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QUANTIFICATION OF ORDER PATTERNS RECURRENCE PLOTS OF EVENT RELATED POTENTIALS

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Abstract

We study an innovative modification of recurrence plots defining the recurrence by the local ordinal structure of a time series. In this paper we demonstrate that in comparison to a recently developed approach this concept improves the analyis of event related activity on a single trial basis.

Keywords: Data Analysis, Recurrence plot, Nonlinear dynamics

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1. Introduction

A basic research in cognitive science deals with the study of the behaviour of the brain after short, surprising stimuli. Such event related changes can be measured as changes in the brain potentials with electroencephalography (EEG), and are called *event related potentials (ERPs)* (Sutton et al., 1965).

Traditionally, ERP waveforms are determined by computing an ensemble average of a large collection of EEG trials that are stimulus time locked. This is based on the following assumptions: (1) the presentation of stimuli of the same kind is followed by the same sequence of processing steps, (2) these processing steps always lead to activation of the same brain structures, (3) this activation always elicits the same pattern of electrophysiological activity, which can be measured at the scalp (Rösler, 1982) and (4) spontaneous activity is stationary and ergodic.

EEG data contain a composition of different effects in the brain. Other signals not related with ERPs are regarded in this context as noise. In order to find characteristic ERPs in such strongly noisy EEG data, EEGs of a number of trials are measured. By averaging the data points, which are time locked to the stimulus presentation, it is possible to filter out the ERP signal of the noise (spontaneous activity). This way, a positive potential 300 ms after the stimulus (P300) was the first ERP discovered. It was inferred that the P300 component varies in dependence on subject internal factors, like attention and expectation, instead on physical characteristics (Sutton et al., 1965). The amplitude of the P300 component is highly sensitive to the novelty of an event and its relevance (surprising moment), so this component is assumed to reflect the updating of the environmental model of the information processing system (context updating) Donchin (1981); Donchin and Coles (1988).

A disadvantage of the averaging is the high number of trials needed to reduce the signal-to-noise-ratio. This disadvantage is crucial for example in clinical studies, studies with children and studies in which repeating a task would influence the performance. Moreover, several high frequency structures of the ERPs are filtered out by using the averaging method. Therefore, new methods for the analysis of event related activity on a single trial basis are highly desirable.

A recently developed approach based on the recurrence quantification analysis has proven its ability to indicate transitions in the brain processes due to the surprising moment and to distinguish ERPs (Marwan and Meinke, 2004). In this paper we demonstrated an improvement in the analysis by an innovative modification of the recurrence plots, where the recurrence is defined by order patterns (Groth, 2005).

This paper is organized as follows. First we briefly review the recurrence plots and its recurrence quantification analysis. Then, the modification of recurrence plots by order patterns is introduced. Finally, we compare both approaches on event related data from the Oddball experiment.

2. Recurrence Plots

We develop a recurrence quantification based on *recurrence plots (RP)*. A RP is a $N \times N$ matrix representing neighbouring states $\vec{x}_i$ in a d-dimensional phase space (Fig. 1) (Eckmann et al., 1987)

$$\mathbf{R}_{i,j}(\varepsilon) = \begin{cases} 1 : \left\|\vec{x}_i - \vec{x}_j\right\| \leq \varepsilon \\ 0 : \text{otherwise} \end{cases} \qquad \vec{x}_i \in R^d, \quad i,j = 1 \ldots N, \tag{1}$$

where N is the number of considered states $\vec{x}_i$; ε is a threshold distance and $\|\cdot\|$ a norm. Hence, (1) is a pairwise test of the closeness of points on a phase space trajectory: points which fall in the neighbourhood of size ε are *recurrence points*. Another definition of RPs does not use such a fixed threshold ε: only the F nearest neighbours are considered to be recurrence points. This is the *fixed amount of nearest neighbours (FAN)* method and coincides with the original definition of RPs by Eckmann et al. (1987). The ratio F/N is the recurrence point density of the RP and we denote it as $\varepsilon_{FAN} = F/N$.

In RPs we obtain different structures: If the phase space trajectory returns to itself and runs close for some time we obtain diagonal lines. Vertical lines or areas indicate phase space trajectory which remain in the same area of the phase space for some time, and single dots indicate that the phase space trajectory heavily fluctuates. The phase space vectors can be reconstructed with the Taken's time delay method $\vec{x}_i = (u_i, u_{i+\tau}, \ldots, u_{i+(m-1)\tau})$ from one-dimensional time series u_i (observation) with embedding dimension $m = 2(d+1)$ and delay τ (Takens, 1981; Kantz and Schreiber, 1997).

