Understanding the Interrelationship Between Commodity and Stock Indices Daily Movement Using ACE and Recurrence Analysis

Kousik Guhathakurta, Norbert Marwan, Basabi Bhattacharya and A. Roy Chowdhury

Abstract The relationship between the temporal evolution of the commodity market and the stock market has long term implications for policy makers, and particularly in the case of emerging markets, the economy as a whole. We analyze the complex dynamics of the daily variation of two indices of stock and commodity exchange respectively of India. To understand whether there is any difference between emerging markets and developed markets in terms of a dynamic correlation between the two market indices, we also examine the complex dynamics of stock and commodity indices of the US market. We compare the daily variation of the commodity and stock prices in the two countries separately. For this purpose we have considered commodity India along with Dow Jones Industrial Average (DJIA) and Dow Jones-AIG Commodity (DJ-AIGCI) indices for stock and commodities, USA, from June 2005 to August 2008. To analyse the dynamics of the time variation of the indices we use a set of analytical methods based on recurrence plots. Our studies show that the dynamics of the Indian stock and commodity exchanges have a lagged correlation while those of US market have a lead correlation and a weaker correlation.

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1 Introduction

The relationship between the temporal evolution of the commodity market and the stock market is of utmost importance for investors and other market participants. It has long term implications for policy makers, and particularly in the case of emerging markets, the economy as a whole. The history of commodity markets dates back much more than that of the stock market. Traditionally, commodity markets had been primarily consumption markets with some investment opportunities as opposed to the stock market which is an investment market only. The commodity as an option of financial investment and an alternative to traditional assets, had always been attractive because of its ability to add to diversification benefits. However, the majority of investments in commodity markets took place in over the counter markets as opposed to the exchange traded stock markets. The dynamics of the two markets were, therefore, governed by different characteristics. This fact coupled with the lack of consistent and reliable data from the over-the-counter markets has made reliable studies of the interrelationship between the dynamics of the two markets difficult, especially in emerging markets like India where exchange based spot and derivatives trading of commodities is relatively new. But with the evolution of the exchange based trading of commodity spot and derivatives markets it is now possible to study and capture the complex dynamics of both stock and commodity markets and compare their interrelationship. In a recent work by Reddy and Sebastin, the dynamics of information transfer among the commodity spot, commodity derivatives, and stock markets in India are studied, using the information theoretic concept of entropy, which captures complex relationships as well [1].

In spite of having a long history of derivatives trading in commodity markets, the history of exchange traded commodity futures trading in India is rather short. The first commodity trading exchange of India Multi-commodity exchange (MCX) started operating from November 2003. However, since then there has been a phenomenal growth in volume and turnover in this exchange. From a mere 9 million Indian rupees in 2004 the trading volume has increased to over 1 billion Indian rupees in 2013. By 2008 it had already grown by up to 438 million Indian rupees. This growth is not unique to the commodity sector; stock market volume has also continued its growth during the same period. The market capitalisation of the National stock exchange grew from 8.63 trillion to 21.23 trillion Indian rupees between 2003 and 2008. This concurrent growth is of importance from the investment portfolio perspective as the inclusion of commodities adds depth and diversification to the portfolio.

The importance of this study, therefore, lies in investigating the utility of the commodity as a diversification tool. Under normal circumstances, we would expect the correlation between the stock and commodity markets to be low. Only then can commodities be used as an avenue for diversification. But in India, we may expect a different scenario. If we look at traditional investment practices in India, people tend to invest a constant proportion of their portfolios in gold. Thus, when investment goes up, both commodity prices and stock prices will rise, due to the increased complementary demand for these assets. Recently, with the introduction of commodity futures, oil has also taken an important position in the Indian portfolio (incidentally, in 2007, 28 % of the trading volume of commodity futures in MCX was contributed by gold, while 56 % was contributed by crude oil). We would thus expect a different relationship to exist between the stock and commodity markets in India when compared to some other developed economy like that of the USA.

The complex dynamics of two investment markets is best understood by analyzing a time series representing the price movement of the respective markets using tools of nonlinear dynamics. Keeping this in mind we analyze the movement of the daily close value of two indices of stock and commodity exchange, respectively, of India. To understand whether there is any difference between emerging markets and developed markets in terms of the dynamic correlation between the two market indices, we compare it with the US market. We use a set of methods based on maximal correlation [2] and recurrence plots [3, 4].

A simple cross-correlation study based on Pearson correlation measures could indicate a degree of correlation between the time series under study. To start with, we will compute the Pearson correlation for both US and Indian markets. Despite the true nature of the dynamics, this linear approach might yield some initial interesting results in a first order approximation. However, in order to get deeper insights we have to consider potential nonlinear properties, because there is ample empirical evidence against the assumption of simple linear dynamics in economics. Theoretically, there is no reason to believe that economic systems must be intrinsically linear (cp. [5–7]). Empirically, a great number of studies show that financial time series exhibit nonlinear dependencies (cp. [8–16]). Hence, "A natural frontier for financial econometrics is the modelling of nonlinear phenomena" [6]. Testing for nonlinearity has become popular in the financial econometrics literature in recent years, though the focus is on financial markets of developed countries. In principle, testing for nonlinearity can be viewed as a general test of model adequacy for linear models [17] and it has been argued that if the underlying generating process for a time series is nonlinear in nature, then it would be inappropriate to employ linear methods. For instance, most of the widely applied statistical tests like the unit root or stationary tests, the Granger causality test, and the cointegration test are all built on the basis of a linear autoregressive model. [18, 19], among others, illustrated that the adoption of linear stationarity tests are inappropriate in detecting mean reversion if the true data generating process is in fact a stationary nonlinear process. On the other hand, the Monte Carlo simulation evidence in [20] indicated that the standard linear cointegration framework presents a mis-specification problem when the true nature of the adjustment process is nonlinear and the speed of adjustment varies with the magnitude of the disequilibrium. Thus, if the underlying process of a time series is indeed nonlinear in nature, we would have to resort to empirical methods the like non-parametric cointegration test due to [20], nonlinear stationarity tests [19, 21, 22], and nonlinear causality tests [23].

