



## EXTENDED RECURRENCE PLOT ANALYSIS AND ITS APPLICATION TO ERP DATA

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We present new measures of complexity and their application to event-related potential data. The new measures are based on structures of recurrence plots and makes the identification of chaos–chaos transitions possible. The application of these measures to data from single-trials of the Oddball experiment can identify laminar states therein. This offers a new way of analyzing event-related activity on a single-trial basis.

*Keywords:* Recurrence quantification analysis; laminarity; event-related potentials.

### 1. Introduction

Neurons are known to be nonlinear devices because they become activated when their somatic membrane potential crosses a certain threshold [Kandel *et al.*, 1995]. This nonlinearity is one of the essentials in neural modeling which leads to the sigmoidal activation functions of neural networks [Amit, 1989]. The activity of large formations of neurons is macroscopically measurable as the electroencephalogram (EEG) at the human scalp which results from a spatial integration of postsynaptic potentials [Nunez, 1981]. However, it is an unsolved problem whether the EEG should be treated as a time series stemming from a linear or a nonlinear dynamical system. Applying nonlinear techniques of data analysis to EEG measurements has a long tradition. Most of these efforts have been done by computing the correlation dimension of spontaneous EEG (e.g. [Babloyantz *et al.*, 1985; Rapp *et al.*, 1986; Gallez & Babloyantz, 1991; Lutzenberger *et al.*, 1992; Pritchard & Duke, 1992]). Theiler *et al.* [1992] applied the technique of surro-

gate data to correlation dimensions of EEG and reported that there is no evidence of low-dimensional chaos but of significance for nonlinearity in the data.

While correlation dimensions are only well defined for stationary time series generated by a low-dimensional dynamical system moving around an attractor, these measures fail in investigating event-related brain potentials (ERPs), [Sutton *et al.*, 1965] since they are nonstationary by definition. Traditionally, ERP waveforms are determined by computing an ensemble average over a collection of stimulus time-locked EEG trials. This is based on the following assumptions: (1) the presentation of stimuli of the same kind is followed by the same sequence of processing steps, (2) these processing steps always lead to activation of the same brain structures, (3) this activation always elicits the same pattern of electrophysiological activity, which can be measured at the scalp [Rösler, 1982] and (4) spontaneous activity is stationary and ergodic [beim Graben *et al.*, 2000].

By averaging the data-points time-locked to the stimulus presentation (cf. Oddball experiment) it is

possible to filter out the signal (ERP) of the noise (spontaneous activity). In the next step the functional significance of a component is assessed. Antecedent conditions of the occurrence of a component and variables, which influence its parameters are defined. Now the commonalities of these factors are identified. The generalization of all empirically found influencing factors leads to a more abstract cognitive theory of the functional meaning of an event-related potential component and makes it usable for the validation of models of cognitive processes.

The disadvantage of the averaging method is the high number of trials needed to reduce the signal-to-noise-ratio [Kutas & Petten, 1994]. This is crucial for example for clinical studies, for studies with children and for studies where repeating a task would influence the performance. So it is desirable to find new ways of analyzing event-related activity on a single-trial basis. Applying nonlinear methods to electrophysiological data could be one way of dealing with this problem.

To compute dimensions of ERPs, Molnár *et al.* [1995] used the pointwise dimensions and reported a drop of the pointwise dimension as a function of time corresponding to the P300 component observed in the Oddball experiment. Recently, concepts of information theory have been introduced to analyze ERPs. On one hand this is the wavelet entropy of Quiroga *et al.* [2001] and on the other hand symbolic dynamics of EEG and ERP [beim Graben *et al.*, 2000; Frisch *et al.*, 2004; Steuer, 2004; Schack, 2004].

A further promising approach is the recurrence quantification analysis (RQA), which is based on the quantification of the diagonal oriented structures in recurrence plots (RPs, [Webber Jr. & Zbilut, 1994; Zbilut & Webber Jr., 1992]). The RQA was broadly applied in a wide field of the analysis of physiological data (e.g. [Casdagli, 1997; Faure & Korn, 1998; Thomasson *et al.*, 2001; Marwan *et al.*, 2002]). The important advantage of methods based on the quantification of RPs is that the required data length can be relatively short. However, the measures of the classical RQA are only able to recognize transitions between periods and chaos and vice versa [Trulla *et al.*, 1996]. In this work, we will use recently introduced additional measures based on RPs in order to find chaos-chaos transitions in physiological data. These new measures use the vertical structures in the RP and are able to identify laminar states [Marwan *et al.*, 2002].

In the first section we will give a short introduction into RPs and their quantification analysis. In the next section we will introduce the new measures and finally we will apply them to event-related potential data gained from the Oddball experiment.

## 2. Recurrence Plots and Their Quantification

The method of recurrence plots (RP) was introduced to visualize the time dependent behavior of the dynamics of systems, which can be pictured as a trajectory in the phase space [Eckmann *et al.*, 1987]. It represents the recurrence of the  $m$ -dimensional phase space trajectory  $\mathbf{x}_i \in \mathcal{R}^m$  ( $i = 1, \dots, N$ , time discrete) to a certain state. The main step of this visualization is the calculation of the  $N \times N$ -matrix

$$\mathbf{R}_{i,j} := \Theta(\varepsilon_i - \|\mathbf{x}_i - \mathbf{x}_j\|), \quad i, j = 1, \dots, N, \quad (1)$$

where  $\varepsilon_i$  is a state dependent cut-off distance,  $\|\cdot\|$  is the norm of vectors,  $\Theta$  is the Heaviside function and  $N$  is the number of states. The phase space vectors for one-dimensional time series  $u_i$  from observations can be reconstructed with the Taken's time delay method  $\mathbf{x}_i = (u_i, u_{i+\tau}, \dots, u_{i+(m-1)\tau})$  with dimension  $m$  and delay  $\tau$  [Kantz & Schreiber, 1997]. The recurrence plot exhibits characteristic large-scale and small-scale patterns which are caused by typical dynamical behavior [Eckmann *et al.*, 1987; Webber Jr. & Zbilut, 1994], e.g. diagonals (similar local time evolution of different parts of the trajectory) or horizontal and vertical black lines (state does not change for some time).