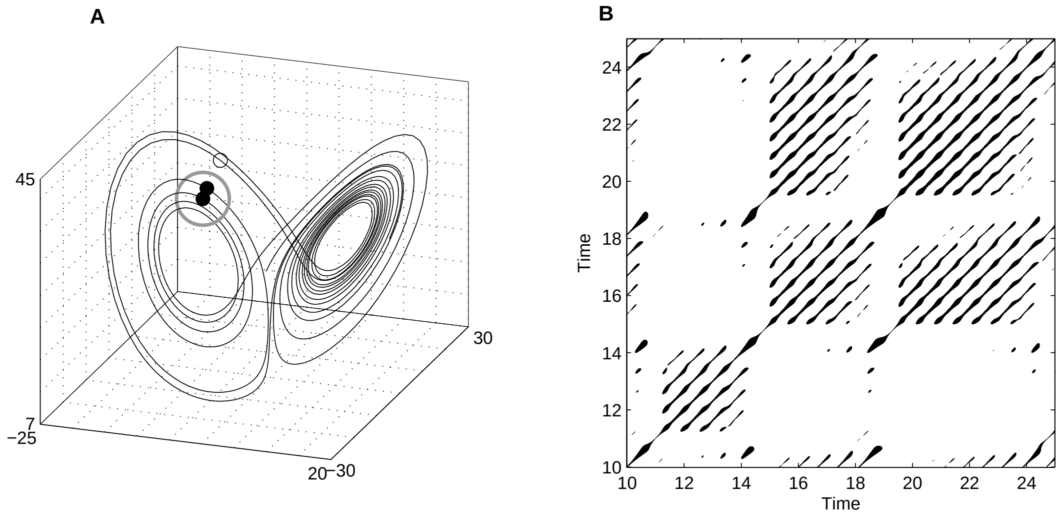

Figure 1. (A) Segment of the phase space trajectory of the Lorenz system (for standard parameters $r = 28$, $\sigma = 10$, $b = \frac{8}{3}$; Lorenz, 1963) by using its three components and (B) its corresponding recurrence plot. A point of the trajectory at j which falls into the neighbourhood (grey circle in (A)) of a given point at i is considered as a recurrence point (black point on the trajectory in (A)). This is marked with a black point in the RP at the location (i, j). A point outside the neighbourhood (small circle in (A)) causes a white point in the RP. The radius of the neighbourhood for the RP is $\varepsilon = 5$.

To characterize the dynamics of the underlying system several measures were introduced (Webber Jr. and Zbilut, 1994; Marwan et al., 2002; Marwan, 2003). Here we focus on the following four measures. We denote the frequency distribution of the lengths of diagonal lines by $P(l)$ and that of vertical lines by $P(v)$.

The *determinism* is the amount of recurrence points forming diagonal lines with regard to the total amount of recurrence points

$$DET(\varepsilon) = \frac{\sum_{l=l_{min}}^{N} l\, P(\varepsilon, l)}{\sum_{i,j}^{N} \mathbf{R}_{i,j}(\varepsilon)}. \tag{2}$$

Processes with stochastic behaviour cause none or very short diagonals, and thus we get low DET. Deterministic processes cause longer diagonals and less single, isolated recurrence points, and we get higher DET. The threshold l_{min} excludes the diagonal lines which are formed by the tangential motion of the phase space trajectory. For $l_{min} = 1$ the $DET = 1$, therefore l_{min} should be at least 2. To exclude the tangential motion, l_{min} can be, e. g., determined with the autocorrelation time (Theiler, 1986), but it has to be taken into account that a too large l_{min} can worsen the histogram $P(l)$ and thus the reliability of the measure DET.

Diagonal structures indicate segments of the trajectory which are close to another segment of the trajectory at different time. Thus these lines are related to the divergence of the trajectory segments. The *average diagonal line length*

$$L(\varepsilon) = \frac{\sum_{l=l_{min}}^{N} l\, P(\varepsilon, l)}{\sum_{l=l_{min}}^{N} P(\varepsilon, l)} \tag{3}$$

is the average time that two segments of the trajectory are close to each other, and can be interpreted as the mean prediction time. Although several authors stated that the inverse of the length of the diagonal lines correlates with the largest positive Lyapunov exponent (e. g. Trulla et al., 1996), it is important to note that this relationship is more complex.

Analogous to the definition of the determinism (2), we define the ratio between recurrence points forming vertical structures and the entire set of recurrence points as

$$LAM(\varepsilon) = \frac{\sum_{v=v_{min}}^{N} v P(\varepsilon, v)}{\sum_{v=1}^{N} v P(\varepsilon, v)}, \tag{4}$$

the *laminarity*. The computation of LAM is realized for those v that exceed a minimal length v_{min} in order to decrease the influence of sojourn points. For maps, $v_{min} = 2$ is used. LAM represents the occurrence of laminar states in the system without describing the length of these laminar phases. If the RP consists of more single recurrence points than vertical structures LAM decreases.

The average length of vertical structures (cp. Eq. (3)) is defined as

$$TT(\varepsilon) = \frac{\sum_{v=v_{min}}^{N} v P(\varepsilon, v)}{\sum_{v=v_{min}}^{N} P(\varepsilon, v)}, \tag{5}$$

and is called *trapping time*. With TT we measure the mean time that the system will abide at a specific state (how long a state will be trapped). The computation also uses the minimal length v_{min} as for LAM.

Note that these measures can be computed from an entire RP or in moving windows (i. e. sub-RPs) covering the main diagonal of the RP. The latter allows us to study the change of these measures with time, which can reveal transitions in the system. Whereas the diagonal-wise defined measures are able to find chaos-order transitions (Trulla et al., 1996), the vertical-wise defined measures indicate chaos-chaos transitions (Marwan et al., 2002).

