



EDITORIAL

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Researchers in different subject areas are tackling similar questions that require a complex systems approach: Can we develop indicators that serve as warning signals for impending regime shifts or critical thresholds in the system behavior? What is the characteristic observation scale that allows for an optimal description of the system dynamics in space and time? How can the resilience (return time to equilibrium) and stability (resistance to external forcing) of a system subjected to disturbance regimes be quantified? Can we derive generalities on how natural and man-made systems develop in time? Amongst interacting units of the system, which ones are the keystones for its global functioning? In this context, recurrence plots are useful tools as they provide a common language for the study of complex systems [Webber, Jr. *et al.*, 2009]. Furthermore, since the method does not require fitting a specific model to data, recurrence plots logically preserve a maximum of the information that is embedded in the sequence of observations. Recurrence plot based methods are also powerful visual and quantitative tools for pattern detection and, as such, continue to birth novel research hypotheses as well as challenge established ones. The ability of these methods to cope with nonlinear and transient behaviors, as well as observational noise is of particular interest to experimentalists and empiricists.

The recurrence plot (RP) was introduced more than twenty years ago in order to visualize the dynamics of complex systems by their recurrences. The first natural extension was the quantification of small-scale structures in an RP by means of recurrence quantification analysis (RQA). RQA has turned out to be a powerful tool for distinguishing different types of dynamics, detecting dynamical transitions, or describing changes in the complexity of spatial objects [Marwan *et al.*, 2007; Marwan, 2008].

Because RPs and RQA have inherent properties that transcend subject areas [Webber, Jr. *et al.*, 2009], recurrence applications keep finding utility in ever expanding scientific fields such as: ecology [Lange, 2011], climate research [Zhao & Li, 2011] neuro and cognitive sciences [Ghomashchi, 2011; Wallot & Van Orden, 2011; Dale *et al.*, 2011], cardiology [Webber, Jr. *et al.*, 2011], molecular sciences [Kulkarni *et al.*, 2011], or economics [Saiki & Yamada, 2011; Crowley & Schultz, 2011]. This is supported by the fact that these methods apply to any sequence of observations: time-series, images, depth and elevation gradients, protein and genome codes, computer and text structures, nested hierarchies, and industrial production chains, to name a few [Marwan, 2008].

This application-driven development of RP and RQA has been enriched in the last decade by fundamental research trying to draw a theory-based link between the heuristic approach of RQA and a system's dynamical properties. Although binary, an RP contains enough information to infer dynamical properties, like existence of unstable periodic orbits (UPOs), correlation dimension, or correlation entropy (which is related to Lyapunov exponents). Under certain conditions, it is even possible to reconstruct the phase-space trajectory of the dynamical system [Marwan *et al.*, 2007]. Moreover, recurrence plots have led to the development of twin surrogates which can be used for statistical tests [Romano *et al.*, 2009]. The idea of twin surrogates has recently inspired new approaches for overcoming the problems of short times series [Komalapriya *et al.*, 2010] or gaps in time series [Facchini & Mocenni, 2011].

Based on the idea of joint recurrences, the detection of synchronization and coupling directions is possible [Marwan *et al.*, 2007; Romano *et al.*, 2007]. However, the question on indirect couplings is generally still a challenging task, to which research on RPs may contribute [Zou *et al.*, 2011].

Nevertheless, there is still a gap between RQA as a heuristic and powerful tool and its theoretical explanation. Similarly important is the knowledge about pitfalls related with RPs and RQA [Marwan, 2011], e.g. the significance of the variation of RQA measures or the choice of the parameters. First approaches for statistical tests have been developed and suggested in the last years [Schinkel *et al.*, 2009; Hirata & Aihara, 2011]. A crucial parameter for construction of RPs is the recurrence threshold. Although, several suggestions appear in the literature (e.g. [Schinkel *et al.*, 2008]), a systematic and theory-based solution to this problem is lacking. More complicated is the fact that different purposes of recurrence based studies need different choices of the recurrence threshold. An alternative solution could be to use a different kind of recurrence criterion, e.g. local minima, as suggested by Schultz *et al.* [2011].

Besides these obvious theoretical implications, the recurrence analysis is beginning to benefit from another theoretical approach: graph theory or complex networks theory. An unweighted and undirected complex network can be described by a binary, square matrix, representing links between nodes. The similarity to the recurrence plot is obvious: we just consider the phase space vectors as nodes and the recurrences as links [Marwan *et al.*, 2009; Donner *et al.*, 2010]. Complex network theory offers a set of powerful measures for the characterization of the topology and structure of the network. We can just apply the complex network measures as new measures of complexity to characterize the RP, hence, the complex dynamics of the phase space trajectory. The first studies in this direction reveal promising new implications for future applications in many fields [Donner *et al.*, 2011; Strozzi *et al.*, 2011].

In 2009, the field lost a giant of a scientist who contributed so much originality and creativity to the study of dynamical systems, and in particular, paved the way for the success of RPs and RQA. Dr. Joseph Peter Zbilut suddenly died an untimely death on Saturday, January 10th, 2009, after shoveling snow at his home on a cold winter afternoon in Skokie, Illinois. Joe was just 15 days shy of his 61st birthday and at the prime of his

career. The *Third International Symposium on Recurrence Plots* in Montréal (Canada) 2009 was dedicated to the memory of Joe Zbilut.

Joe (Rush University Medical Center) and Chuck (Loyola University Medical Center) first met over twenty years ago in 1988 and the two physiologists soon became close friends and strong collaborators on dynamical systems. When asked how to pronounce his last name Joe reminded inquirers to simply say “Spill-It” to keep things easy. Joe was born in East Germany, studied the classics in college, became a Franciscan Friar in the Catholic Church, and earned his first doctorate in Slavic languages (PhD 1973, Northwestern University). In his curriculum vitae under languages mastered, he lists: Russian, Polish, Ukrainian, Old Church Slavic, Czech, French, German, and Latin (with English implied). His second earned doctorate was in mathematical modeling (D.N.Sc. 1987, Rush University). Joe used this medical degree to teach nurse students and treat clinical patients.

Joe and Chuck first studied various physiological systems using Fast Fourier Transforms and methods from chaos, but were never satisfied with outcomes. Everything changed when they turned their attention to the new recurrence plots (RP) of Eckmann *et al.* [1987]. Joe and Chuck expanded the concept of qualitative recurrences by defining variables extracted from RPs. The two investigators coined the term Recurrence Quantification Analysis (RQA) in two key papers [Zbilut & Webber, Jr., 1992; Webber, Jr. & Zbilut, 1994]. As echoed by the cliché, the rest is history.

Joe’s scientific contributions can be summarized into three major categories as captured by his numerous papers and four books. First, Joe was an originator of recurrence quantifications. Second, Joe spearheaded the concept of dynamical singularities in which smooth trajectories are interrupted (stopped) by intervening pauses. Third, Joe explained how complex systems can be derived from very simple building blocks, implying that simplicity underlies complexity. Surely we have lost not only an imaginative and dynamic scientist and colleague, but we have also lost a mentor, debater, lecturer, friend, brother, husband, and father. We all miss you, Joe!

Joe Zbilut would have been thrilled to see this special issue, and the broad range of applications in which recurrence plots are currently being used. Understanding and quantifying the complex dynamics of natural and human systems is one of the key challenges of scientific



Fig. 1. Joe Zbilut, as drawn by a street artist in Rome circa 1997, captures his personality to a tee.

research today. We hope that the work reported here will inspire others to apply recurrence plots to describing and predicting the complexity of an even wider range of complex systems.

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