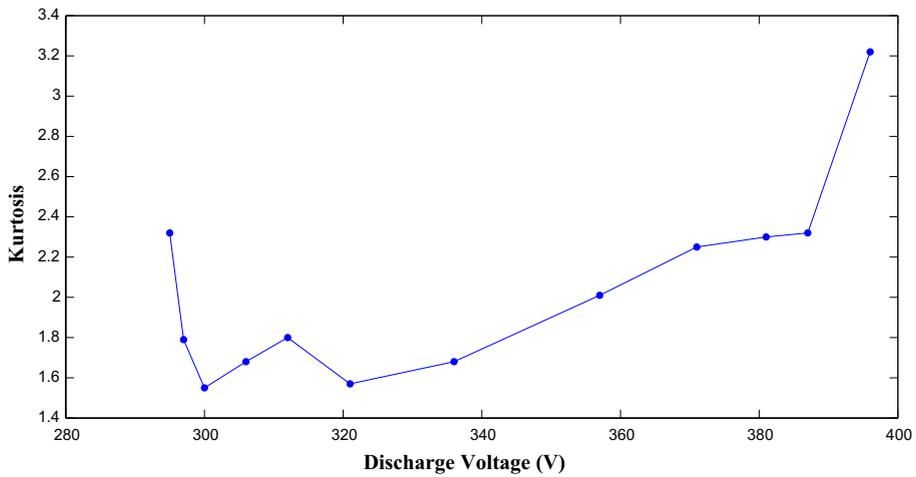


Fig. 8. Kurtosis of floating potential fluctuations with variation in discharge voltages.

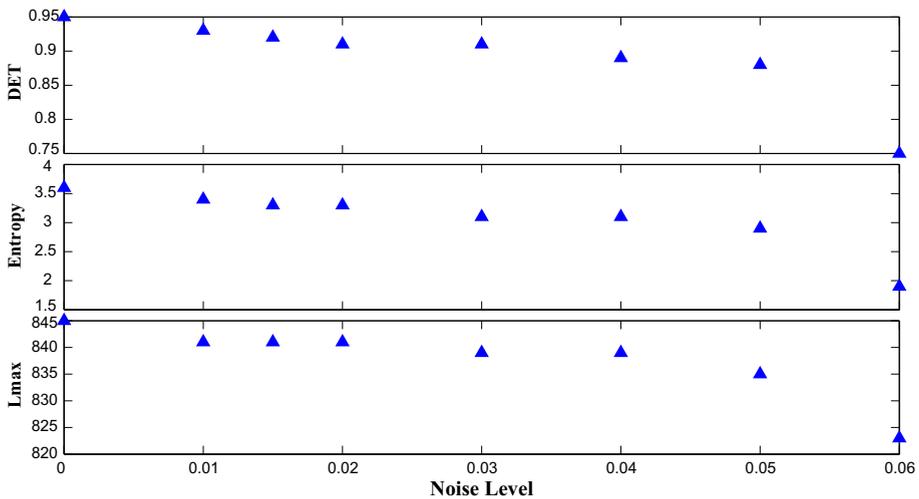


Fig. 9. Variation of recurrence quantification variables DET, entropy and $line_{max}$ at different noise level.

of the plasma oscillations. In Fig. 5(a) distance between two small consecutive and large consecutive lines are estimated and the time period are obtained as 8.5 kHz and 33 kHz which are almost equal to the value estimated from power spectrum. Fig. 5(a)–(k) reflects long diagonal lines from $V_d = 295$ V to $V_d = 371$ V which indicate that the dynamics of FPF remains in ordered regime. Fig. 5(j) is showing several long and small diagonal lines and the distance between each diagonal lines are not equal which indicate quasi periodicity. Fig. 5(k) and (l) reflect small interrupted diagonal lines which visually appeals that at 387 V and 396 V the plasma oscillation is showing chaotic nature.