3. Order Patterns Recurrence Plots

In (1) a recurrence is defined by spatial closeness between phase space trajectories $\vec{x}_i$ or embedded time series u_i. Now we neglect the norm $\|\cdot\|$ and define a recurrence by the local order structure of a trajectory. Given a one-dimensional time series $(u_1, \ldots, u_i, \ldots, u_N)$ we start to compare $d = 2$ time instances and define the order patterns as

$$\pi_i = \begin{cases} 0 : u_i < u_{i+\tau} \\ 1 : u_i > u_{i+\tau} \end{cases} \tag{6}$$

with the scaling parameter τ (tied ranks $u_i = u_{i+\tau}$ are assumed to be rare). Next, for $d = 3$ there are six order patterns between u_i, $u_{i+\tau}$ and $u_{i+2\tau}$ possible (Fig. 2). In general the

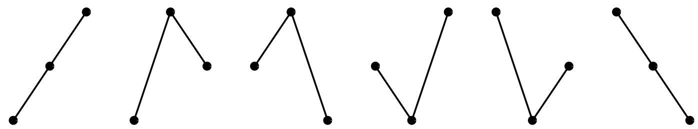

Figure 2. Order patterns of dimension $d = 3$ (tied ranks $u_i = u_{i+\tau}$ are assumed to be rare).

d components in $\vec{x}_i = (u_i, u_{i+\tau}, \ldots, u_{i+(d-1)\tau})$ can form $d!$ different patterns. On systems with continuous distribution of the values the equality has measure zero and we neglect this. From these order patterns we get a new symbolic time series π_i and define the *order patterns recurrence plot (OPRP)* as (Groth, 2005)

$$\mathbf{R}_{i,j}(d) = \begin{cases} 1 : \pi_i = \pi_j \\ 0 : \text{otherwise} \end{cases} \qquad i, j = 1 \ldots N. \tag{7}$$

The order patterns decompose the phase space $\vec{x}$ into $d!$ equivalent regions and recurrence is given if the trajectory runs throw the same region at different time. A main advantage of this symbolic representation is the well-expressed robustness against non-stationary data. The order patterns are invariant with respect to an arbitrary, increasing transformation of the amplitude. A common approach to overcome the problem of a non-stationary amplitude is the decomposition of a signal into instantaneous phase and amplitude, where only the phase is studied. In (Groth, 2005) relations between phase and order patterns are represented.

Furthermore a robust complexity measure based on this symbolic dynamics was already proposed (Bandt and Pompe, 2002) and successfully applied to epileptic seizure detection(Cao et al., 2004).

4. Event Related Potentials

4.1. The Oddball Experiment

The Oddball experiment studies brain potentials during a stimulus presentation. In the present Oddball experiment accoustic stimuli were used. Test subjects were seated in front of a monitor and had to count tones of high pitch using the cursor keys of the keyboard. During these tests, the EEG of the subjects was recorded. The experiment was repeated in nine blocks containing at least 30 target tones. The blocks varied in the probability of occurrence of the higher tones from 10 to 90 %. The accoustic stimuli were computer-generated beeps of 100 ms length and of either high (1400 Hz) or low (1000 Hz) pitch. They were presented with an interstimulus interval of 1000 ms.

The measurement of the EEG was performed with 31 electrodes/ channels (Tab. 1), where electrodes 26-31 were reference electrodes. The sample interval for the measurements was 4 ms (250 Hz).

We focus on the ERP data for an event frequency of 90% (ERP90) and 10% (ERP10). For ERP90, a set of 40 trials and for ERP10 a set of 31 trials are measured. The averaging of the potentials of ERP90 and ERP10 over the trials reveals the P300 ERP component,

Table 1. Notation of the electrodes and their numbering as used in the figures (electrodes 26–31 are reference electrodes).

#	Electrode	#	Electrode	#	Electrode	#	Electrode
1	F7	8	T7	15	P7	22	POZ
2	FC5	9	CP5	16	PZ	23	PO3
3	F3	10	C3	17	P3	24	CPZ
4	FZ	11	FCZ	18	CZ	25	PO4
5	F4	12	C4	19	P4		
6	FC6	13	CP6	20	P8		
7	F8	14	T8	21	OZ		

where its amplitude is higher for ERP10 (higher surprise moment, Fig. 3) (Marwan and Meinke, 2004). This confirms the knowledge about this ERP, that is related on subject-internal factors like attention and expectation instead of physical characteristics (Sutton et al., 1965) and its amplitude is sensitive to novelty of an event and its relevance (context updating, Donchin, 1981; Donchin and Coles, 1988).