This requires the application of alternative methods over and above simple Pearson correlation measure to understand the degree of nonlinear correlation between the time series. We used the alternating conditional expectation algorithm (ACE) to test the correlation between the time series [24]. The ACE algorithm showed that the functional form of the data sets under examination are clearly nonlinear. This encouraged us to use nonlinear methods to study the interrelationship between the time series. This also motivated us to go for a study of the entire time evolution of the time series to understand whether they co-evolve or not, by using the cross recurrence plot and recurrence quantification analysis [3, 4], which has already been successfully applied to financial and economic data [16, 25, 26]. Recent extensions in recurrence network analysis allow us to estimate topological dimensions from time series like the transitivity dimensions [27]. Motivated by this, we construct the transitivity dimensions of the two markets and see whether topological measures reinforce our findings in recurrence analysis.

The structure of the paper is as follows. In Sect. 2, we briefly describe the source and nature of the data. In Sect. 3 we discuss all the tests performed, giving the background theory of our analysis before commenting upon the results. In Sect. 4 we perform the comparative analysis of the results of our tests. Finally we summarise our conclusions in Sect. 5.

2 Data

Our analysis is based on daily time series of the S and P CNX NIFTY (NIFTY) and MCX-COMDEX (commodity) index of India as well as Dow Jones Industrial Average (DJIA) stock index and Dow Jones-AIG Commodity Index (DJ-AIGCI) of USA (Fig. 1). From the respective exchange web sites (www.nseindia.com, www.mcxindia.com, www.djaverages.com, www.nasdaq.com/symbol/ucd) historical data respectively for NIFTY, NCX, DJIA and DJ-AIGCI have been collected for the period from June 2005 to August 2008 (both months inclusive). Considering this time period, the sub-prime period is not completely excluded from the analysis. The initial few months when the first shock affected the market are still included, but the long period of global recession after this initial period are not. The idea was to capture the beginning of the stock market crash and see whether the commodity market was reacting in a correlated manner. Our purpose was to understand whether there is anything endogenously different in the two markets. The S and P CNX Nifty is a well-diversified 50 stock index (traded in the National Stock Exchange, India) accounting for 25 sectors of the Indian economy. MCX-COMDEX is a composite futures index comprising of commodity futures of diversified sectors traded in MCX India. MCX-COMDEX is based on futures prices of 15 different commodities, comprising of three sub-indices which represent the major sector groupings in commodity trading: metals, energy, and agricultural products. DJIA is a composite index computed from stock prices of 30 largest and most widely held public companies in the USA. DJ-AIGCI is a composite index composed of future contracts on 19 physical commodities traded on US exchanges.

For our analysis, we have used the z-score of the original index time series, i.e., we have normalized all time series to have a mean of zero and standard deviation of one (no log change or change data used).

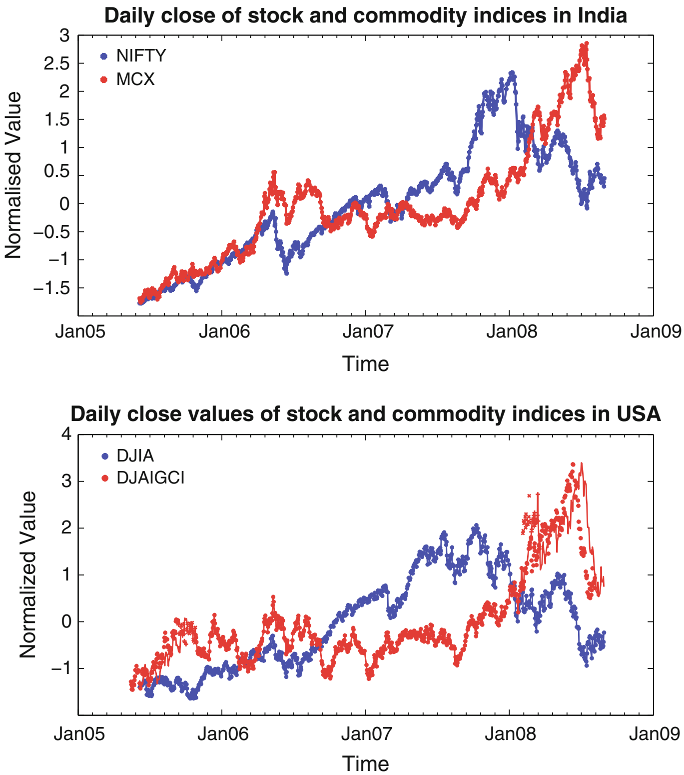


Fig. 1 Normalized values of daily close of NIFTY and MCX-COMDEX (*top*), as well as of daily close of DJIA and DJ-AIGCI (*bottom*)

3 Background of Examinations

3.1 Alternating Conditional Expectation (ACE) Algorithm and Maximum Correlation Function (MCF)

The ACE algorithm [24] estimates the transformations $\Phi(x)$ and $\Theta(y)$ giving rise to the maximal multiple correlation of a response y and a set of predictor variables x:

$$MC = \frac{\langle \Phi(x)\Theta(y)\rangle}{\sqrt{\langle \Phi^2(x)\rangle\langle \Theta^2(y)\rangle}} \stackrel{!}{=} \max\,. \tag{1}$$

These real-valued measurable mean-zero functions $\Theta(y)$ and $\Phi(x)$ (with $\stackrel{!}{=}$ max meaning that the correlation is maximal for the found $\Theta(y)$ and $\Phi(x)$) are then called *optimal transformations*, and *MC* is called *maximal correlation*. A study of these transformations can give the insight into the relationships between these variables.

