

Introduction to focus issue: Recurrence quantification analysis for understanding complex systems

Norbert Marwan, Charles L. Webber, Elbert E. N. Macau, and Ricardo L. Viana

Citation: *Chaos* **28**, 085601 (2018); doi: 10.1063/1.5050929

View online: <https://doi.org/10.1063/1.5050929>

View Table of Contents: <http://aip.scitation.org/toc/cha/28/8>

Published by the [American Institute of Physics](#)

Articles you may be interested in

[Recurrence quantification analysis for the identification of burst phase synchronisation](#)

Chaos: An Interdisciplinary Journal of Nonlinear Science **28**, 085701 (2018); 10.1063/1.5024324

[Optimizing the detection of nonstationary signals by using recurrence analysis](#)

Chaos: An Interdisciplinary Journal of Nonlinear Science **28**, 085703 (2018); 10.1063/1.5022154

[Extended recurrence plot and quantification for noisy continuous dynamical systems](#)

Chaos: An Interdisciplinary Journal of Nonlinear Science **28**, 085722 (2018); 10.1063/1.5025485

[Deriving inferential statistics from recurrence plots: A recurrence-based test of differences between sample distributions and its comparison to the two-sample Kolmogorov-Smirnov test](#)

Chaos: An Interdisciplinary Journal of Nonlinear Science **28**, 085712 (2018); 10.1063/1.5024915

[Recurrence threshold selection for obtaining robust recurrence characteristics in different embedding dimensions](#)

Chaos: An Interdisciplinary Journal of Nonlinear Science **28**, 085720 (2018); 10.1063/1.5024914

[Phase space reconstruction for non-uniformly sampled noisy time series](#)

Chaos: An Interdisciplinary Journal of Nonlinear Science **28**, 085702 (2018); 10.1063/1.5023860



Introduction to focus issue: Recurrence quantification analysis for understanding complex systems

Norbert Marwan,¹ Charles L. Webber, Jr.,² Elbert E. N. Macau,³ and Ricardo L. Viana⁴

¹Member of the Leibniz Association, Potsdam Institute for Climate Impact Research (PIK), P.O. Box 60 12 03, D-14412 Potsdam, Germany

²Health Sciences Division, Department of Cell and Molecular Physiology, Loyola University Chicago, Maywood, Illinois 60153-3328, USA

³National Institute for Space Research, São José dos Campos, São Paulo 12227-010, Brazil

⁴Departamento de Física, Universidade Federal do Paraná, Curitiba, Paraná 80060-000, Brazil

(Received 3 August 2018; accepted 6 August 2018; published online 31 August 2018)

<https://doi.org/10.1063/1.5050929>

In 1987, recurrence plots were first introduced by Eckmann, Oliffson-Kamphorst, and Ruelle as a simple graphical tool to visualize basic dynamical characteristics of time series.¹ This present Focus Issue is dedicated to the 30th anniversary of recurrence plots and constitutes a unique collection of diverse papers on advanced recurrence plots, their extensions and ramifications, as well as their broad applications and utility. In the last three decades, an analytical framework based on recurrence plots has been developed, demonstrating an unanticipated but huge potential stemming from the original conceptualizations.^{2,3} In its brief history, thousands of recurrence publications over numerous disciplines spanning these three decades have permeated the scientific literature. In addition, regular scientific meetings continue to attract and recruit new members to the “recurrence community” indicating lively growth and expansion into new disciplines of inquiry. For example, our most recent meeting in Brazil at the Escola Politécnica Universidade De São Paulo (August 23–25, 2017) focused on disciplines of engineering, earth science, and life and social sciences.

A recurrence plot visualizes the times when the phase space trajectory of a dynamical system recurs to previous (or later) states up to a small error (the recurrence threshold).^{1,2} The recurrence plot is the ground base for quantification of the recurrence structures, e.g., using recurrence quantification analysis⁴ or recurrence networks.⁵ Recurrence analysis of time series can be used to classify different signals or states, to identify transitions in the dynamics, or even to investigate interrelations, coupling directions, and synchronization between different time series.