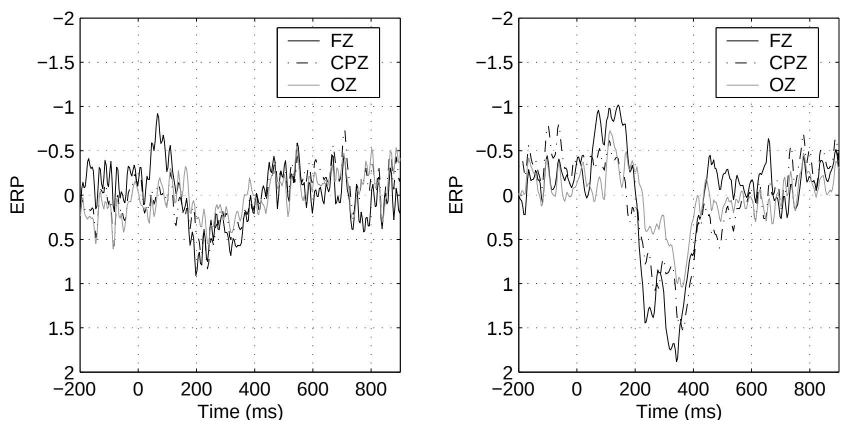

Figure 3. Mean event related potentials for event frequencies of 90 % (left, 40 trials) and 10 % (right, 31 trials) at selected electrodes. The P300 component is well pronounced for the frequencies of 10 %.

4.2. Recurrence Quantification

Recurrence quantification measures were already successfully applied to ERP data (Marwan and Meinke, 2004). In this work it has been shown that especially the measures DET, L, LAM and TT can be used for discrimination the events on a single trial bases.

In order to uncover transitions in the brain processes during unexpected stimulation on a single trial basis, we firstly compute common RPs and their quantification similar as presented in (Marwan and Meinke, 2004). The quantification is applied on moving windows of size 240 ms (60 samples) with a shifting step of 8 ms, which allows us to study the time dependence of the recurrence measures. We use the embedding parameters $m = 3$ and $\tau = 12$ ms and a neighbourhood criterion of $\varepsilon_{FAN} = 10\,\%$ (fixed amount of nearest neighbours). The embedding parameters dimension and delay were estimated by the standard methods false nearest neighbours and mutual information, respectively (Kantz and Schreiber, 1997). The neighbourhood criterion of 10 % nearest neighbours was found heuristically to be reliable even for non-stationary data.

The RPs of the ERP90 and ERP10 data sets contain diagonal lines and extended white areas (Fig. 4). One white band is located at the time of the stimulus. Other white bands which are located around 250 and 400 ms, occur almost only for ERP10 data and correspond with the P300 component. Moreover, clustered black points around 300 ms occur also only in RPs of the ERP10 data set.

The application of the recurrence quantification measures to these ERP data discriminates the single trials with a distinct P300 component resulting from a low surprise moment (high frequent events) in favour of such trials with a high surprise moment (less frequent events). In a previous study it was found that *LAM* is the most distinct parameter for this discrimination (Marwan and Meinke, 2004). In the ERP data, the *LAM* reveals transitions from less laminar states to more laminar states after the occurrence of the event and a transition from more laminar states to less laminar states after approximately 350 ms. These transitions occur around bounded brain areas (parietal to frontal along the central axis). The comparable measures *DET* and *LAM* as well as *L* and *TT* reveal similar results, because extended black areas contain also a high amount of diagonal lines (Figs. 5 and 6).

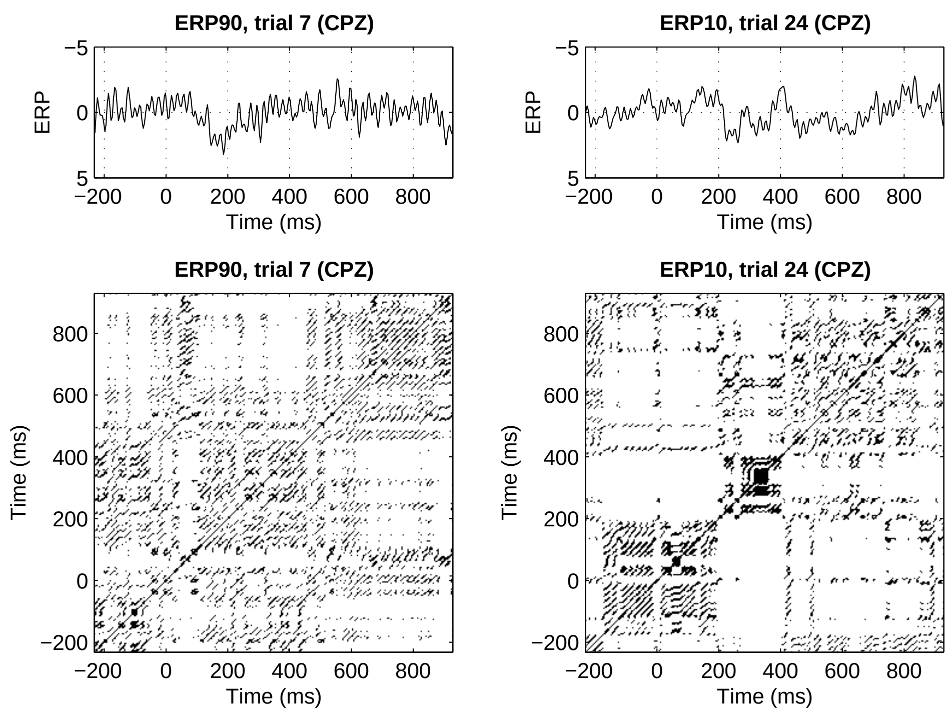

Figure 4. Recurrence plots (RPs) for the ERP90 and ERP10 measured at the central-parietal electrode (CPZ). For the ERP10, more cluster of recurrence points occur around 300 ms.