Zbilut and Webber have developed the recurrence quantification analysis (RQA) to quantify an RP [Webber Jr. & Zbilut, 1994; Zbilut & Webber Jr., 1992]. They defined measures using the recurrence point density and *diagonal* structures in the recurrence plot, the recurrence rate *RR* (density of recurrence points), the determinism *DET* (ratio of recurrence points forming diagonal structures to all recurrence points), the maximal length of diagonal structures  $L_{\max}$  (or their averaged length  $L$ ), the Shannon entropy *ENT* of the distribution of the diagonal line lengths and the trend *TREND* (paling in the RP). The computation of these measures in shifted windows along the main diagonal of the RP enables one to find characteristic excursions of the trajectory in the phase space of the considered system.

Trulla *et al.* have applied these measures in order to find transitions in dynamical systems

[Trulla *et al.*, 1996]. They have showed, that the RQA is able to find transitions between chaos and order (periodical states). But they could not find the chaos-chaos transitions.

### 3. Laminarity and Trapping Time

We have recently introduced two additional measures which are based on the *vertical* structures in the RP [Marwan *et al.*, 2002]. We define these measures analogous to the definition of *DET* and *L*, but we consider the distribution  $P(v)$  of the length of the vertical structures in the RP.

First, the laminarity *LAM*

$$LAM := \frac{\sum_{v=2}^N vP(v)}{\sum_{v=1}^N vP(v)}, \quad (2)$$

is the ratio of recurrence points forming vertical structures to all recurrence points and represents the probability of occurrence of laminar states in the system, but it does not describe the length of these laminar phases. It will decrease if the RP consists of more single recurrence points than vertical structures.

Next, the trapping time *TT*

$$TT := \frac{\sum_{v=2}^N vP(v)}{\sum_{v=2}^N P(v)}, \quad (3)$$

is the averaged length of the vertical structures. The measure *TT* contains information about the amount and the length of the laminar phases.

The difference between these measures and the traditional RQA measures is their ability to find transitions between chaos and chaos [Marwan *et al.*, 2002]. For example, such transitions can be found in the logistic map  $x_{n+1} = ax_n(1 - x_n)$  with increasing control parameter  $a \in [0, 4]$  and  $x_n \in [0, 1] \subset \mathcal{R}$ . For such trajectories  $x(a)$  which contain laminar states (e.g.  $a = 3.678, 3.791, 3.927$ ), *LAM* and *TT* show pronounced maxima (Fig. 1). The application of these measures to heart rate variability data, has shown, that they are able to detect and quantify laminar phases before a life-threatening cardiac arrhythmia and, thus, to enable a prediction of such an event [Marwan *et al.*, 2002]. These findings can

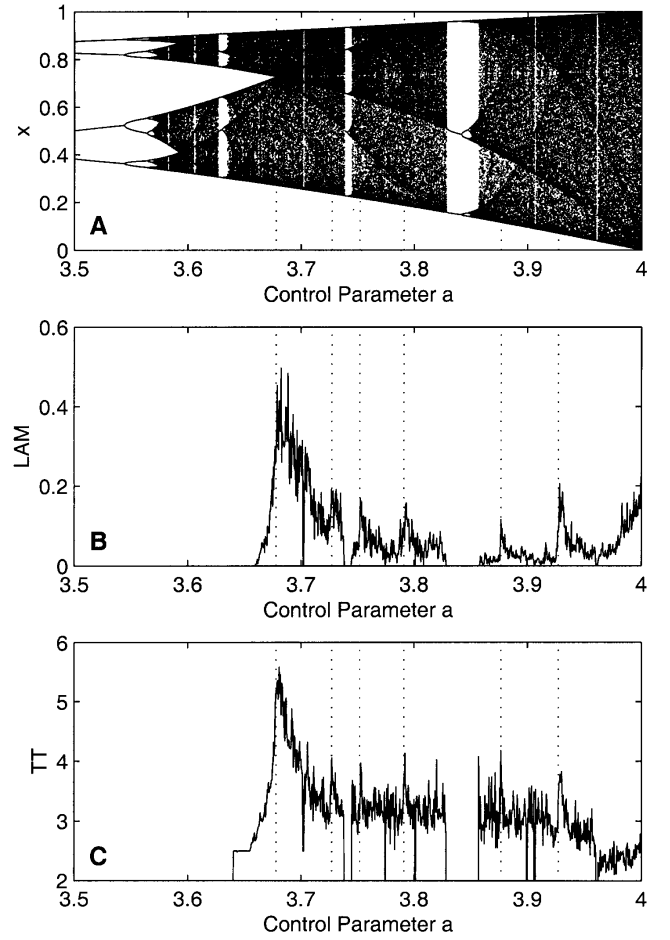


Fig. 1. Laminarity (B) and trapping time (C) of time series gained from the logistic map for various control parameters (A). These measures reveal laminar and intermittent states. The vertical dotted lines show a choosing of points of band merging and laminar behavior ( $a = 3.678, 3.727, 3.752, 3.791, 3.877, 3.927$ ). The length of the data were  $N = 1000$  and the embedding parameters were  $m = 1$ ,  $\tau = 1$  and  $\varepsilon = 0.1$ .

be of importance for the therapy of malignant cardiac arrhythmias.