To calculate the optimal transformations, we have used an adaptive partitioning algorithm as described in [2]. The ACE and MCF functions of the CRP toolbox [28] use this algorithm except for two differences:

1. The output is not normalized with respect to the mean values of the optimal transformations, so the mean values may not necessarily be zero.
2. The data are rank ordered before the calculation of the optimal transformations. This leads to a simpler computation of conditional expectation values.

The algorithm (but not the particular subroutine of estimating the conditional expectation values) is described in [2, 29, 30].

Although this algorithm is meant for exploring whether there is a relationship between response and predictor variables, we have used the stock and commodity index data as if a predictive relationship exists between them, and then found whether in such a case the programme detects a correlation or not.

3.2 Recurrence Analysis

Natural processes can have a distinct recurrent behavior (e.g., Milankovich cycles, El Niño phenomenon, extreme flooding events, epileptic seizures). Recurrence of states $\mathbf{x_i} \in \mathbb{R}^m$ (with m the dimension of the phase space), in the meaning that states are arbitrary close after some time, is a fundamental property of deterministic dynamical systems.

Eckmann et al. have introduced a tool which visualizes the recurrence of states $\mathbf{x}_i$ in phase space [3]: the recurrence plot. A recurrence plot (RP) is a visualisation of state- space dynamics that shows all those times at which a state of the dynamical system recurs:

$$R_{i,j} = \Theta(\varepsilon - \parallel \mathbf{x}_i - \mathbf{x}_j \parallel), \quad \mathbf{x}_i \in \mathbb{R}^m, \quad i, j = 1, \ldots, N, \tag{2}$$

where $\mathbf{R}$ is the recurrence matrix, N is the number of considered states x_i, ε is a threshold distance, $\parallel \cdot \parallel$ a norm, and $\Theta(\cdot)$ the Heaviside function. A recurrence of a state at time i at a different time j is, thus, marked within a two-dimensional squared matrix with ones and zeros. Both axes of the recurrence matrix are time axes. This representation is called recurrence plot (RP). RPs has shown to be useful for analysing short and non-stationary data [4].

In our study we apply the RP in order to reveal the characteristics of the dynamics of the economic time series under investigation. For an economic time series, the patterns over time tell us whether the series is disrupted, non-stationary or nonlinear in nature. By comparing RPs of two economic time series we can visually infer whether the dynamical systems governing the time series are similar, or not.

3.2.1 Embedding Parameters

If only one observable is available, the phase space can be reconstructed using time-delay embedding [4]. Thus, we need to choose an appropriate value for the time delay d and the embedding dimension m. Several methods have been developed to best estimate m and d. Frequently used methods are the Average Mutual Information Function (AMI) for the time delay [31] and the False Nearest Neighbors (FNN) method for the embedding dimension [32]. As for the embedding delay, we chose such a value where the mutual information has its first minimum or changes its scaling behavior, and for the embedding dimension, we use such a value for m where the number of false nearest neighbours in the phase space vanishes.

3.2.2 Structures in Recurrence Plots

The initial purpose of RPs was the visual inspection of recurrences of phase space trajectories. The view on RPs gives hints about the time evolution of these trajectories. RPs exhibit characteristic large scale *typology* and small scale patterns (*texture*). The typology offers a global impression which can be characterized as homogeneous, periodic, drift, and disrupted [4]. Small scale structures are single dots, diagonal lines as well as vertical and horizontal lines (the combination of vertical and horizontal lines obviously forms rectangular clusters of recurrence points). For a recurrence analysis, the diagonal and vertical line structures are important.

A diagonal line $R_{i+k,j+k} = 1$ (for $k = 1, \ldots, l$, where l is the length of the diagonal line) occurs when a segment of the trajectory runs parallel to another segment, i.e., the trajectory visits the same region of the phase space at different times. The length of this diagonal line is determined by the duration of such similar local evolution of the trajectory segments and can give an idea about its divergence behavior, i.e., the faster the trajectory segments diverge, the shorter are the diagonal lines.

A vertical (horizontal) line $R_{i,j+k} = 1$(for $k = 1, \ldots, v$, where v is the length of the vertical line) marks a time length in which a state does not change or changes very slowly. It seems, that the state is trapped for some time. This is a typical behavior of laminar states (intermittency).

These small scale structures are the base of a quantitative analysis of the RPs.

Though the visual interpretation of RPs requires some experience, their quantification offers a more objective way for the investigation of the considered system. A detailed discussion on the application and interpretation of RPs and the various structures in a RP can be found in [4].

3.2.3 Cross Recurrence Plot

The cross recurrence plot (CRP) is a bivariate extension of the RP and was introduced to analyze the similarity and synchronization of the states of two different dynamical systems [33]. Suppose we have two dynamical systems, each one represented by the trajectories $\mathbf{x}_i$ and $\mathbf{y}_i$ in a d-dimensional phase space. Analogously to the RP, Eq. (2), the corresponding cross recurrence matrix is defined by

$$CR_{i,j}^{\mathbf{x},\mathbf{y}}(\varepsilon) = \Theta(\varepsilon - \|\mathbf{x}_i - \mathbf{y}_i\|), \quad i = 1, \ldots, N, \quad j = 1, \ldots, M \tag{3}$$

where the length of the trajectories of $\mathbf{x}_i$ and $\mathbf{y}_i$ are not required to be equal, and hence the matrix **CR** is not necessarily square. Note that both systems are represented in the same phase space, because a CRP looks for those times when a state of the first system recurs to one of the other system. If the embedding parameters are estimated from both time series, but are not equal, the higher embedding should be chosen. However, the data under consideration should be from a comparable process (physically the same). Here, we consider the different markets as physically very similar. Moreover, we do not compare a specific variable (like trading volume of a specific good) from these markets but a generalized index value. This additionally supports our consideration of using physically similar variables in our application. For a detailed discussion on RPs and CRPs we refer to [4, 33].