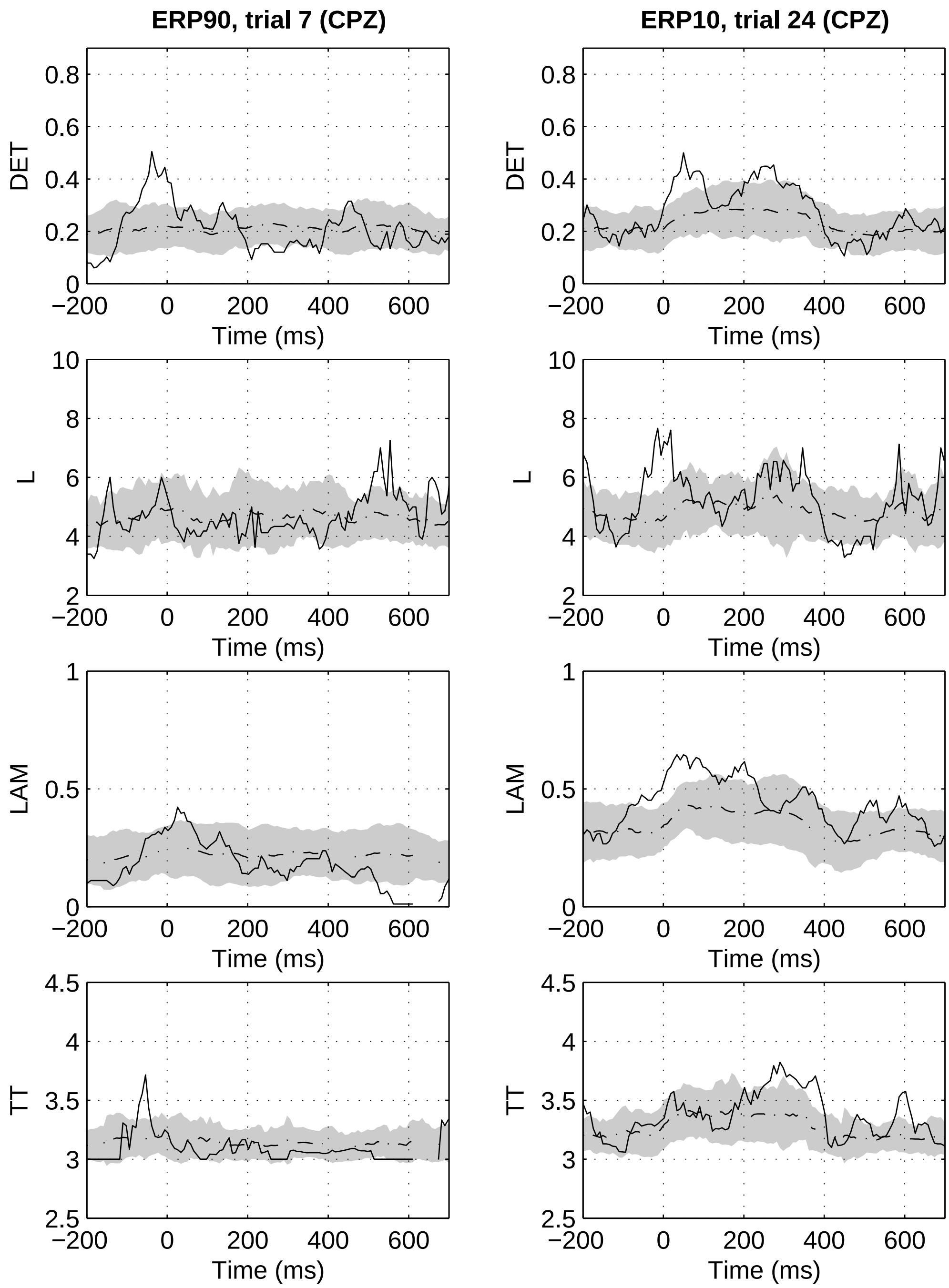

Figure 5. RQA measures for selected single trials and the central-parietal electrode (solid line). The trial-averaged RQA measures for the same electrode is shown with a dashed line (the light grey band marks the 95 % significance interval).