In the next section we will apply this extended RQA to physiological data.

## 4. Event-Related Potentials

### 4.1. The Oddball experiment

As mentioned in the Introduction, the Oddball experiment studies brain potentials during a stimulus presentation.

The measurement of the EEG was done with 31 electrodes/channels (Table 1). The first 25 electrodes were localized as shown in Fig. 2; the others were reference electrodes. The sample interval for the measurements was 4 ms.

Table 1. Notation of the electrodes and their numbering as used in the figures (electrodes 26–31 are reference electrodes).

| #  | Electrode | #  | Electrode |
|----|-----------|----|-----------|
| 1  | F7        | 14 | T8        |
| 2  | FC5       | 15 | P7        |
| 3  | F3        | 16 | PZ        |
| 4  | FZ        | 17 | P3        |
| 5  | F4        | 18 | CZ        |
| 6  | FC6       | 19 | P4        |
| 7  | F8        | 20 | P8        |
| 8  | T7        | 21 | OZ        |
| 9  | CP5       | 22 | POZ       |
| 10 | C3        | 23 | PO3       |
| 11 | FCZ       | 24 | CPZ       |
| 12 | C4        | 25 | PO4       |
| 13 | CP6       |    |           |

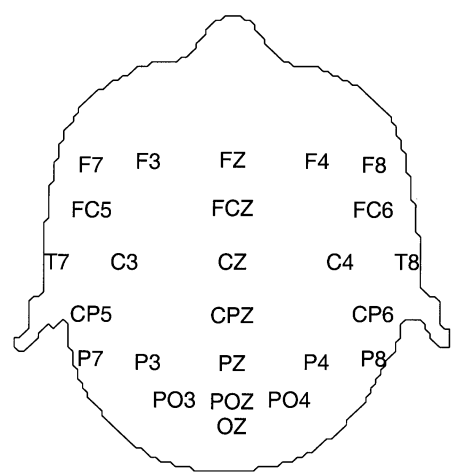


Fig. 2. Localization of the electrodes on the head.

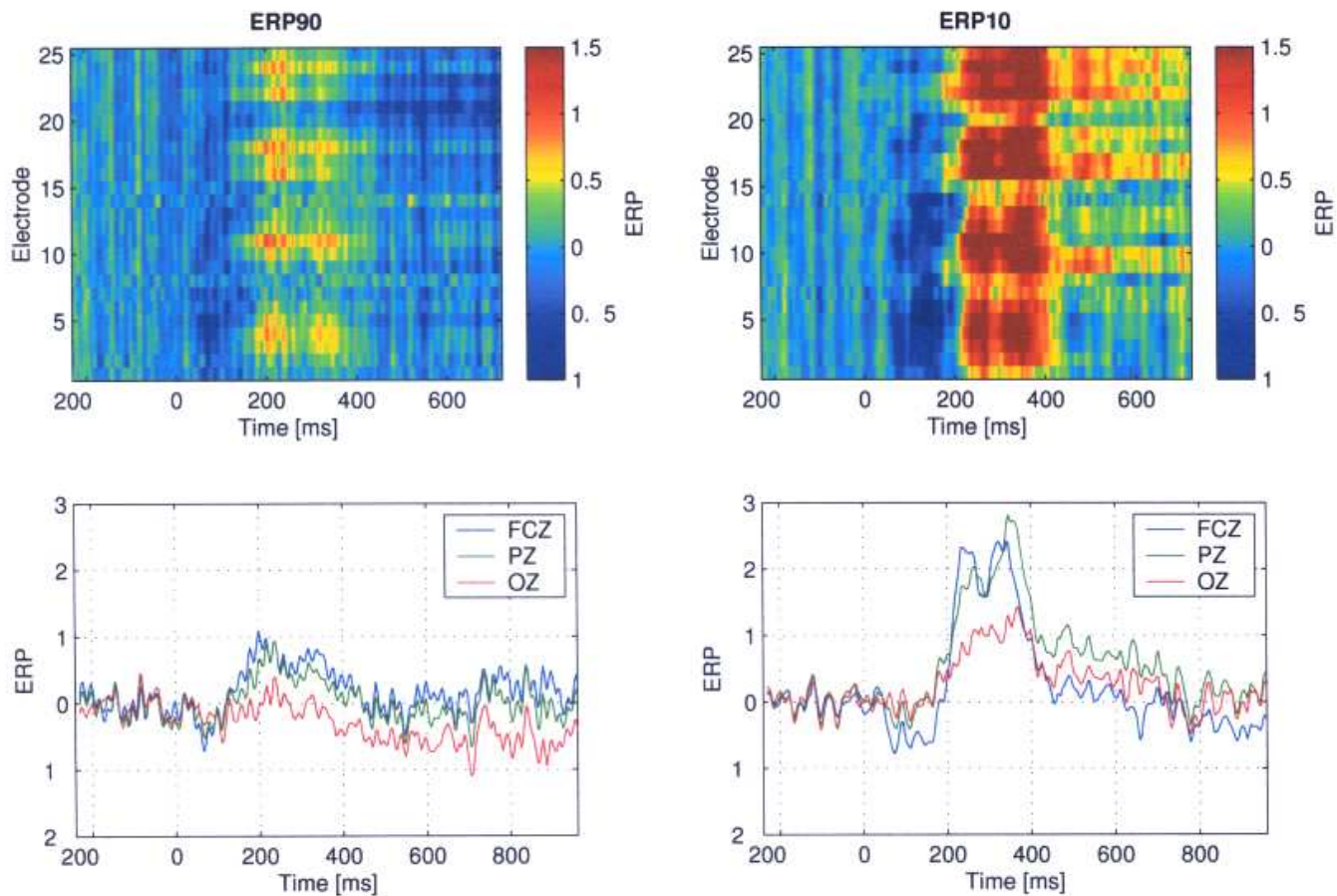


Fig. 3. Mean event-related potentials for event frequencies of 90% (left, 40 trials) and 10% (right, 31 trials). The N100 and P300 components are well pronounced for the frequencies of 10%. The lower plots show the ERP of selected electrodes. The reference of the electrode numbers is given in Table 1.