The CRP of two identical trajectories coincides with the RP of one of the trajectories and contains the main diagonal or *line of identity* (LOI). However, if the trajectories are not equal or their evolution happens on different time scales, the LOI will be somewhat displaced, disrupted or bowed and is called *line of synchronisation* (LOS).

For our analysis, we use the CRPs to visually inspect the interrelationship between the two economic time series under investigation. By looking at the pattern, i.e., the LOS, we can infer whether the two systems are completely uncorrelated or a relationship exists between them with some lead or lag. If the LOS is shifted to the right then we can conclude that there is a delayed relationship between the two time series. The other possible bivariate extension of RPs, the joint recurrence plot, is not applicable here because it tests for simultaneous recurrences, but we are also interested in changes of time scales. The potential of recurrence plot based approaches for analyzing financial and economics data was shown in [16, 25, 26].

3.3 Quantification of Recurrence Plots

A quantification of recurrence plots (Recurrence Quantification Analysis, RQA) was developed in order to distinguish between different appearances of RPs [34, 35]. Measures which base on diagonal structures are able to find chaos-order transitions, whereas measures based on vertical (horizontal) structures are able to find chaos-chaos transitions (laminar phases) [4].

Using the histogram of diagonal line lengths (see Sect. 3.2.2), we define the fraction of recurrence points forming diagonal lines as a measure called *determinism* DET,

$$DET = \frac{\sum_{l=l_{\min}}^{N} l\,P(l)}{\sum_{l=1}^{N} l\,P(l)}, \tag{4}$$

where $P(l)$ is the histogram of the diagonal lines of exactly length l, and $l_{\min}$ is a minimal length a diagonal structure should have to be counted as a line. Processes with uncorrelated or weakly correlated, stochastic or irregular chaotic behaviour cause none or very short diagonals, hence, small DET. In contrast, regular deterministic processes lead to longer diagonals and less isolated recurrence points, resulting in higher values of DET. This measure can also be interpreted as characterizing the predictability of the system.

The *average diagonal line length*

$$L = \frac{\sum_{l=l_{\min}}^{N} l\,P(l)}{\sum_{l=l_{\min}}^{N} P(l)} \tag{5}$$

gives the average time that two segments of the trajectory are close to each other, and can be interpreted as the mean prediction time.

Analogously to the definition of the determinism in Eq. (4), we can use the histogram of the vertical lines of exactly length v, and define the fraction of recurrence points forming vertical structures in the RP as the *laminarity LAM*

$$LAM = \frac{\sum_{v=v_{min}}^{N} vP(v)}{\sum_{v=1}^{N} vP(v)}. \tag{6}$$

The computation of LAM is realized for those v that exceed a minimum length v_{min} in order to decrease the influence of the tangential motion (time-continuous systems that are discretized with sufficiently high sampling rate and an appropriately large threshold ε result in a large amount of recurrences coming from succeeding states $\mathbf{x}_i, \mathbf{x}_{i+1}, \mathbf{x}_{i+2}, \ldots$). LAM represents the occurrence of laminar states in the system without describing the length of these laminar phases. In particular, LAM decreases if the RP consists of more isolated recurrence points than vertical structures.

The average length of vertical structures is given by

$$TT = \frac{\sum_{v=v_{min}}^{N} vP(v)}{\sum_{v=v_{min}}^{N} P(v)}, \tag{7}$$

and is called *trapping time*. As in the case of LAM, the computation of TT requires the consideration of a minimal length v_{min} as well. The trapping time estimates the mean time that the system will abide at a specific state, i.e., how long the state will be trapped.

Both LAM and TT have been proven to be useful for describing the dynamics of discrete systems and studying chaos-chaos transitions. RQA consists of further measures which are not used in this study. RQA as the whole is a very powerful technique for quantifying differences in the dynamics of complex systems and has meanwhile found numerous applications, e.g., in astrophysics, biology, engineering, geo- and life sciences, or protein research [35].

Recent developments combined recurrence analysis with the complex network approach [36, 37]. By considering the recurrence plot $\mathbf{R}$ as the adjacency matrix of a network $\mathbf{A} = \mathbf{R} - \mathbb{1}$, several measures from complex networks statistics can be applied and used as alternative measures of complexity characterizing the geometrical properties of the phase space trajectory. For example, the *transitivity coefficient*,

$$\mathscr{T} = \frac{\sum_{i,j,k=1}^{N} A_{j,k} A_{i,j} A_{i,k}}{\sum_{i,j,k=1}^{N} A_{i,j} A_{i,k} (1 - \delta_{j,k})}, \tag{8}$$

measuring the fraction of closed triangles in the network, is a good measure to distinguish regular from irregular dynamics [38]. Based on geometric considerations, $\mathscr{T}$ can be used to construct a dimensionality measure, the *transitivity dimension*,

$$D_{\mathscr{T}} = \frac{\log \mathscr{T}}{\log(3/4)}, \tag{9}$$

providing a theoretically understandable measure for complexity, as more complex/irregular behaviour belongs to higher dimensional dynamics than periodic/regular behavior [39].