The nonlinear variables DET, $line_{max}$ and entropy are plotted in Fig. 6 with respect to the discharge voltage. It has been noticed that the value of DET is gradually increasing reaches almost unity which reflects the dynamics is becoming periodic and at 381 V it starts to fall and becomes minimum at 396 V. An understanding of the recurrence plot in conjunction with the physical interpretation of the DET results reveals that the dynamics of plasma oscillation is altering from a periodic to quasi periodic and then a chaotic state. The variable entropy reflects the complexity of the system using the statistics of diagonal lines. The value of entropy is increasing with respect to discharge voltage. It has been noticed that the value of entropy is low in periodic regime and high in chaotic regime. In general inverse of $line_{max}$ is proportional to positive Lyapunov exponent. $line_{max}$ is high in periodic regime and becoming low in chaotic regime. Small and large values of $line_{max}$ generally indicate chaotic and periodic dynamics of plasma oscillation, respectively.

In Figs. 7 and 8 skewness and kurtosis of floating potential fluctuations are plotted with the variation of discharge voltages. It has been observed that at 295 V the skewness value is 0.165. When the discharge voltage is increased to 297 V the skewness becomes negative (0.114) and it remains negative up to 357 V. The skewness value turns to positive at 371 V (0.2274) and remains positive up to 396 V. The skewness value is maximum at 396 V. It has been observed that skewness is positive when the nature of oscillation is irregular and complex. As the plasma discharge is struck at 295 V, so initially the power of the signal is less (1.83) compared to the other signal. Although periodicity is present at 295 V but still the skewness value is positive due to the strong impact of noise that has been observed in state space plot also. This transition of skewness value from negative to positive with the variation in discharge voltages reflect that plasma oscillation is transiting from periodic to chaotic state via quasi periodicity.

The value of kurtosis is 2.32 at 295 V. The kurtosis value decreases when the discharge voltage increases to 297 V. It starts to increase when the discharge voltage is 371 V and it reaches its maximum (3.22) at 396 V which is due to the chaotic oscillation that has been verified by both RP and LLE technique.

Largest Lyapunov exponent (LLE) is computed for the twelve time series by using Tisean Package [30]. The LLE of 387 V and 396 V are showing positive value of 0.026 and 0.105 which indicate chaotic dynamics and for other voltage it is showing negative value (periodic) which supports the previous analysis.

We introduce noise to each time series represented in Fig. 2 to investigate the sensitivity of the RQA measures in presence of various noise levels. We increase the signal to noise ratio (SNR) from 20,000 to 100,000. So the scaling factor multiplied to the generated noise is reducing from 0.06 to 0.01. When the signal to noise ratio is increasing the noise level in the signal is decreasing. It has been observed that with the increase of the noise level RQA measures are decreasing but the rate of decrement of the RQA values are very low. But when the noise level is becoming high (20,000) the RQA measures are changing abruptly for all the signals. We filtered the noise to observe the changes of RQA variables in the presence of zero noise level. We observe that the RQA estimated for the raw signal is nearly similar to the filtered signal. The noise level verses RQA measures are shown in Fig. 9. It can be elucidated from the sensitivity analysis that the RQA technique is robust to noise. While increasing the noise level we found that value of LLE is changing abruptly. It has been investigated that noise has a strong impact on LLE computation and a noise free data set is an essential criteria to get correct LLE result.

5. Conclusion

Experimental investigation has been carried out to study the dynamical behaviour of glow discharge plasma oscillations. The floating potential fluctuations are obtained by the Langmuir probe. The fluctuations are studied qualitatively with the help of recurrence plots, and quantitatively by recurrence quantification (RQA) variables. These observations are compared with well known existing techniques based on statistical analysis. Recurrence plot provides a qualitative impression and recurrence quantification analysis gives a quantitative measure of a dynamical system. RQA investigates the recurrence pattern within the phase space trajectory. This pattern elucidates important information about the system dynamics that is distinctly different from that obtained from other analysis techniques. The different recurrence variables reflect the system's predictability. All the quantification variables are changing with respect to discharge voltage and indicate the gradual transition of plasma oscillations exhibiting initially periodic, quasi periodic and finally a chaotic nature. It is discerned that the technique with its visual appeal collaborated by quantitative analytical tools is capable of distinguishing many hidden phenomena of a dynamical system.

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