Probands were seated in a dimly lit room in front of a monitor and were instructed to count tones of high pitch. Each subject was tested in nine blocks. The blocks varied in the probability of occurrence of the higher tones from 10 to 90%. Each block contained at least 30 target tones. Response was given in a three-alternative-choice (using cursor keys of the keyboard). During the test, the EEG was recorded. The stimuli were computer-generated beeps of 100 ms length. Tones were either high (1400 Hz) or low (1000 Hz). They were presented with an interstimulus interval of 1000 ms.

After computing event-related voltage averages for the experimental manipulations (10% up to 90% target probability) one can observe a P300 ERP component whose amplitude is anti-correlated to the probability of the stimuli (surprise ERP, Fig. 3).

The P300 component of the ERP was the first potential discovered to vary in dependence on subject-internal factors like attention and expect-

tation instead of physical characteristics [Sutton *et al.*, 1965]. The amplitude of the P300 component is highly sensitive to novelty of an event and its relevance. So this component is assumed to reflect the updating of the environmental model of the information processing system (context updating), [Donchin, 1981; Donchin & Coles, 1988].

#### 4.2. Data analysis

Our focus will be directed to the ERP data of two extreme event probabilities. Henceforth, the time (measured in ms) is denoted as  $t$ , the trial number as  $i$  and the electrode as  $e$  (the allocation of the electrode numbers with their notion, see Fig. 2).

The first set ERP90 contains 40 trials of ERP data for an event frequency of 90% and the second set ERP10 contains 31 trials for an event frequency of 10%. Both data sets can be rather well discriminated in the N100 and P300 components by the

