Eur. Phys. J. Spec. Top. (2023) 232:1–3
https://doi.org/10.1140/epjs/s11734-023-00766-z

THE EUROPEAN PHYSICAL JOURNAL SPECIAL TOPICS

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Editorial

Special Issue "Trends in recurrence analysis of dynamical systems"

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Published online 15 February 2023

More than a decade has passed since the publication of the special issue "20 Years of Recurrence Plots: Perspectives for a Multi-purpose Tool of Nonlinear Data Analysis" in the *European Physical Journal—Special Topics* [1]. The hope for further developments inspired by the interesting contributions in this special issue was fully realized. We see an amazing development in the field of recurrence plots (RPs), recurrence quantification analysis (RQA), and recurrence networks. Recurrence analysis is not just one method; it has emerged as an entire framework with many extensions, special recurrence definitions, and specifically designed methods and tools. It has found spreading applications in diverse and growing scientific fields. Recurrence analysis has become a widely accepted concept, even referred to in studies that are actually not using it as a method, but rather using it as a reference or alternative tool. It continues to be an active area of research and development today. An attempt to provide an overview of the most significant technical developments of this recurrence-plot-based framework in the past decade is included in this special issue [2].

Nevertheless, the methodical developments have not been exhausted, as we can see by further numerous new ideas and approaches collected in this issue. Shiro and Hirata suggest a pseudo-basis approach based on the recurrence matrix for better understanding and improving the conversion between time series and recurrence matrix and vice versa [3]. Improvements for the reliability of measures of the RQA are suggested by Özdes and Eroglu [4] and Pánis et al. [5]. Both approaches suggest an averaging schema: one is based on averaging over many scales (such as when using an edit distance metric for the recurrence definition), and the other one is averaging over a range of recurrence thresholds. Averaging over the recurrence thresholds has the potential to finally omit the selection of this threshold. An interesting novel measure for recurrence quantification is suggested by Braun et al. [6], which has its roots in the succolarity measure developed in the field of fractal geometry. This novel measure is modified to study the characteristic geometric structures in a RP by measuring a virtual flow through channels. It can be used to study interdependencies between different time series or to find the embedding delay for time-delay embedding. Comparison between time series usually asks how much similarity they share; the opposite question, how two almost identical time series would differ, is not so common but important in some special applications (e.g., for material testing). A modification of the recurrence definition to consider the divergence of the spatial distances in phase space instead of the spatial distances leads to difference RPs that allow amplification and detection of tiny differences between time series that are virtually identical [7].

Numerous applications of the recurrence framework illustrate the potential and applicability for a wide spread of problems and research questions. RQA can be used to investigate the dynamics of systems described with fractional orders [8]. Historically, RP and RQA are widely used to analyze problems in cognitive science or cardiology. A number of applications discuss cardiological states related to specific problems of hemodialysis [9, 10] and the relationship between heart rate and cognitive performance [11]. A new methodical modification based on recurrence analysis of short-term Fourier

Trends in Recurrence Analysis of Dynamical Systems. Guest editors: Norbert Marwan, Charles Webber Jr., Andrzej Rysak.

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transforms of time series is used to study electroencephalographic (EEG) data [12]. RQA can also be used to study the differences in trained and untrained athletes, as shown in the study on martial art movement patterns [13]. Recurrence time entropy is a rarely used RQA measure, although it is powerful to discriminate different dynamics. In a study on event-related potentials, a common problem in quantitative psychology, it is used to indicate ambiguous visual stimuli [14]. Using the cross-RP approach, Corbin and Davis have shown the potential of using the multivariate dataset for constructing the phase space and analyzed the interpersonal motor coordination of joint actions between individuals [15].

The application potential of recurrence analysis in the field of engineering is growing as well. In this special issue we have collected various applications, ranging from material damage testing, over communication and hydrological applications, to analyzing flows. RQA has been used to develop nondestructive tests based on ultrasonic waves for composite materials, such as those used in airplanes [7, 16]. A specific variation of the recurrence analysis method can be used to study the digital modulations in signals even in noisy environments [17]. Cross-RP was used to indicate the synchronization between the inlet and outlet channels of boiling flows [18]. Applications of RP-based methods in the field of hydrology are rare. Here we have an example investigating the impact of dam constructions on the dynamics of (mountain) river flows [19].

The final examples of successful applications of the recurrence analysis cover the field of climate science. RQA has helped to identify multiple transitions in the dynamics of the El Niño/Southern Oscillation [20]. The already mentioned new average scheme for RQA of irregularly sampled time series was used to study important transitions in the climate of the past [4].

Original recurrence plots and recurrence quantifications have a long history dating back some three decades [21–24], and this computational methodology has found utility in numerous fields of inquiry [2, 25]. Software developments continue to improve as new strategies centered on recurrence analyses keep evolving. One can never be removed for the fundamental concepts of what it means for systems and states to recur as reviewed in the past [24].

Funding Information Open Access funding enabled and organized by Projekt DEAL.