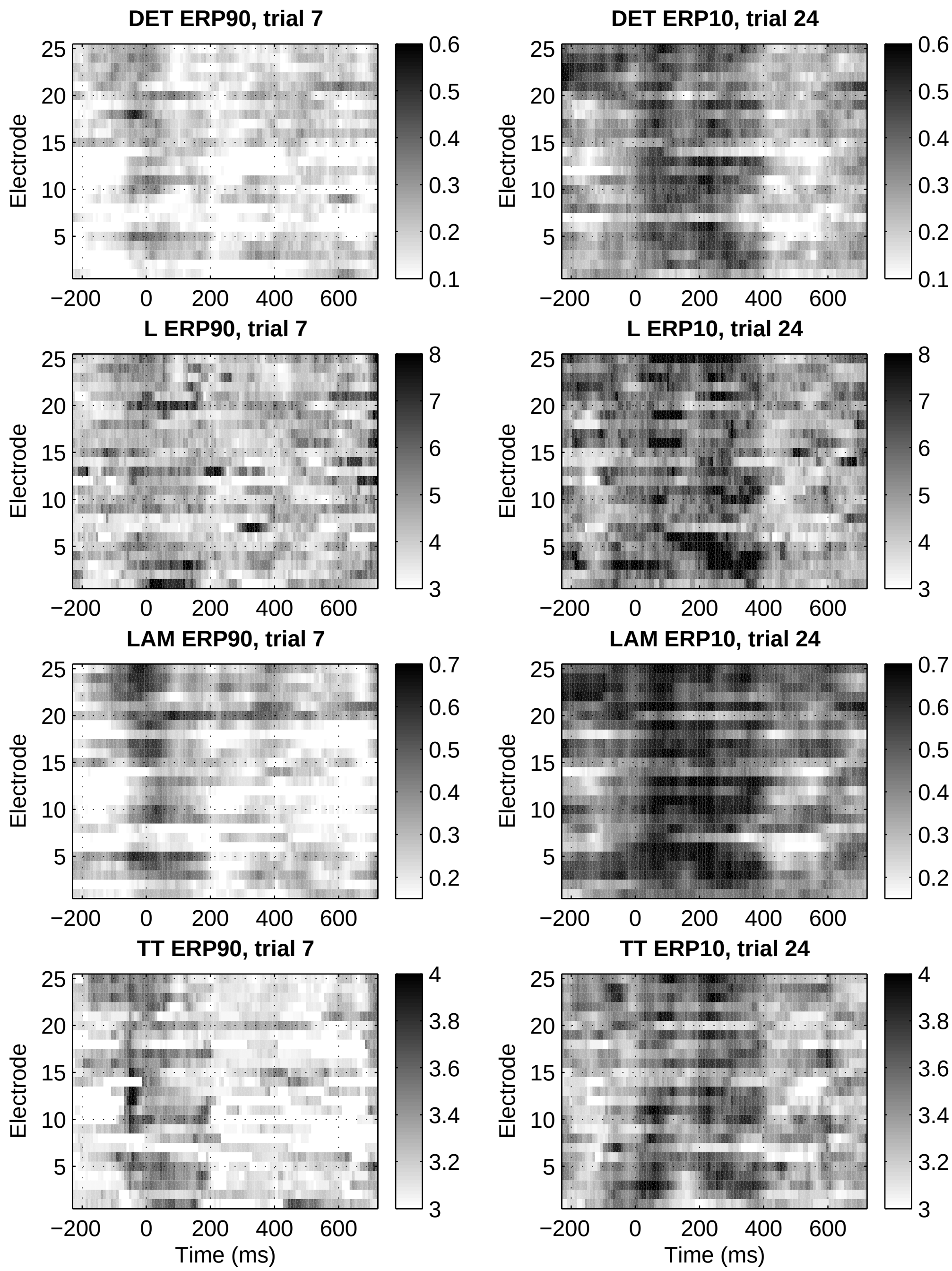

Figure 6. RQA measures for the same trials as in Fig. 5, but shown for all electrodes.

Next we compute OPRPs and quantify them by using the same moving windows as for the common RPs. We use the dimension $d = 3$, i. e. six order patterns and a delay of $\tau = 20$ ms.

The OPRPs are different in comparison to the common RPs (Fig. 7). They are more homogeneous and do not reveal such "disruption" as shown in Fig. 4. This is due to the robustness of OPRPs with respect to non-stationarity.

All measures gained from OPRPs reveal significant differences between ERP90 and ERP10. For the same trial, we find a more distinct difference using OPRPs than common RPs (Fig. 8). The quantification measures for ERP10 reveal high amplitudes at approximately 300 ms after the stimulus, wheras for ERP90 they vary within their standard deviation.

In contrast to the analysis with common RPs, the measures based on OPRPs are more different for different channels (Figs. 6 and 9). Electrodes in the frontal-central area (FZ, FCZ, CZ) reveal high amplitudes in DET, L, LAM and TT between 100 and 400 ms. Electrodes in the right frontal to parietal area (F4, C4, CP6, P4, PO4) reveal high amplitudes in these measures around 300 to 400 ms after the stimulus.