RQA measures can be computed in moving windows along the main diagonal (sub-RPs). This allows us to study their time dependence and can be used for the detection of transitions. Yet, one key question in empirical research concerns the confidence bounds of the calculated RQA measures. Schinkel et al. have suggested a bootstrap method to estimate the confidence of the RQA measures [40]. This method is based on the bootstrapping of line structures from the RP (or the sub-RP), allowing to estimate an empirical test distribution of all of the used RQA measures. We have used 95 % confidence level for the statistical evaluation of these measures.

The measures *DET*, *LAM*, *L*, and *TT* are not used as absolute indices of the dynamic state (i.e., chaotic, random, laminar, etc.). Instead we will consider their relative movement over time when comparing the two systems,i.e., the stock market index and commodity market index. By comparing their movement, we try to detect whether they move concurrently or absolutely independent of each other.

All analysis was performed by using the *Cross Recurrence Plot Toolbox*, Version 5.15 (R28.4) 21-Jul-2009 (http://tocsy.pik-potsdam.de).

4 Results

First, we compute the Pearson correlation for the two markets separately. We find that the correlation between DJIA and DJ-AIGCI indices were much lower (0.23) as compared to that between NIFTY and MCX (0.62). Next we proceed with the nonlinear analysis in order to get further details of the nature of correlation. We have calculated the MCF for a maximal lag of 20 days (the length of the boxcar window was 11). For the calculation of the CRPs, the important parameters are embedding dimension and time delay. If the embedding parameters are estimated from both time series, but are not equal, the higher embedding was chosen [4]. Using the methods mentioned in Sect. 3.2.1, we got the same parameters for both Indian data sets, i.e., we found an embedding dimension of 5 and a time delay of 4. For the US data set we used the same embedding parameters because those were the higher embedding parameters amongst the two time series, viz., the DJIA time series. We used the CRP and maximum norm method for finding out the neighbours of the plot. For the RQA we used the same embedding parameters and a threshold parameter of 0.1, kept the bootstrapping sample size at 500, used a 95 % level of confidence. The used window size was 100 days and step size was 10 days.

4.1 ACE and Maximum Correlation Function

A close look at the ACE and MCF results of Nifty and MCX time series (Fig. 2), and those of DJ-AIGCI and DJI, respectively (Fig. 3), reveals a clear difference in the two markets. The optimal transformations for Nifty and MCX time series are a monotonous function, with a linear part for low values (Fig. 2). In contrast, the optimal transformations for DJ-AIGCI and DJI reveal a non-monotonous and

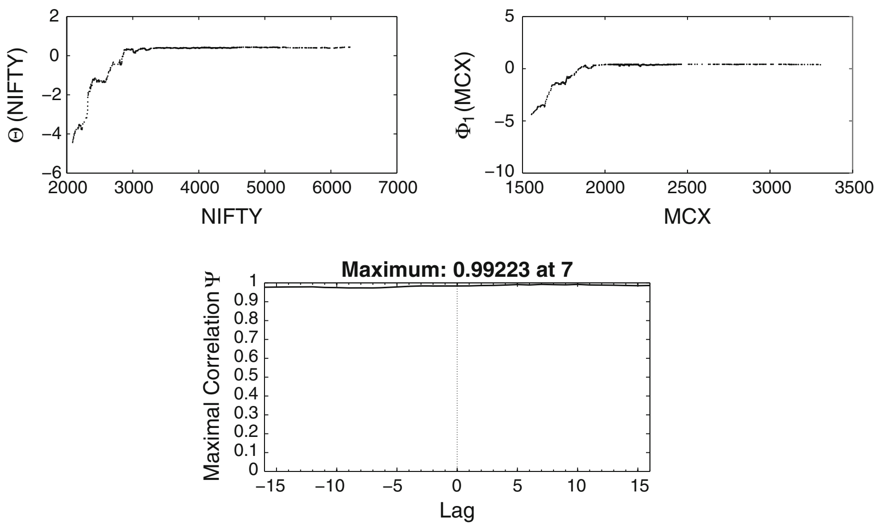


Fig. 2 Optimal transformations for NIFTY and MCX-COMDEX (*top*). Maximal correlation function for NIFTY and MCX-COMDEX (*bottom*) with rather constant value of 0.99, indicating a high level of nonlinear correlation

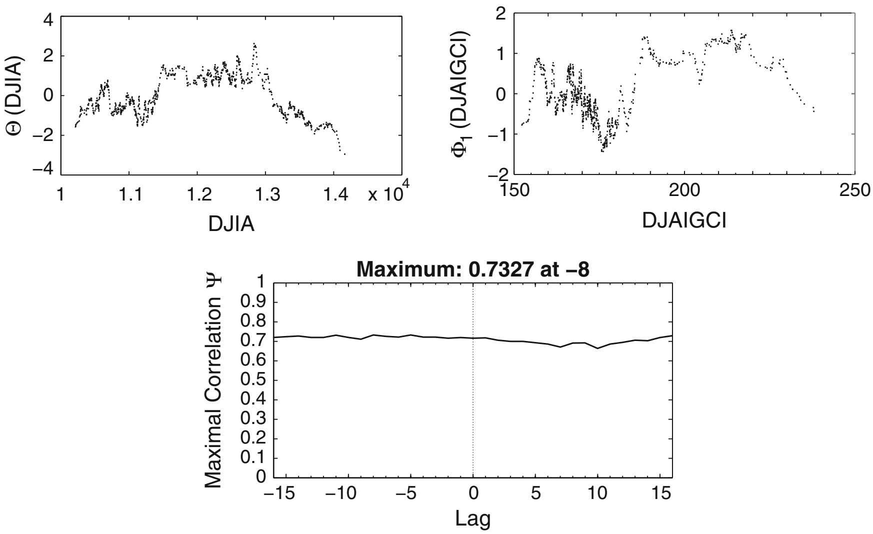


Fig. 3 Optimal transformations for DJIA and DJ-AIGCI (*top*). Maximal correlation function for DJIA and DJ-AIGCI (*bottom*) with rather constant value of 0.73, indicating a rather law correlation in the US markets ("low" from the view point of the maximal correlation)