References

1. N. Marwan, A. Facchini, M. Thiel, J.P. Zbilut, H. Kantz, 20 years of recurrence plots: perspectives for a multi-purpose tool of nonlinear data analysis. Eur. Phys. J. Spec. Top. **164**(1), 1–2 (2008). https://doi.org/10.1140/epjst/e2008-00828-2
2. N. Marwan, K.H. Kraemer, Trends in recurrence analysis of dynamical systems. Eur. Phys. J. Spec. Top. (2023). https://doi.org/10.1140/epjs/s11734-022-00739-8
3. M. Shiro, Y. Hirata, A pseudo-basis using a recurrence plot. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00702-7
4. C. Özdes, D. Eroglu, Transformation cost spectrum for irregularly sampled time series. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00512-x
5. R. Pánis, K. Adámek, N. Marwan, Averaged recurrence quantification analysis—method omitting the recurrence threshold choice. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00686-4
6. T. Braun, K.H. Kraemer, N. Marwan, Recurrence flow measure of nonlinear dependence. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00687-3
7. C. Brandt, N. Marwan, Difference recurrence plots for structural inspection using guided ultrasonic waves: a new approach for evaluation of small signal differences. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00701-8
8. A. Rysak, Revealing fractionality in the Rössler system by recurrence quantification analysis. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00740-1
9. M. Calderón-Juárez, D.A. Gutiérrez Alvarado, G.H. González Gómez, J.C. Echeverría, J. Arellano-Martínez, E. Pérez-Granados, S. López-Gil, I.D. Campos-González, L.A. Mariscal-Ramírez, D.L. Pérez-Negrete, C. Lerma, Recurrence plot analysis of heart rate variability in end-stage renal disease treated twice-weekly by hemodialysis with or without intradialytic hypotension. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00682-8
10. M. Calderón-Juárez, G.H. González Gómez, J.C. Echeverría, J. Arellano-Martínez, V.H. Gómez-Suárez, I.D. Campos-González, C. Lerma, Recurrence quantitative analysis of heart rate variability during intradialytic hypotension. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00688-2
11. D. López Pérez, A.L.W. Bokde, C.M. Kerskens, Complexity analysis of heartbeat-related signals in brain MRI time series as a potential biomarker for ageing and

cognitive performance. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00696-2
12. Ł Furman, W. Duch, L. Minati, K. Tołpa, Short-time Fourier transform and embedding method for recurrence quantification analysis of EEG time series. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00683-7
13. B.G. Straiotto, N. Marwan, D.C. James, P.J. Seeley, Recurrence analysis discriminates martial art movement patterns. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00684-6
14. N. Frolov, E. Pitsik, V. Maksimenko, A. Hramov, Applying recurrence time entropy to identify changes in event-related potentials. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00743-y
15. S.F. Corbin, T.J. Davis, Comparing bivariate and multivariate timeseries analysis in joint action using cross-recurrence quantification analysis. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00745-w
16. B. Maack, C. Brandt, M. Koerdt, C. Polle, A.S. Herrmann, Continuous baseline update using recurrence quantification analysis for damage detection with guided ultrasonic waves. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00685-5
17. D. Stanescu et al., Characterization of digital modulations using the phase diagram analysis. Eur. Phys. J. Spec. Top. (2023). https://doi.org/10.1140/epjs/s11734-022-00744-x
18. G. Rafałko, H. Grzybowski, P. Dzienis, I. Zaborowska, R. Mosdorf, G. Litak, Recurrence analysis of phase distribution changes during boiling flow in parallel minichannels. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00741-0
19. M. Kędra, Dam-induced changes in river flow dynamics revealed by RQA. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00689-1
20. S. Das, R. Bhardwaj, V. Duhoon, Chaotic dynamics of recharge–discharge El Niño-Southern Oscillation (ENSO) model. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00742-z
21. J.-P. Eckmann, S.O. Kamphorst, D. Ruelle, Recurrence plots of dynamical systems. Europhys. Lett. (EPL) **4**, 973–977 (1987). https://doi.org/10.1209/0295-5075/4/9/004
22. J.P. Zbilut, C.L. Webber Jr., Embeddings and delays as derived from quantification of recurrence plots. Phys. Lett. A **171**, 199–203 (1992). https://doi.org/10.1016/0375-9601(92)90426-M
23. C.L. Webber Jr., J.P. Zbilut, Dynamical assessment of physiological systems and states using recurrence plot strategies. J. Appl. Physiol. **76**, 965–973 (1994). https://doi.org/10.1152/jappl.1994.76.2.965
24. N. Marwan, M.C. Romano, M. Thiel, J. Kurths, Recurrence plots for the analysis of complex systems. Phys. Rep. **438**, 237–329 (2007). https://doi.org/10.1016/j.physrep.2006.11.001
25. N. Marwan, A historical review of recurrence Plots. Eur. Phys. J. Spec. Top. **164**, 3–12 (2008). https://doi.org/10.1140/epjst/e2008-00829-1