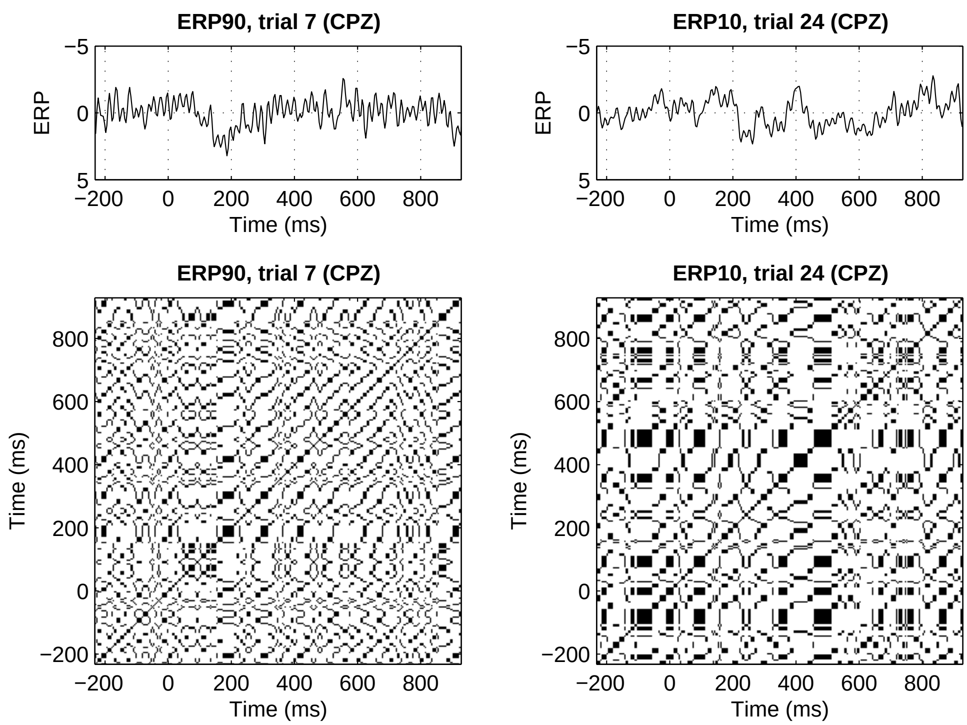

Figure 7. Order patterns recurrence plots (OPRPs) for the ERP90 and ERP10 measured at the central-parietal electrode (CPZ). Their appearance differs from those of common recurrence plots (Fig. 4)

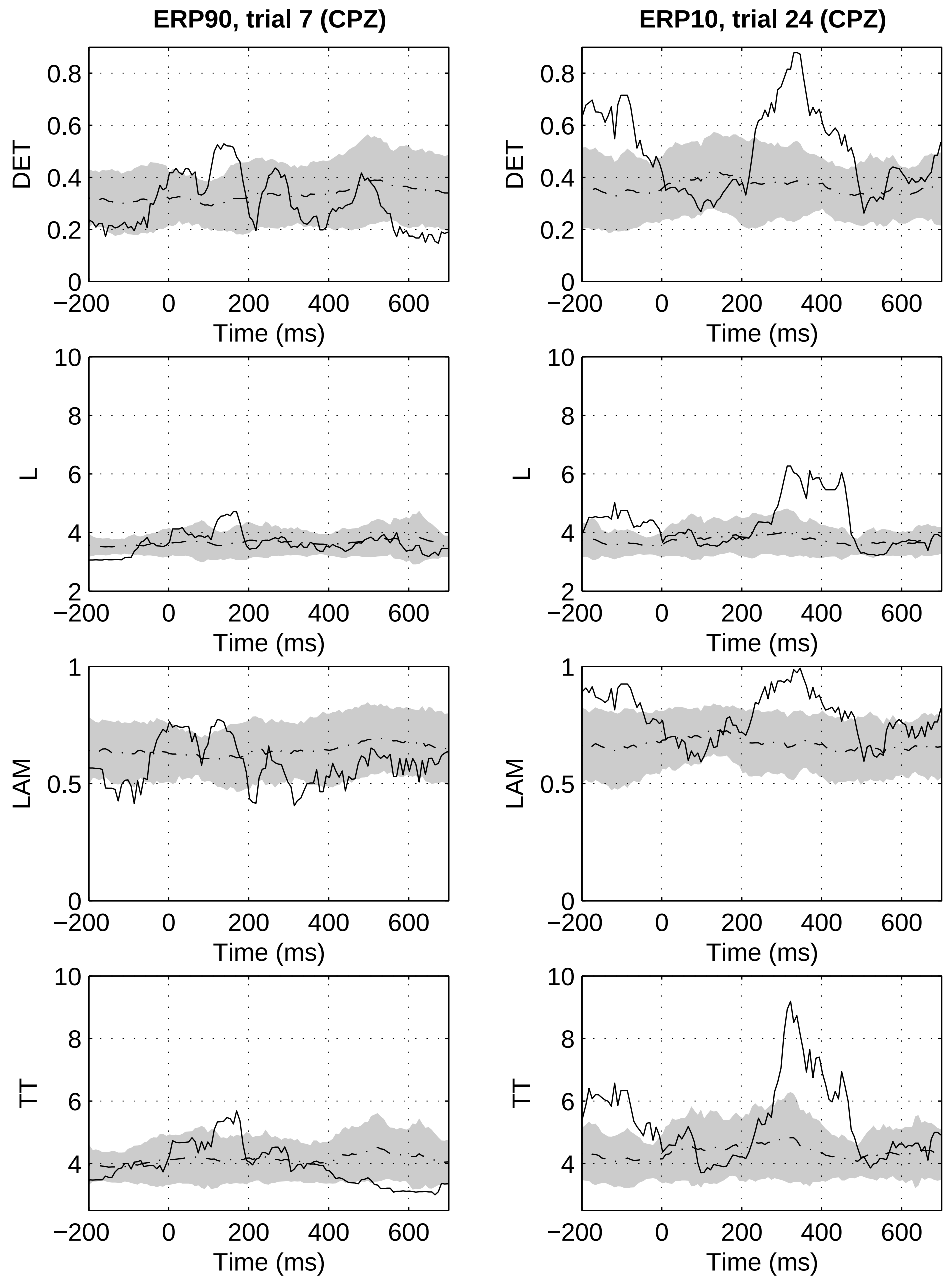

Figure 8. RQA measures for selected single trials and the central-parietal electrode (solid line). The trial-averaged RQA measures for the same electrode is shown with a dashed line (the light grey band marks the 95 % significance interval).

