Recurrence Plot Based Measures of Complexity to Predict Life-Threatening Cardiac Arrhythmias

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Abstract — **We present recently introduced new recurrence plot based measures of complexity and illustrate their potential with applications to the logistic map and heart rate variability data. These new measures make the identification of chaos-chaos transitions possible and identify laminar states. The application to the heart rate variability data detects and quantifies the laminar phases before a life-threatening cardiac arrhythmia occurs; thereby facilitating a prediction of such an event.**

1 INTRODUCTION

Numerous scientific disciplines use data analysis techniques to get an insight into the complex processes observed in nature which show generally a nonstationary and complex behaviour. As these complex systems are characterized by different transitions between regular, laminar and chaotic behaviours, the knowledge of these transitions is necessary for understanding the process. Linear approaches of time series analysis are often not sufficient and most of the nonlinear techniques (cf. [4]), such as fractal dimensions or Lyapunov exponents, suffer from the curse of dimensionality and require rather long data series.

To overcome the difficulties with nonstationary and rather short data series, the method of *recurrence plots (RP)* has been introduced [3]. An additional quantitative analysis of recurrence plots has been developed to detect transitions (e. g. bifurcation points) in complex systems [12]. However, these measures can identify only transitions between chaos and order. Therefore, we present here three other measures basing on RPs and demonstrate their potentials for a prototypical nonlinear model and for cardiac data [9].

2 RECURRENCE PLOTS

The method of recurrence plots (RP) was firstly introduced to visualize the time dependent behaviour of the dynamics of systems, which can be pictured as a trajectory $\vec{x}(t) = \vec{x}_i \in \mathcal{R}^n$ ($i = 1, \ldots, N$, $t = i\,\Delta t$, where Δt is the sampling rate) in the n-dimensional phase space [3]. It represents the recurrence of the phase space trajectory to a certain state, which is a fundamental property of deterministic dynamical systems. The main step of this visualization is the calculation of the $N \times N$-matrix

$$\mathbf{R}_{i,\,j} := \Theta(\varepsilon - \|\vec{x}_i - \vec{x}_j\|), \quad i, j = 1 \ldots N,$$

where ε is a cut-off distance, $\|\cdot\|$ a norm (e. g. L_2 or L_∞ norm; in this work the L_2 norm is used) and $\Theta(x)$ the Heaviside function. The phase space vectors for one-dimensional time series u_i from observations can be reconstructed by using the Taken's time delay method $\vec{x}_i = (u_i, u_{i+\tau}, \ldots, u_{i+(m-1)\,\tau})$ with dimension m and delay τ [10], whereby the dimension m can be estimated by using methods basing on false nearest neighbours (cf. [4]). The binary values in $R_{i,\,j}$ can be simply visualized by a matrix plot with the colours black (1) and white (0).

The recurrence plot exhibits characteristic large-scale and small-scale patterns which are caused by typical dynamical behaviour [3, 14], e. g. diagonals (similar local evolution of different parts of the trajectory) or horizontal and vertical black lines (state does not change for some time). Recently introduced extensions to cross recurrence plots use the diagonal structures and their distortions, respectively, for finding similarities and time transfer functions between two different systems [6, 8].

Zbilut and Webber have developed the recurrence quantification analysis (RQA) to quantify an RP [14]. They defined measures using the recurrence point density and *diagonal* structures in the recurrence plot, e.g. the recurrence rate RR (percent recurrences, density of recurrence points), the determinism DET (ratio of recurrence points forming diagonal structures to all recurrence points), the maximal length of diagonal structures L_{max} (or their averaged length L). A theoretical approach to the RQA including the effect of observational noise was recently published by Thiel et al. [11].

Trulla et al. have applied the RQA in order to find transitions in dynamical systems [12]. They have showed, that the RQA measures are able to find transitions between chaos and order (periodical states). But they could not find chaos-chaos transitions, which are very typical in dynamical systems.

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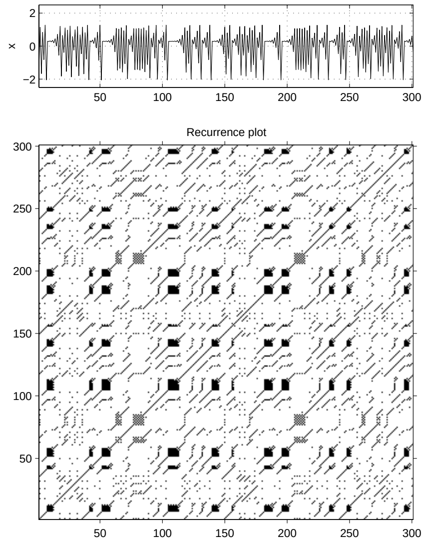

Figure 1: Exemplary recurrence plot of the logistic map for the band merging $a = 3.679$; RP parameters are $m = 1$, $\tau = 1$ and $\varepsilon = 0.1\sigma$.