A fundamental parameter in recurrence analysis is the recurrence threshold. When considering systems with changing dynamics, the threshold selection deserves special attention. Kraemer *et al.* discuss the effect of time-delay embedding and the consequences for threshold selection.⁶ Prado *et al.* suggest an approach of optimizing the selection of the threshold in order to better find regime transitions in non-stationary systems or to infer coupling between dynamical systems.⁷ As for any time series analysis, recurrence-based methods are also affected by certain challenges, such as noisy data or time series with irregular sampling. For the latter, Lekscha and Donner show how to use Legendre polynomials for

reconstructing the phase space trajectory, which is then used for the recurrence analysis.⁸ Wendi and Marwan reconsider an alternative criterion of recurrence⁹ and extend it for noisy data; moreover, some of the basic elements of recurrence quantification analysis are redefined in order to overcome the challenges with noise.¹⁰ Further challenges are related to special use cases, such as analyzing spatial data, event data, and discourses. These challenges are solved by a combination of complex networks and recurrences that allow for investigating spatial data,¹¹ introducing a new windowing concept for recurrence analysis,¹² and extending the conceptual recurrence quantification analysis for the discourse data.¹³ All these new developments emphasize the applicability of the recurrence approach. The potentials of this approach are also best demonstrated by prototypical examples and selected physical problems. Ramdani *et al.* apply recurrence analysis to investigate the correlation structure of stochastic processes, such as fractional Gaussian noise;¹⁴ Santos *et al.* study the regime changes in the standard nontwist map;¹⁵ and Lameu *et al.* use recurrence analysis to identify chaotic burst phase synchronization in networks.¹⁶ Moreover, recurrence plots contain enough information about time series that they can be used for nonparametric inferential statistics, as worked out by Wallot and Leonardi.¹⁷

The recurrence plot framework can be applied to diverse research questions in various scientific disciplines. The collection of studies in this Focus Issue gives an overview about this. By using recurrence quantification analysis as test statistics in surrogate data tests, the hypothesis that musical compositions arose from a Markov chain is tested.¹⁸ Recurrence analysis is further used to identify events, regime transitions, and bifurcations, such as in social media streams (concept drifts in Twitter sentiments),¹⁹ in the nonlinear magnetospheric dynamics of the earth's magnetosphere (magnetic storms),²⁰ in electrochemical systems (corrosion processes),²¹ in the cardiovascular system to detect physiological stress,²² and in polysomnography data for sleep-wake detection.²³ Applications of recurrence plots in medicine have a long tradition and were one of the drivers of certain developments, e.g., leading to the recurrence quantification analysis. Therefore, there is no surprise that recurrence plot methods are widely applied for different medical purposes: to

identify certain physiological or pathological states, e.g., voice disorder,²⁴ atrial fibrillation,²⁵ or patients' response to ventilator treatment;²⁶ to analyze the quality of surgeons using dual eye tracking;²⁷ or to classify certain states, e.g., for an emotion recognition system,²⁸ or to distinguish between mental fatigue and normal mental state (interesting for brain-computer interfaces).²⁹ Further applications presented here and completing the disciplines refer to the detection of unstable periodic orbits in mineralizing geological systems³⁰ and to investigate the coupling between the Pacific and the tropical North Atlantic by an atmosphere-land bridge (via the Amazonian).³¹ The diversity of topics in general and in this Focus Issue speaks to the wide applicability of recurrence plots across many disciplines of inquiry.