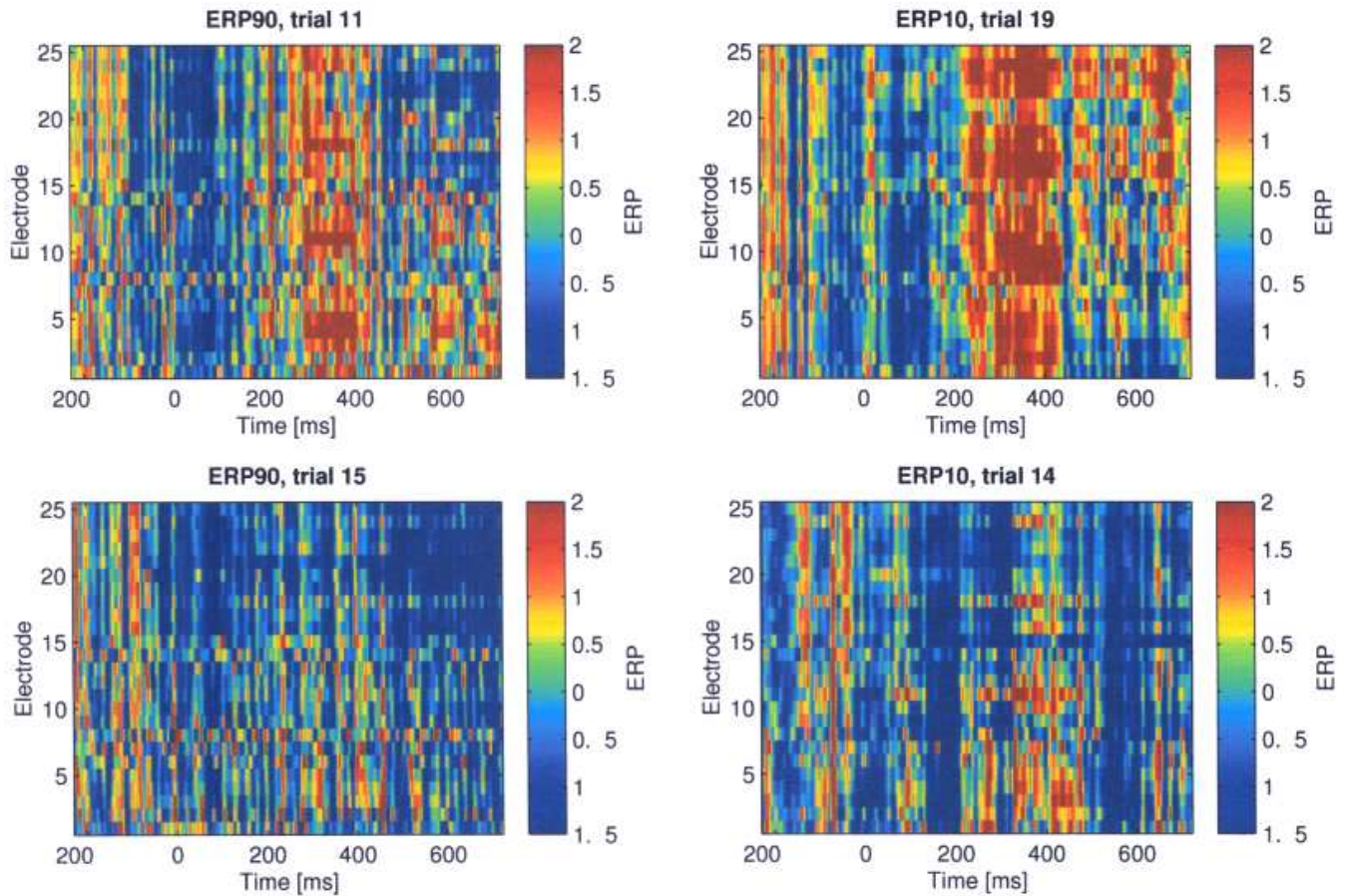


Fig. 4. Event-related potentials for selected trials of the event frequencies of 90% (left) and 10% (right). Both, ERP10 and ERP90 of single trials can be strongly or weakly pronounced, respectively, which makes their discrimination difficult. The reference of the electrode numbers are given in Table 1.

average over all trials (Fig. 3). As expected, both components have increased for lower event probabilities (ERP10). The maxima of the P300 are located around the central and central-parietal electrodes. However, the single trials do not obtain such a clear result. The P300 component is only well pronounced in 15 trials. When the single trials are observed, then extreme values can also occur in the ERP90 data and vanish in the ERP10 data (Fig. 4). We applied also a statistical variance-based T-test to the single trial ERP data. However, this method could also not clearly distinguish the single trials.

The recurrence quantification analysis (RQA) is based on the structures obtained by recurrence plots (RPs). The RPs were firstly computed for the

means of ERP90 and ERP10 over all trials and then for the single trials. This was done with the embedding parameters  $m = 3$ ,  $\tau = 3$  and  $\varepsilon = 10\%$  (fixed amount of nearest neighbors). The embedding parameters were estimated by using the standard methods false nearest neighbors (dimension) and mutual information (delay) [Kantz & Schreiber, 1997]. Due to the N100 and the P300 components in the data, the RPs show varying structures changing in time (Fig. 5). Diagonal structures and clusters of black points occur. The nonstationarity of the data around the N100 and P300 causes extended white bands along these times in the RPs. However, the clustered black points around 300 ms occur in almost all RPs of the ERP10 data set.

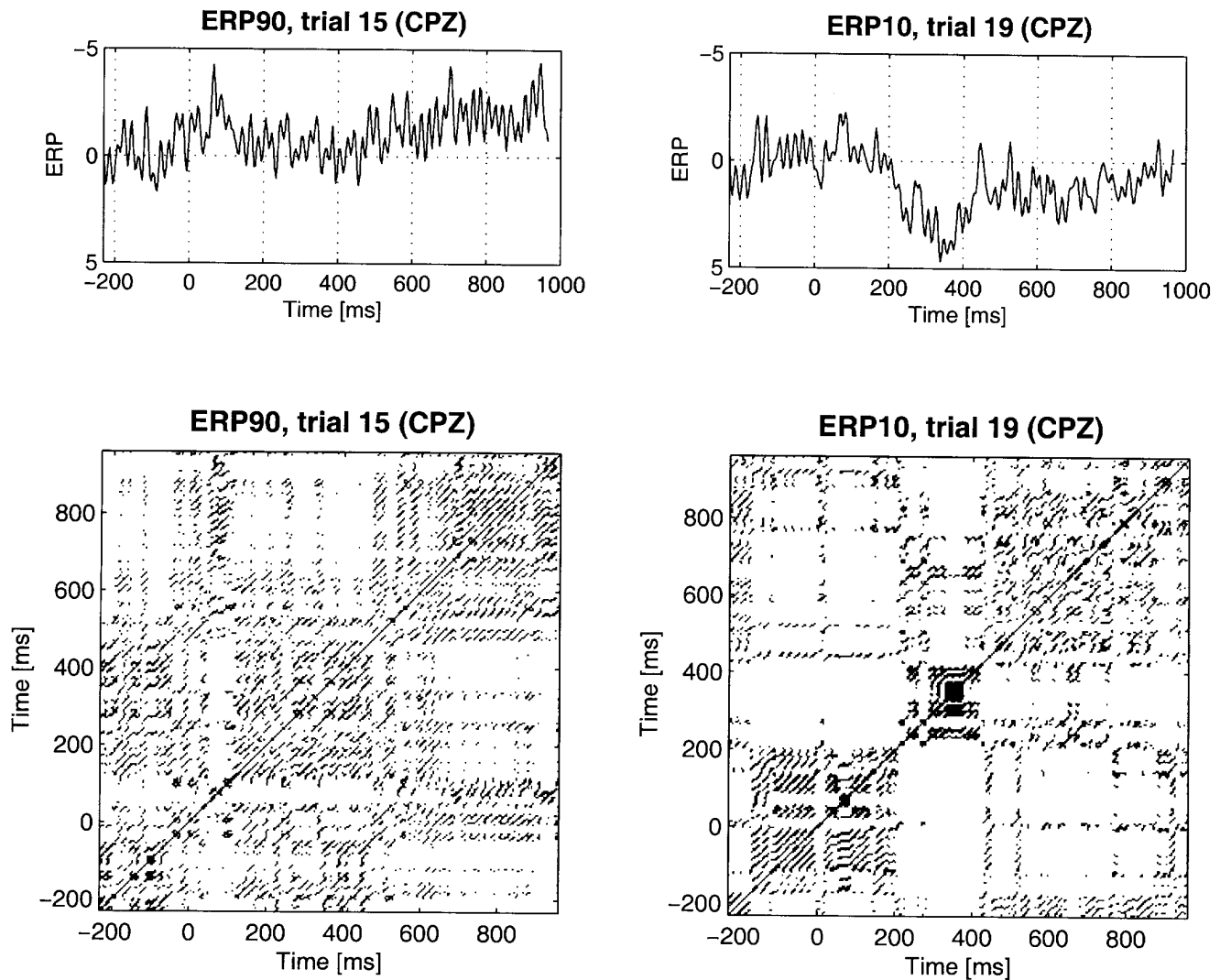


Fig. 5. ERP data for event frequencies of 90% (upper left) and 10% (upper right), and their corresponding recurrence plots (lower plots). For the lower event frequency, more cluster of recurrence points occur at 100 ms and 300 ms.