3 MEASURES OF COMPLEXITY

Therefore, we have recently introduced two additional measures which are based on the *vertical* structures in the RP [9, 7]. We define these measures analogous to the definition of DET and L (and L_{max}), but we consider the distribution $P(v)$ of the length of the vertical structures in the RP.

First, the laminarity LAM

$$LAM := \frac{\sum_{v=2}^{N} vP(v)}{\sum_{v=1}^{N} vP(v)},$$

is the ratio of recurrence points forming vertical structures to all recurrence points and represents the probability of occurrence of laminar states in the system, but it does not describe the length of these laminar phases. It will decrease if the RP consists of more single recurrence points than vertical structures.

Next, the trapping time TT

$$TT := \frac{\sum_{v=2}^{N} vP(v)}{\sum_{v=2}^{N} P(v)},$$

is the averaged length of the vertical structures. The measure TT contains information about the amount and the length of the laminar phases.

Finally, we use the maximal length of the vertical structures in the RP

$$V_{max} := \max\left(\{v_l\, ;\ l = 1, 2, \ldots L\}\right)$$

as a measure, which is the analogue to the standard RQA measure L_{max}.

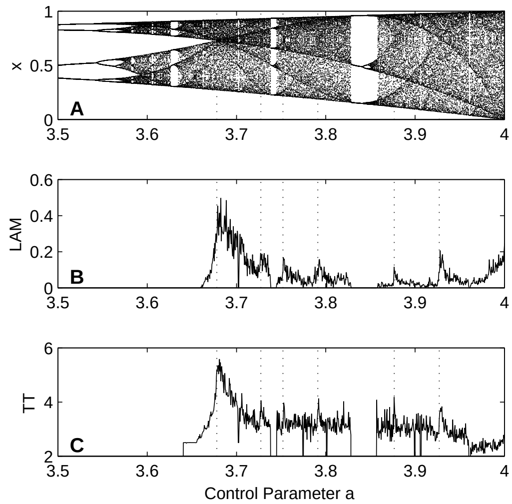

Figure 2: Laminarity (B) and trapping time (C) of time series gained from the logistic map for various control parameters (A). These measures reveal laminar and intermittent states. The vertical dotted lines show a choosing of points of band merging and laminar behaviour (a = 3.678, 3.727, 3.752, 3.791, 3.877, 3.927).

The distinction between these measures and the traditional RQA measures is their ability to find transitions between chaos and chaos [9].

4 APPLICATION TO THE LOGISTIC MAP

In order to illustrate the potentials of LAM, TT and V_{max}, we firstly apply them to the logistic map $x_{n+1} = a\, x_n\, (1 - x_n)$, especially the interesting range of the control parameter $a \in [3.5, 4]$. We generate for each control parameter a a separate time series. In the analyzed range of a various regimes and transitions between them occur, e. g. accumulation points, periodic and chaotic states, band merging points, period doublings, inner and outer crisis [1].

We compute the RP with a cut-off distance of $\varepsilon = 0.1$ (in units of the standard deviation σ); an embedding is not necessary here (i.e. $m = 1$ and $\tau = 1$). The cut-off distance ε is selected to be 10 percent of the diameter of the reconstructed phase space. Smaller values would lead to a better distinction of small variations (e. g. the range before the accumulation point consists of small variations), but the recurrence point density decreases in the same way and thus the statistics of continuous structures in the RP becomes soon insufficient.

For various values of the control parameter a the RPs exhibits specific features (an exemplary RP is