strongly nonlinear function (Fig. 3). For a lag of seven days, we found a maximum correlation of 0.98 for Nifty and MCX-COMDEX, while for the DJIA and DJ-AIGCI the maximum correlation is only 0.70, at a lead of eight days. These results suggest a simpler relationship between the commodity exchange index and stock exchange index of India, which is also confirmed by its strong correlation, whereas the US market is much more complex and unpredictable, i.e., much weaker correlation in the US market (as the maximum correlation is significant only for very high values, i.e., larger than 0.95). This points towards a greater interrelation within the Indian markets. In India the lagged relationship suggests the commodity market follows the stock market. Since the maximum correlation in the US is not significant we are not too concerned about the nature of such correlation, i.e., lead or lag.

Next we look for a lagged maximal correlation (MCF). We have found a maximum in the MCF at lag 7 for the stock and commodity market in India (Fig. 3). This suggests a delayed relationship between the markets. The low (non-significant) values of MCF for the US market do not allow a conclusion about delayed relationships (Fig. 4).

We now proceed with the recurrence analysis of the data set to capture the relative changes of the dynamics of the respective indices as also to find out the interrelationship between the indices.

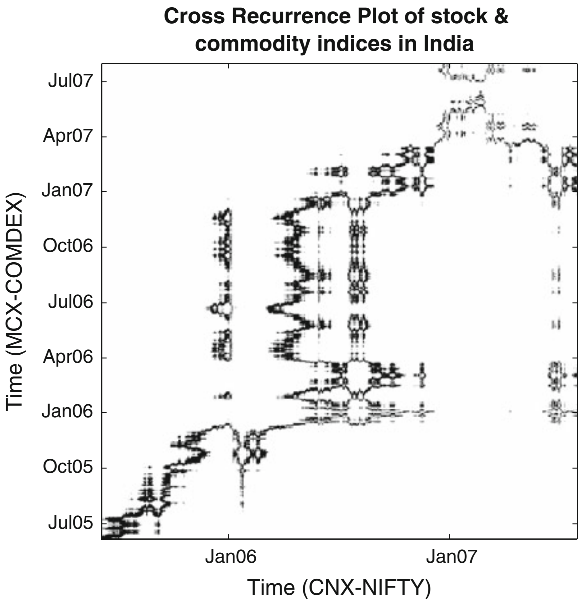


Fig. 4 Cross Recurrence Plot of time series representing daily close values of the stock market index CNX-NIFTY and commodity market index MCX-COMDEX in India

4.2 Cross Recurrence Plot

The CRP of NIFTY and MCX-COMDEX reveal a pattern which suggests a relationship between these two indices for India (Fig. 4). The pattern indicates partly a co-evolution of the two time series, as indicated by the connected structure of low values in the CRP, which we consider to be the LOS. The shifting of the LOS is an indication of a lagged relationship. The LOS is changing with time, suggesting that the relationship between the stock and commodity markets is not constant. The CRP contains two disruptions at May 2006 and January 2008 which correspond to stock market crashes.

In contrast, the CRP of the DJIA and DJ-AIGCI does not show a well connected structure of low distance values, which could be interpreted as a LOS (Fig. 5). Only between March 2007 and January 2008, some lagged relationship seems to be apparent. This suggests a weaker relationship between the NIFTY and MCX-COMDEX what is in line with the correlation results.

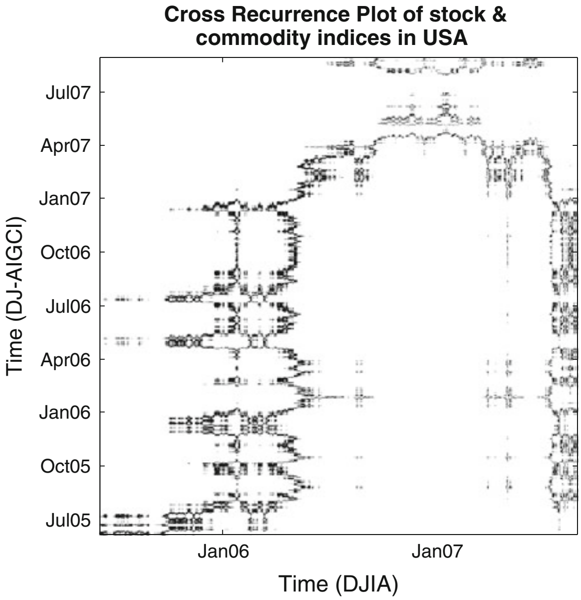


Fig. 5 Cross Recurrence Plot of time series representing daily close values of the stock market index DJIA and commodity market index DJ-AIGCI in the USA

4.3 Recurrence Quantification Analysis

Next we compare the time variation of the RQA measures in order to check for similar dynamics. The variations of the *DET*, *L*, *LAM*, and *TT* values of the index data reveal changes between different dynamics, e.g., from more predictable to less predictable (DET). Such dynamical changes in the NIFTY data are well concurrent to that of the MCX-COMDEX data but with a small lag (Fig. 6). The values, though not identical in absolute terms, are increasing and decreasing in a similar fashion, except during March 2008, where the two time series appear to be out of sync. This means that the respective change of states of the two systems, i.e., the corresponding markets' dynamics, is closely related to each other. If one looks carefully at the DET values, one will notice that the DET values for both NIFTY and MCX are almost concurrent during July 2005 to October 2006, and also June 2007 onwards. We also find similar regime specific behaviour in other RQA variables as well. The concurrence departs in 2006 after a crash in the stock markets and again reappears when the market rebounds. We can infer that the stock market crash affects the two markets differently, with the commodity market recovering faster than the stock market, therefore leading to the departure of the concurrence.