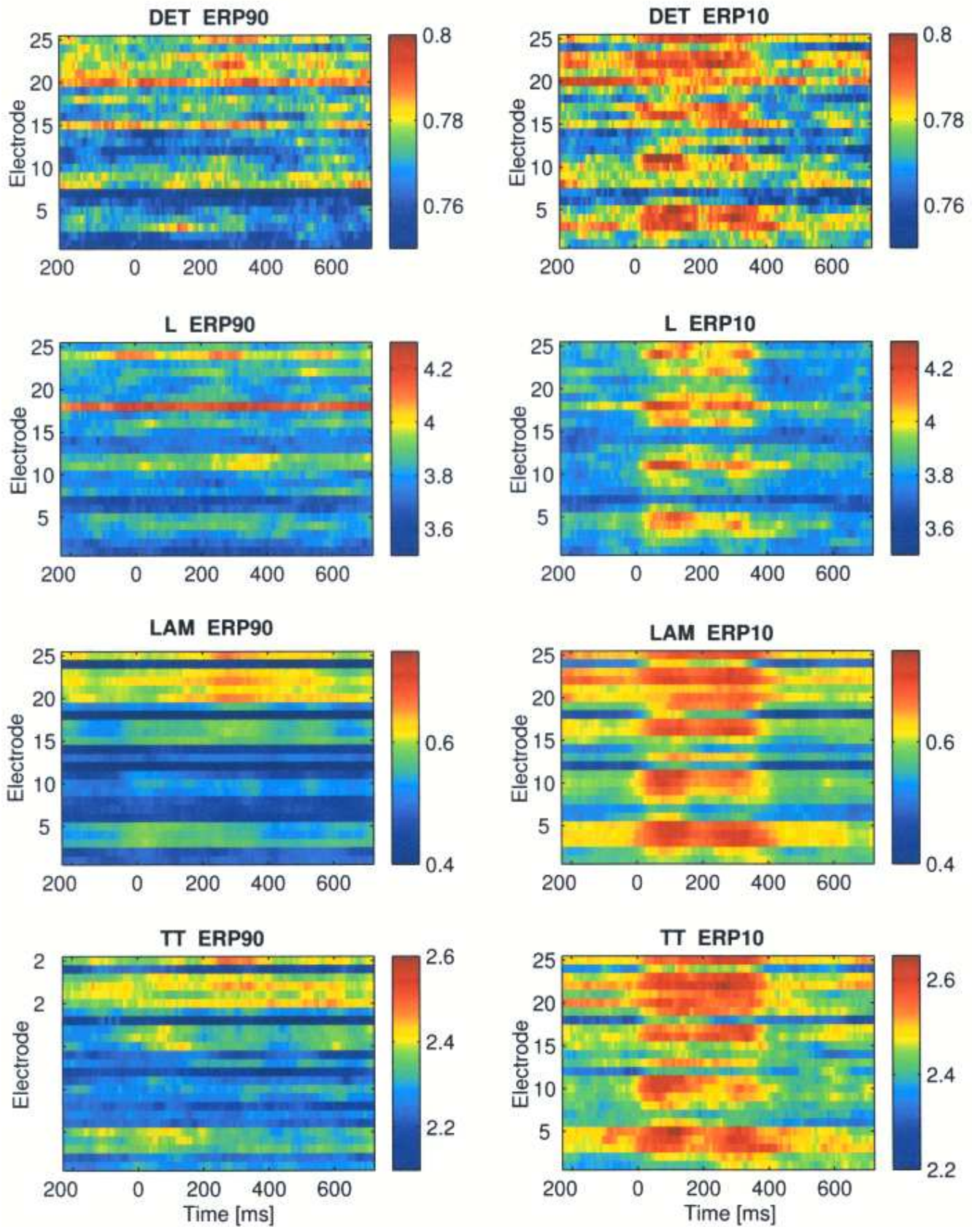


Fig. 6. Averaged RQA measures for the ERP data of both event frequencies (averaged over all trials). Whereas the measures do not reveal any transitions in the ERP90 data, they clearly recognize the transitions for the ERP10 data.