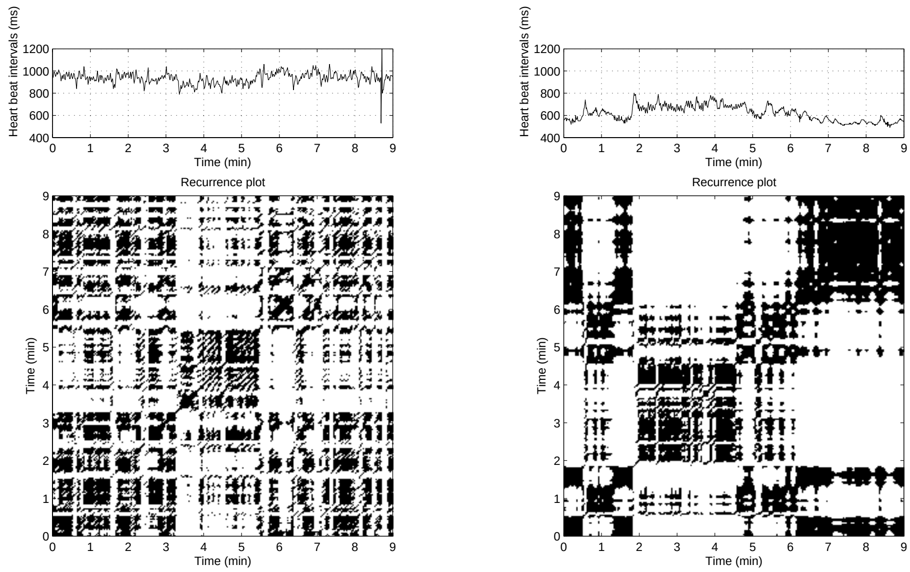

Figure 3: Recurrence plots of the heart beat interval time series at a control time (left) and before a VT (right) with $m = 6$ and $\varepsilon = 170$. The RP before a life-threatening arrhythmia is characterized by big black rectangles whereas the RP from the control series shows only small rectangles.

shown in Fig. 1). Periodic states cause continuous and periodic diagonal lines in the RP but no vertical or horizontal lines. Band merging points and inner crisis represent states with short laminar behaviour and cause vertically and horizontally spread black areas in the RP. Fully developed chaotic states ($a = 4$) cause a rather homogeneous RP with numerous single points and rare short diagonal or vertical lines.

Therefore, the measures LAM, TT and V_{max}, which base on these vertical structures, find the periodic-chaotic/ chaotic-periodic transitions as well as the laminar states (Fig. 2 B, C). Since vertical lines occur much more frequently at inner crisis and band merging points (i.e. laminar states) than in other chaotic regimes, LAM and TT grows up at those points. Although V_{max} also reveals laminar states, it is quite different from the other two measures, because it gives the maximum of all of the durations of the laminar states [9].

Hence, the vertical length based measures yield periodic-chaotic/ chaotic-periodic as well as chaos-chaos transitions (laminar states).

5 APPLICATION TO HEART RATE VARIABILITY DATA

A major challenge in biological physics is the analysis of cardiac time series. Heart rate variability (HRV) typically shows a complex behaviour and it is difficult to identify disease specific patterns. Implantable cardioverter defibrillators (ICD) are a safe and effective treatment for ventricular tachycardia or fibrillation (VT). These fatal cardiac arrhythmias are the main factors triggering sudden cardiac death. A fundamental challenge in cardiology is to find early signs of VT in patients with an ICD based on HRV data (e.g. [2]). Recently studies applied standard methods, methods basing on symbolic dynamics as well as finite-time growth rates to the HRV parameters from time and frequency domain [2, 5, 13, 15].

The defibrillators used in this study are able to store at least 1000 beat-to-beat intervals prior to the onset of VT (10 ms resolution), corresponding to approximately 9–15 minutes. We studied the ICD stored beat-to-beat intervals before the onset of 24 VT episodes and at 24 control intervals without VT in 17 ICD patients of the Franz-Volhard-Hospital with severe congestive heart failure. The beat-to-beat intervals of the VT at the end of the time series were removed from the tachograms so that we analysed only the dynamics occurring immediately prior to VT.

We calculate all standard RQA parameters described in [14] as well as the new measures LAM, TT and V_{max} for different embedding dimensions m and vicinity threshold radii ε. We find differences between both groups of data for several of the parameters mentioned above. However, the most significant parameters are V_{max} and L_{max} for rather large radii (Tab. 1). The vertical line length V_{max} is more powerful in significantly discriminating both groups than the diagonal line length L_{max}, as can be recognized by the higher p-values for V_{max} (Tab. 1). The RP before a life-threatening arrhythmia is characterized by large black rectangles, whereas the RP from the control series shows only small rectangles (Fig. 3).

Table 1: Results of L_{max} and V_{max} shortly before VT and at control time, nonparametric Mann-Whitney U-test, p – significance; * – $p < 0.05$; ** – $p < 0.01$; ns – not significant $p \geq 0.05$)

m	ε	$\langle L_{max}\rangle$ VT	$\langle L_{max}\rangle$ Ctr.	p	$\langle V_{max}\rangle$ VT	$\langle V_{max}\rangle$ Ctr.	p
3	77	396.6	261.5	ns	261.4	169.2	*
6	110	447.6	285.5	*	283.7	179.5	**
9	150	504.6	311.6	*	342.4	216.1	**
12	170	520.7	324.7	*	353.5	215.1	**

6 CONCLUSIONS

We have demonstrated that our new three measures of complexity basing on recurrence plots are able to identify chaos-chaos transitions and epochs of laminar behaviour.

The application of these measures to heart rate variability data, has shown, that they are able to detect and quantify laminar phases before a life-threatening cardiac arrhythmia and, thus, predict such an event [9]. These findings can be of importance for the therapy of malignant cardiac arrhythmias.

A download of a Matlab implementation is available at: www.agnld.uni-potsdam.de/~marwan.

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