In summary, from their very outset 30 years ago, recurrence plots are not only beautiful to look at (art) but also contain hidden quantitative details that report on dynamical subtleties (science). The ability to embed vector inputs from the temporal and spatial domains into higher dimensional spaces teases out numerical descriptors that have amazing utility in diagnosing real-world dynamics. As evidenced by all the unique contributions to this Focus Issue, recurrence plots and their quantifications are powerful nonlinear tools whose applications run circles around classical linear methodologies. Researchers not familiar with recurrence plots are encouraged to apply this approach to their system of choice. The hope is that new generations of scientists will catch the vision and contribute creatively to the field. Indeed, there seems to be no limit to the type of dynamic that can be viewed from this perspective. In short, recurrence plots are here to stay. Indeed, we are all indebted to Eckmann, Oliffson-Kamphorst, and Ruelle for their astounding contribution and key insights from so many years ago.¹

- ¹J.-P. Eckmann, S. Oliffson-Kamphorst, and D. Ruelle, "Recurrence plots of dynamical systems," *Europhys. Lett.* **4**, 973–977 (1987).
- ²N. Marwan, M. C. Romano, M. Thiel, and J. Kurths, "Recurrence plots for the analysis of complex systems," *Phys. Rep.* **438**, 237–329 (2007).
- ³C. L. Webber, Jr. and N. Marwan, *Recurrence Quantification Analysis—Theory and Best Practices* (Springer, Cham, 2015), 421 pp.
- ⁴J. P. Zbilut and C. L. Webber, Jr., "Embeddings and delays as derived from quantification of recurrence plots," *Phys. Lett. A* **171**, 199–203 (1992).
- ⁵N. Marwan, J. F. Donges, Y. Zou, R. V. Donner, and J. Kurths, "Complex network approach for recurrence analysis of time series," *Phys. Lett. A* **373**, 4246–4254 (2009).
- ⁶K. H. Kramer, R. Donner, J. Heitzig, and N. Marwan, "Recurrence threshold selection for obtaining robust recurrence characteristics in different embedding dimensions," *Chaos* **28**, 085720 (2018).
- ⁷T. D. Prado, G. Z. Lima, B. L. Soares, G. do Nascimento, G. Corso, J. Fontenele-Araujo, J. Kurths, and S. R. Lopes, "Optimizing the detection of nonstationary signals by using recurrence analysis," *Chaos* **28**, 085703 (2018).
- ⁸J. Lekscha and R. Donner, "Phase space reconstruction for non-uniformly sampled noisy time series," *Chaos* **28**, 085702 (2018).
- ⁹A. Schultz, Y. Zou, N. Marwan, and M. T. Turvey, "Local minima-based recurrence plots for continuous dynamical systems," *Int. J. Bifurcat. Chaos* **21**, 1065–1075 (2011).
- ¹⁰D. Wendi and N. Marwan, "Extended recurrence plot and quantification for noisy continuous dynamical system," *Chaos* **28**, 085722 (2018).
- ¹¹C. Chen, H. Yang, and S. Kumara, "Recurrence network modeling and analysis of spatial data," *Chaos* **28**, 085714 (2018).