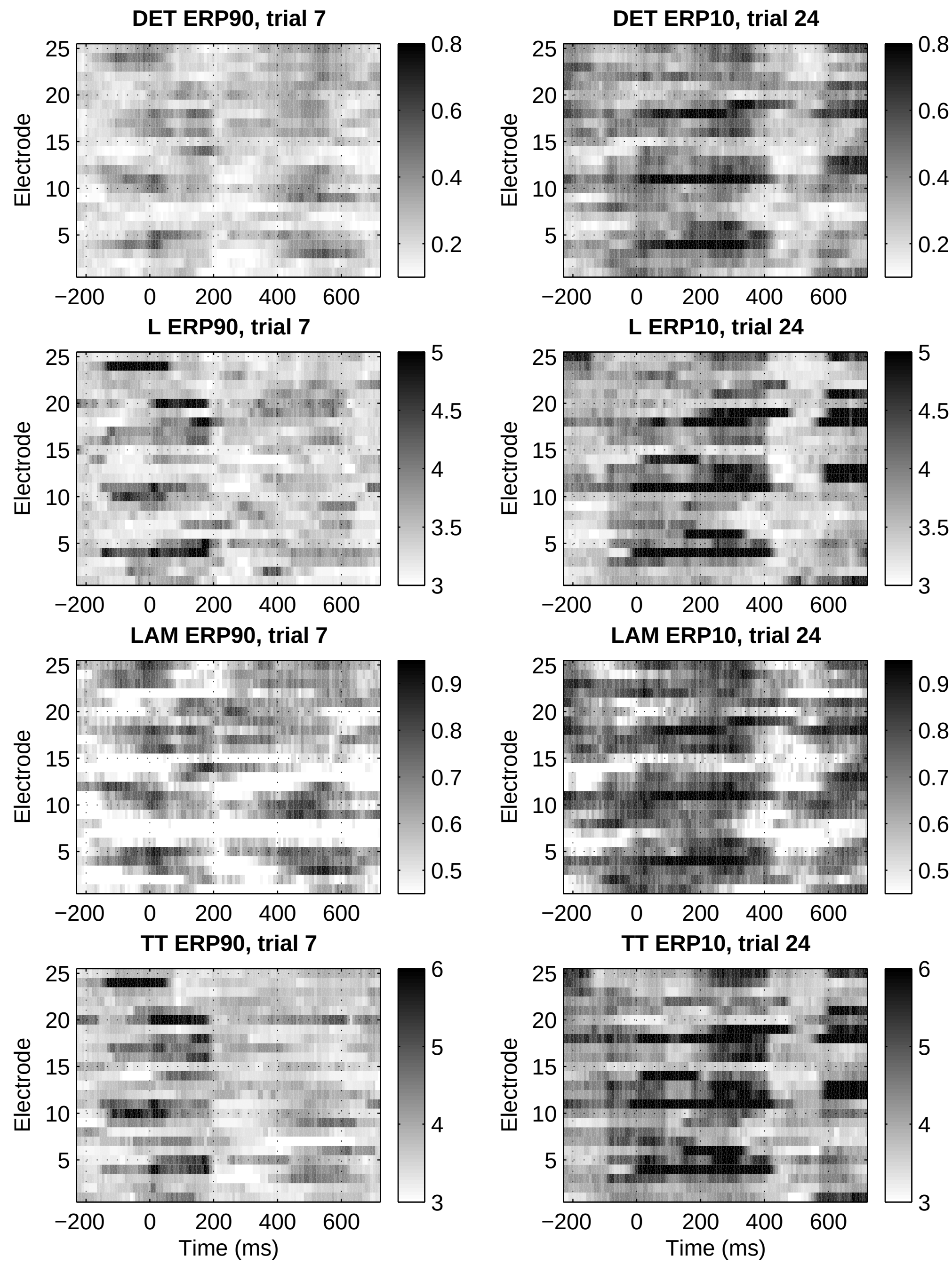

Figure 9. RQA measures for the same trials as in Fig. 8, but shown for all electrodes.

5. Conclusions

From these results we can infer that the application of order patterns is more appropriate in order to study event related potentials on a single trial basis. In comparison with the common recurrence plots, the transition to order patterns has the advantage to reveal more significantly the P300 component and, moreover, differentiates better between single electrodes.

As already found in a previous work, the P300 component is related with specific chaos-chaos transitions where laminar states occur (Marwan and Meinke, 2004). These transitions can also be detected with order patterns. Using OPRPs, these transitions can be localized in the frontal-central and slightly right frontal to parietal regions.

The reliability of this method is currently tested by using EEG data of linguistic experiments Schinkel et al. (subm). A further improvement of this approach could be possible by using a spatio-temporal approach for the reconstruction of the phase space trajectory Mandelj et al. (2001).

Acknowledgments

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