The values of *DET*, *L*, *LAM*, and *TT* of the DJ-AIGCI and DJI data reveal a much less synchronised variation of the dynamical properties than in the Indian markets (Fig. 7). There are more epochs when the two values are out of sync or even negatively

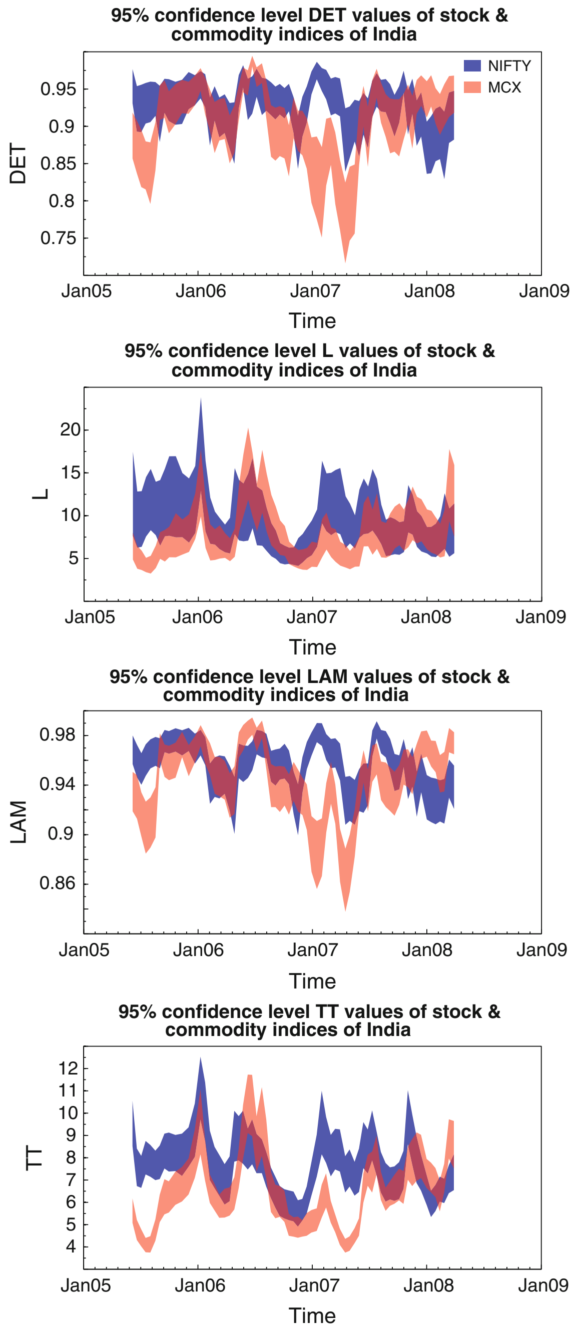
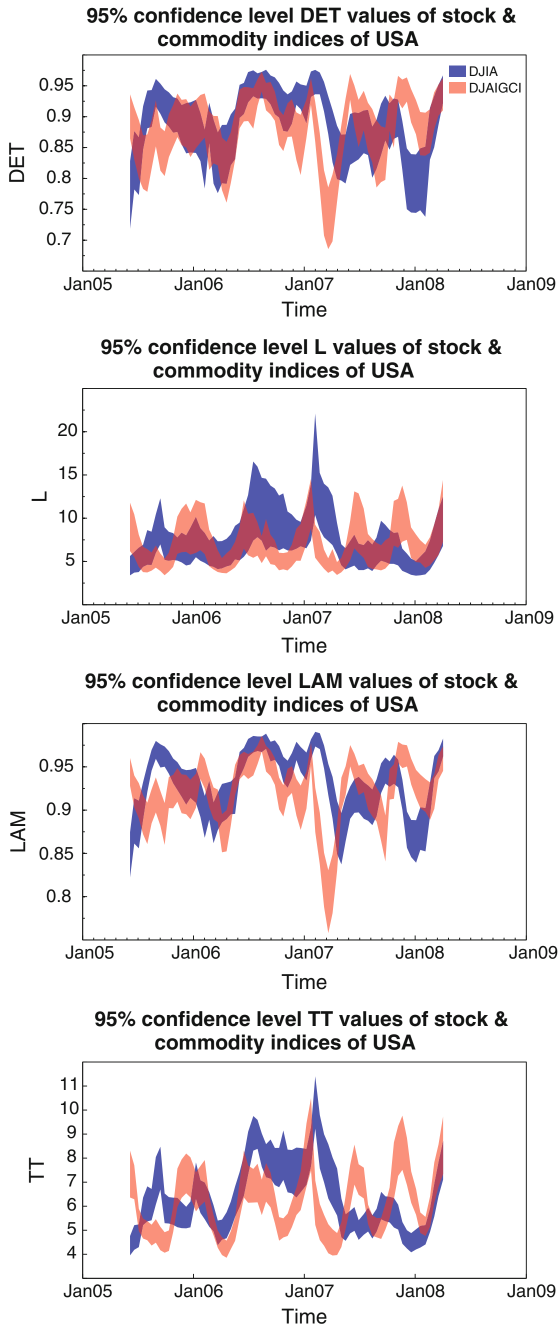


Fig. 6 *DET*, *L*, *LAM*, and *TT* values with 95 % confidence limits for time series representing daily close values of the stock market index CNX-NIFTY and commodity market index MCX-COMDEX in India. The values reveal changes between different dynamics like degree of predictability. We can see how the two indices are concurrently changing in their dynamics except for March 2008