- ¹²J. P. Hummel, J. Akar, A. Baher, and C. Webber, Jr., "New skip parameter to facilitate recurrence quantification of signals comprised of multiple components," *Chaos* **28**, 085718 (2018).
- ¹³D. Angus and J. Wiles, "Social semantic networks: Measuring topic management in discourse using a pyramid of conceptual recurrence metrics," *Chaos* **28**, 085723 (2018).
- ¹⁴S. Ramdani, F. Bouchara, and A. Lesne, "Analyzing the correlations of fractional Gaussian noise processes using diagonal and vertical structures of their recurrence plots," *Chaos* **28**, 085721 (2018).
- ¹⁵M. Santos, M. Mugnaine, J. D. Szezech, Jr., A. M. Batista, I. Luiz Caldas, L. da Silva Baptista, and R. L. Viana, "Recurrence-based analysis of barrier breakup in the standard nontwist map," *Chaos* **28**, 085717 (2018).
- ¹⁶E. L. Lameu, S. Yanchuk, E. E. Macau, F. D. Borges, K. C. Iarosz, I. L. Caldas, P. R. Protachevich, R. R. Borges, R. L. Viana, J. D. Szezech, Jr., A. M. Batista, and J. Kurths, "Recurrence quantification analysis for the identification of burst phase synchronization," *Chaos* **28**, 085701 (2018).
- ¹⁷S. Wallot and G. Leonardi, "Deriving inferential statistics from recurrence plots: A recurrence-based test of differences between sample distributions and its comparison to the two-sample Kolmogorov-Smirnov test," *Chaos* **28**, 085712 (2018).
- ¹⁸J. M. Moore, D. Correa, and M. Small, "Is Bach's brain a Markov chain? Recurrence quantification to assess Markov order for short, symbolic, musical compositions," *Chaos* **28**, 085715 (2018).
- ¹⁹R. Mello, R. A. Rios, P. Pagliosa, and C. Lopes, "Concept drift detection on social network data using cross-recurrence quantification analysis," *Chaos* **28**, 085719 (2018).
- ²⁰R. Donner, V. Stolbova, G. Balasis, J. F. Donges, M. Georgiou, S. M. Potirakis, and J. Kurths, "Temporal organization of magnetospheric fluctuations unveiled by recurrence patterns in the Dst index," *Chaos* **28**, 085716 (2018).
- ²¹D. Valavanis, D. Spanoudaki, C. Gkili, and D. Sazou, "Using recurrence plots for the analysis of the nonlinear dynamical response of iron passivation-corrosion processes," *Chaos* **28**, 085708 (2018).
- ²²G. H. Gonzalez-Gomez, O. Infante, P. Martinez-Garcia, and C. Lerma, "Analysis of diagonals in cross recurrence plots between heart rate and systolic blood pressure during supine position and active standing in healthy adults," *Chaos* **28**, 085704 (2018).
- ²³V. C. Parro and L. Valdo, "Sleep-wake detection using recurrence quantification analysis," *Chaos* **28**, 085706 (2018).
- ²⁴V. J. Vieira, S. Costa, S. Correia, L. Lopes, W. Costa, and F. de Assis, "Exploiting the nonlinearity of the speech production system for voice disorders assessment by recurrence quantification analysis," *Chaos* **28**, 085709 (2018).
- ²⁵T. P. Almeida, F. S. Schlindwein, J. Salinet, X. Li, G. S. Chu, J. H. Tuan, P. J. Stafford, G. A. Ng, and D. C. Soriano, "Characterization of human persistent atrial fibrillation electrograms using recurrence quantification analysis," *Chaos* **28**, 085710 (2018).
- ²⁶N. C. Carvalho, L. L. Portes, A. Beda, L. Tallarico, and L. A. Aguirre, "Recurrence plots for the assessment of patient-ventilator interactions quality during invasive mechanical ventilation," *Chaos* **28**, 085707 (2018).
- ²⁷W. He and B. Zheng, "Dual eye-tracking for the assessment of team cognition in laparoscopic surgery: Evidences from cross recurrence analysis," *Chaos* (to be published).
- ²⁸Y. Yang, Z. Gao, X. Wang, Y. Li, J. Han, N. Marwan, and J. Kurths, "A recurrence quantification analysis-based channel-frequency convolutional neural network for emotion recognition from EEG," *Chaos* **28**, 085724 (2018).
- ²⁹Z. Gao, C. Liu, Y. Yang, Q. Cai, W. Dang, X. Du, and H. Jia, "Multivariate weighted recurrence network analysis of EEG signals from ERP-based smart home system," *Chaos* **28**, 085713 (2018).
- ³⁰S. M. Oberst, R. K. Niven, D. Lester, A. Ord, B. Hobbs, and N. Hoffmann, "Detection of unstable periodic orbits in mineralising geological systems," *Chaos* **28**, 085711 (2018).
- ³¹A. Builes Jaramillo, A. M. de Torres Ramos, and G. Poveda, "Atmosphere-land bridge between the Pacific and tropical North Atlantic SST's through the Amazon River basin during the 2005 and 2010 droughts," *Chaos* **28**, 085705 (2018).