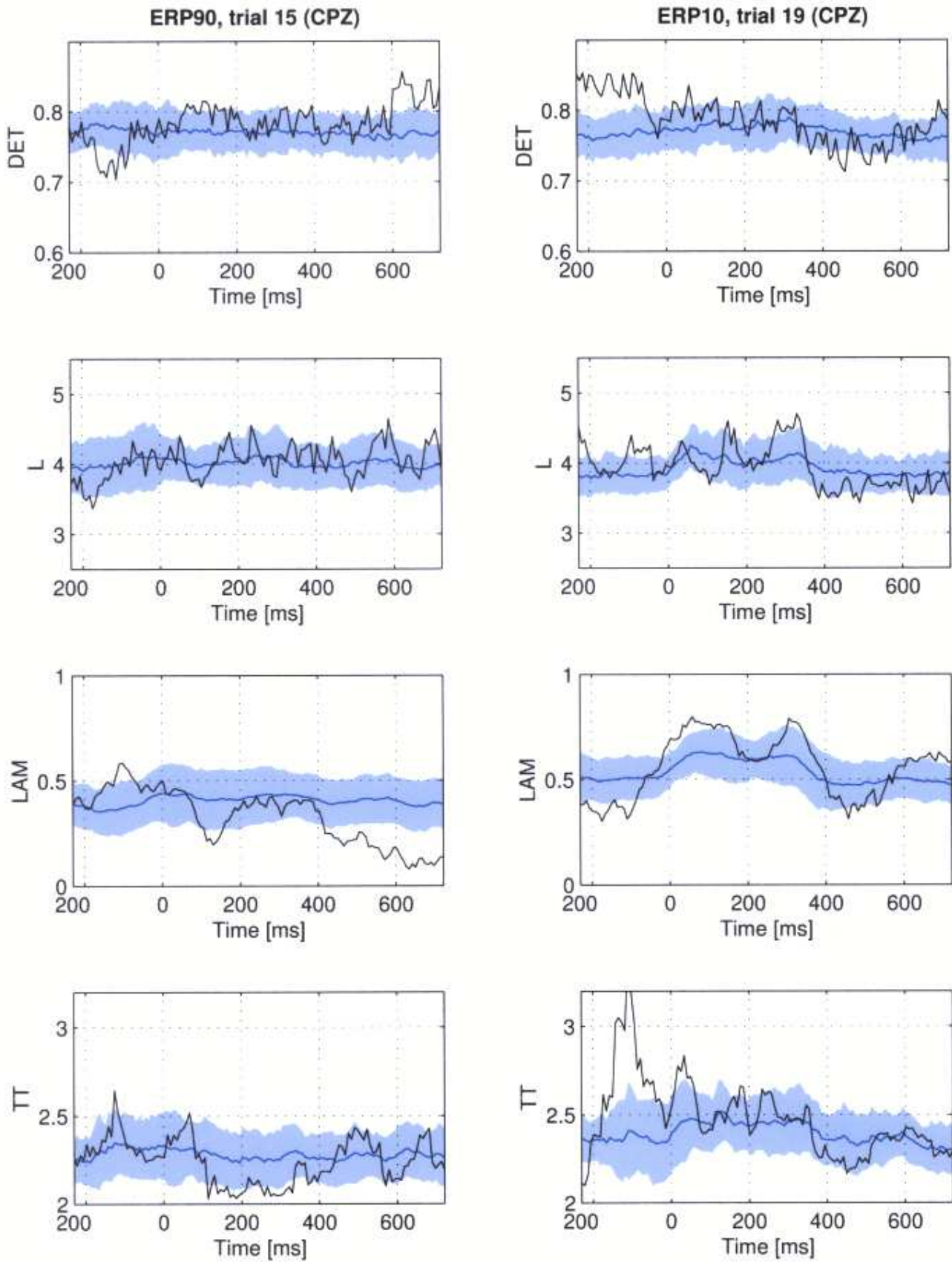


Fig. 7. RQA measures for selected single trials and the central-parietal electrode (black). The trial-averaged RQA measures for the same electrode is shown in blue (the light blue band marks the 95% significance interval).