Fig. 7 *DET*, *L*, *LAM*, and *TT* values with 95 % confidence limits for time series representing daily close values of the stock market index DJIA and commodity market index DJ-AIGCI in USA. The values show that the US indices are significantly different in their dynamical properties



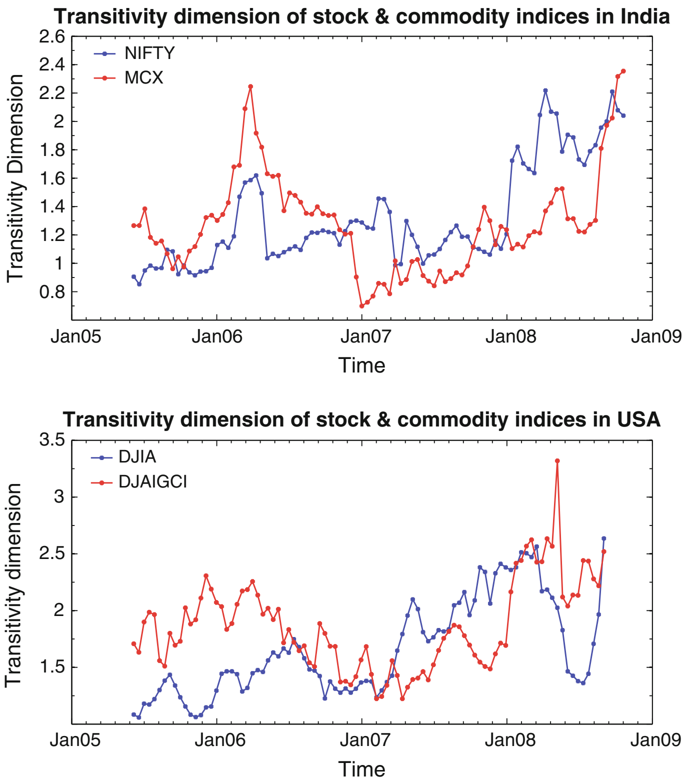


Fig. 8 Transitivity Dimensions of the commodity and stock market indices

correlated. This suggests a weaker link between the dynamics of the two markets and a significant difference in the predictability of the stock and commodity markets. Pearson correlation between the RQA values of NIFTY and MCX ranges between 0.25 and 0.55, while for DJIA and DJ-AIDCI we find correlation between 0.05 and 0.1 only. These findings corroborate our conclusions on the closer interrelation between the Indian exchanges than the US exchanges.

We conclude our analysis by comparing the transitivity dimension of the chaotic dynamics of the two markets. We find two epochs of higher transitivity dimension of the daily close of MCX, one between June 2005 and November 2006 and another after July 2008, with a decrease between November 2006 and July 2008, coinciding with a similar evolution of the close values of NIFTY, the stock market index of India (Fig. 8). The evolution of the transitivity dimensions of daily close of DJIA, the US stock market index, and of DJAIGCI, the US commodity market index, are not as similar as for the Indian counterpart, although we find a decrease between October 2006 and January 2008. In comparison to the Indian markets, the higher values of transitivity dimension in the US indices indicate a more complex/ irregular dynamics.

5 Conclusions

In the research presented, we have compared the variability of stock and commodity markets in India and US and found clear differences between the Indian and the US market behavior. We have used a nonlinear approach to measure correlation based on the alternating conditional expectation (ACE) algorithm and maximum correlation function (MCF). We found that the correlation between the Indian Markets is much stronger than in the US. While the maximal correlation between the stock and commodity markets in India is quite high, the correlation between the US stock and commodity markets is low and negligible. The relationship for the Indian markets is lagged by seven days. Furthermore, we have applied the recurrence plot analysis to look at different aspects of the dynamics. Based on cross recurrence plots, we found distortions in the link between the stock and commodity markets during stock market crashes and when the relationship between the stock and commodity markets is changing or diverging (in terms of lags). The recurrence quantification analysis (RQA) suggested a concurrence between the Indian stock and commodity markets in terms of coinciding predictability while these markets in the US were mostly out of sync. These findings are supported by transitivity dimension which reflects the changes between regular and complex market behavior, which coincides in the Indian markets, but is more divergent in the US markets.

From these findings we can infer that the Indian and the US markets behave differently. In India both markets are probably linked by external factors, like global market behavior, which influences the economic state of India in all sectors, i.e., in stock exchange and commodity markets in a similar way. There is a long tradition in India of investing in metals, particularly gold, in the form of ornaments and jewellry. Such dependance on commodities as a constant source of hedging has created an investment psychology that could have driven the Indian investors to invest a fixed proportion of their wealth always in commodity futures. This has resulted in concurrent investments in stock and commodity leading to stronger correlation. One reason for the lagged relationship could be that after every initial boom in the stock market, investors start piling up a portion of their wealth in commodity and vice versa. In India, since people have routes to diversify in commodity markets through informal channels like ornaments and utensils, they may treat the commodity futures exchange as an alternative to the informal market. That is why they seem to invest in both stock and exchange traded commodities concurrently.

Globally two commodities, namely oil and gold, play important roles in portfolio management. In the case of gold we already see that the Indian market has reasons to behave differently than US market. We can also find from investment trends that oil also, rather than the entire commodity futures, has a similar place in the investment portfolios of India, which means that whenever investment goes up, it goes up almost simultaneously both in stock and commodity markets. Finally, it may be inferred that exchange traded commodities may not be a useful diversification avenue for investors in India as yet. However, the exchange traded commodity market is relatively new in India. The dynamics will probably undergo a change with time as the market in India further develops.

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