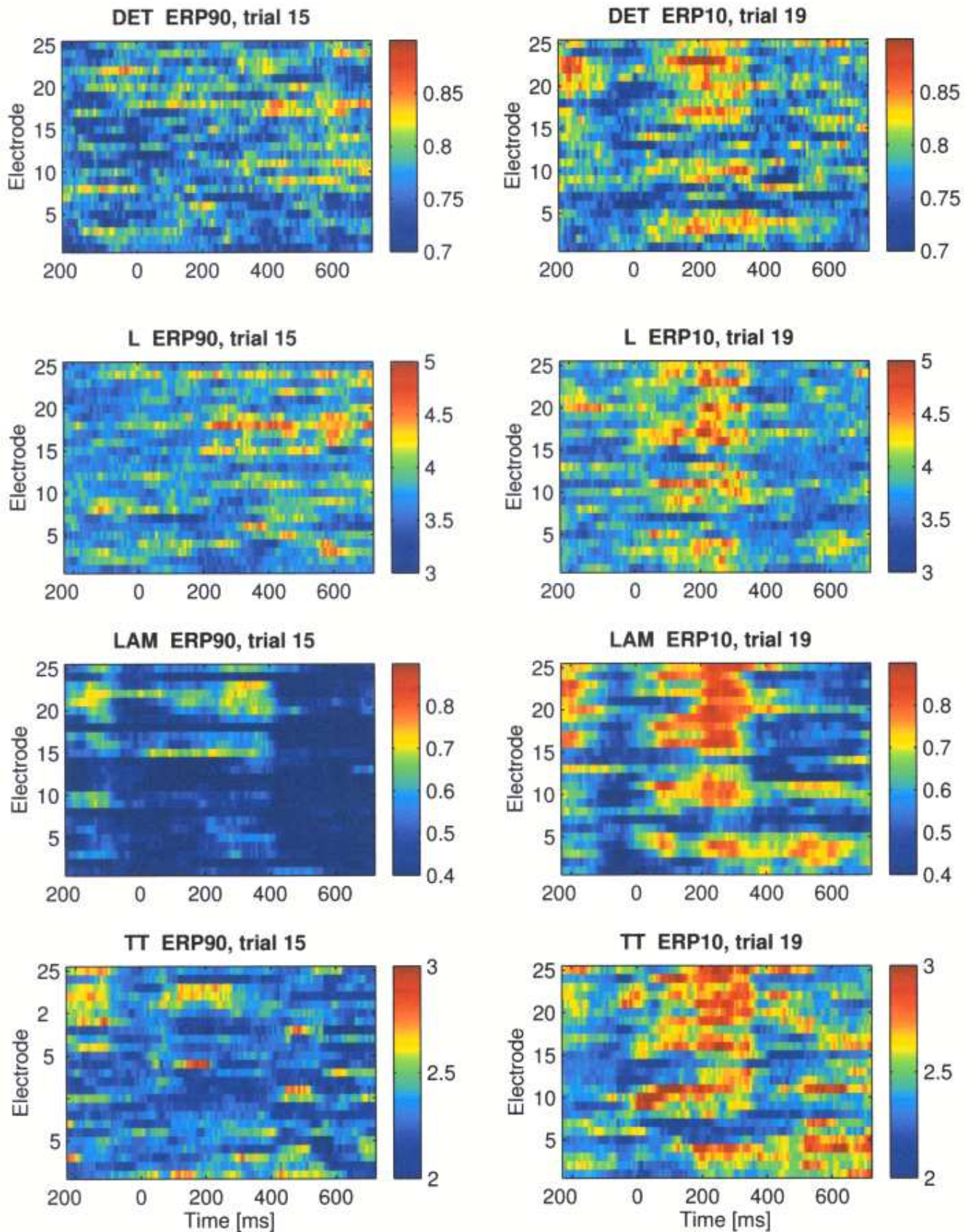


Fig. 8. RQA measures for the same trials as in Fig. 7, but shown for all electrodes.

The RQA was computed from the RPs of ERP90 and ERP10 for the single trials, in sliding windows over the RPs (which have the dimension  $m = 3$ ) with a length of 240 ms and with a shifting step of 8 ms. This window length corresponds with a data length of 60 values.

The mean of all RQA variables of ERP10 reveal typical structures in the data (Fig. 6, right column). They indicate the transitions corresponding to the N100 and P300 components around the central electrodes. The RQA variables for the ERP90 do not reveal these transitions (Fig. 6, left column). The onset of the increasing of the parameter is about 120 ms before the event. This is due to the windowed analysis of the RPs (240 ms windows). We have chosen the middle of the RP window for the time, what results in a 120 ms earlier onset of the RQA variables.

The four RQA variables are quite different, especially in their amplitude. For ERP10, *LAM* and *TT* are the best pronounced parameter and have two distinct maxima at some electrodes; *DET* and *L* reveal these maxima at these electrodes too, but are lesser pronounced (Fig. 6). These maxima occur at the transition around 100 ms and 300 ms after the event and occur at the electrodes F3, F4, FZ, C3, FCZ, PZ, POZ and PO3. Differences between the various transitions found by these measures also occur in time and brain locations (electrodes). But, the study was not detailed enough in order to give reliable results.

The analysis of the single trials achieves similar results (Figs. 7 and 8 show the results for selected trials). The *LAM* clearly found the N100 and P300 components for ERP10 in 26 trials (of 31), but not for the ERP90 trials. The other measures have lesser maxima and, thus, are not suitable for such recognition.

This result indicates that our introduced measures of complexity (especially *LAM*) are able to recognize transitions in brain potentials, which are caused by e.g. stimulative events. These transitions can be found in the single trials, which is an improvement to the classical method of averaging all observations.

## 5. Summary

We have applied an extended recurrence quantification analysis (RQA) to physiological event-related potential data (ERP). The classical RQA consists of measures which are mainly based on diagonal

structures in the recurrence plots (RPs), e.g. the determinism (*DET*), which is the ratio of recurrence points located on connected diagonal structures in the RP, and the averaged diagonal line length (*L*). We have extended the RQA with two recently introduced measures, the laminarity (*LAM*) and the trapping time (*TT*). These measures are analogously defined as *DET* and *L*, but provided by the vertical structures in recurrence plots. Whereas the classical RQA enables the identification of period-chaos transitions, the new measures make the identification of chaos-chaos transitions and laminar states possible.

The classical method to study ERP data is to average them over many trials. Our aim was to study the single trials in order to find transitions in the data.

The application of the extended RQA to ERP data has discriminated the single trials with a distinct P300 component due to a high surprise moment (less frequent events) against such trials with a low surprise moment (high frequent events). Considering the raw ERP10 data, the P300 component can only be found in half of all trials. Also a statistical variance test fails to distinguish clearly the trials. The *LAM* is the most pronounced parameter in this analysis. It measures the ratio of recurrence points located on connected vertical structures in an RP. These structures correspond with laminarity within the underlying process. In the ERP data, the *LAM* reveals transitions from less laminar states to higher laminar states after the occurrence of the event and a transition from higher laminar states to less laminar states after about 400 ms. These transitions occur around bounded brain areas (parietal to frontal along the central axis). The comparable measures *DET/LAM* and *L/TT* are quite different in their amplitude. There should also be differences in time and brain location of the found transitions.

These results show that the measures based on vertical RP structures make the identification of transitions possible, which are not found by the classical RQA measures. They indicate transitions in the brain processes into laminar states due to the surprising moment of observed events.

A future work will be concerned with the development of a statistical evaluation of these results. Furthermore, this investigation has to be extended to ERP data gained from other frequent events and a detailed study of the comparable measures *DET/LAM* and *L/TT* should give hints about the different transitions in the brain processes.

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