Quantification Analysis of Time Series Based on 2D Visibility State Plot*

Chunyu Zhao, Ming Zeng, Erhong Wang, Jingjing Han, Qinghao Meng

Abstract — Time series analysis is of significant importance with various engineering applications. Inspired by the idea of recurrence quantification analysis (RQA), a novel quantification time series analysis method based on 2D visibility state plot is proposed. In particular, if the two arbitrary data satisfy the visibility criterion, and consequently become two connected nodes of the associated visibility state plot (VSP), Next, In order to quantitatively analyze the VSP, six measures, i.e., visibility rate (VR), degree of certainty (DC), mean diagonal length (DL), visibility entropy (VE), layered (LAY), vertical/horizontal line length (VHL), are calculated. Finally, to verify the effectiveness of the proposed method, we compare the performance of our approach to the RQA method using wind speed signals of two different environments. The results show that the quantitative measures of VSP can distinguish indoor and outdoor signals, but the classical RQA method fail to work.

I. INTRODUCTION

Time series refer to the sequence of the same statistical index values in the order of their occurrence time. It is the main external manifestation of working status of many complex systems, and contains rich information of system structure and dynamic evolution rules. An effective tool of time series analysis is crucial to a variety of scientific and technical applications ranging from meteorology [1-3], economics [4] to medical science [5-8]. Therefore, time series analysis elicits a great deal of attention from various disciplines.

Some researches mainly focused on complexity of time series. Donner et al. [9] demonstrated that quantification characteristics, i.e., average path length, clustering coefficient, and centrality measures of the recurrence network, are sensitive to transitions of different dynamical behaviors of time series. Gao et al. [10] proposed a modality transition-based network for mapping the experimental multivariate measurements into a directed weighted complex network. The results show that the multivariate complex network analysis allows quantitatively uncovering the transitions of distinct flow patterns and yields deep insights into the nonlinear dynamic behavior underlying gas-liquid flows. Zeng et al. [11] proposed a directed weighted complex networks based on time series symbolic pattern representation. This technique was applied to investigate the

natural wind field signals. The results indicate that the values of three network parameters show consistent increase or decrease trend with the spatial regular arrangement of the nine anemometers.

Some researches mainly focused on fractal properties of time series. Vassoler et al. [12] proposed a method of DCCA cross-correlation coefficient to quantitatively measure the cross-correlations levels between climatological data. Zeng et al. [13] proposed an approach of multivariate multifractal detrended fluctuation analysis (MV-MF DFA) to study the fractal dynamics of multichannel data in natural near-surface wind fields. Kavasseri et al. [14] systematically analyzed hourly wind speed data obtained from four potential wind generation sites in North Dakota. It is seen that the records at all four locations exhibit similar scaling behavior which is also reflected in the multifractal spectrum. Zeng et al. [15] develop a new method called asymmetric multiscale multifractal analysis (A-MMA) to explore the multifractality and asymmetric autocorrelations of the signals with a variable scale range.

Quite Recently, there has been an increasing interest in analyzing the time series form the complex network point of view. A very effective tool, called visibility graph (VG) invented by Lacasa et al. [16], has been extensively used for characterizing time series. The main advantages of the VG are high computational efficiency and easy implementation. Nunez et al. [17] proposed horizontal visibility graph (HVG). This algorithm stands as an effective method to discriminate randomness in time series. Zhou et al. [18] introduced the conception of visibility distance and developed an improved visibility algorithm, called limited penetrable visibility graph (LPVG).

The recurrence plot (RP) firstly introduced by Eckmann et al. [19] is very effective tool which visualize the dynamics of phase space trajectories of time series. The recurrence quantification analysis (RQA) proposed by Zbilut and Webber is a quantitative extension of RP method. This technique defines several measures mostly based on diagonal oriented lines in the RP. In addition, the RP method has many derivative forms, such as Cross Recurrence Plot (CRP) [20], Joint Recurrence Plot (JRP) [21], and Order Recurrence Plot (ORP) [22]. The main advantage of the RP based methods is that it can provide useful information even for short and non-stationary data, where other methods fail.

In this paper, we aim to develop a new quantification time series analysis approach which combines the advantages of two state-of-art methods, i.e., visibility graph (VG) and recurrence quantification analysis (RQA).

The content of this paper is arranged as follows: Section II gives a brief introduction of the VG method, quantification
analysis of visibility graph and algorithm steps. In section III, we introduce how to acquire the wind speed data. In section IV, indoor and outdoor wind speed time series are selected to generate the visibility graph, and we compare the quantification analysis of visibility graph and recurrence plot. Finally, some conclusions are given in section V.

II. METHODOLOGY

A. Visibility graph (VG)

Visibility graph method can map time series into complex networks. For the discrete time series, the data point is defined as the network node, and the connection between the data points satisfying the visibility criterion is defined as the network edge. As shown in Fig. 1, straight bars represent the 20 data points in a periodic time series (Fig. 1(a)), whose height corresponds to the true value of the data. If the tops of two bar are visible to each other, the two points are considered to be connected in the network. Fig. 1(b) is the network generated by the method, in which the real points that are sequentially arranged are in one-to-one correspondence with the discrete time series data points, and the real lines between the real points correspond to the visible lines between the straight lines. It is worth noting that the visible line cannot be repeated, omitted or connected with itself, the line of sight between two straight bars cannot cross the other bars.

Visibility Criteria: If two discrete points, named \((t_a, y_a)\) and \((t_b, y_b)\), are connected, then for any point \((t_c, y_c)\):

\[
\frac{y_c - y_a}{y_c - y_b} < \frac{t_c - t_a}{t_c - t_b}
\]

Where \(t_a < t_b < t_c\).

The visibility graph network has the following properties: every point is connected with at least the left neighbor and the right neighbor; the network is an undirected right-of-way network; the scales change or affine transform of horizontal and vertical coordinates doesn’t change the visibility. The visibility graph method inherits some characteristics of the original time series, that is, the periodic time series is transformed into a regular network, and the random time series is transformed into a random network, and the fractal time series is transformed into a scale-free network.

B. Visibility State Plot (VSP)

Inspired by the recurrence plot, we can draw a visibility state plot based on the visibility of the corresponding points. According to our method, if the two points are visible, the corresponding point in the visibility state matrix is 1, otherwise is 0, so that we can draw a visibility state plot. Fig. 2 shows a visibility state plot from wind speed time series. Fig. 3 shows a recurrence plot of the same signal.

C. VSP quantification analysis measures

In order to quantitatively analyze the two-dimensional visibility state plot, the present method defines the quantification measures of two-dimensional visibility state plot from the macro and micro perspectives to extract the non-linear characteristics of the time series, and realize the quantification analysis of two-dimensional visibility state plot. Quantification measures include the visibility rate (VR), the degree of certainty (DC), the mean diagonal length (DL), the visibility entropy (VE), the layered (LAY), the vertical/horizontal line length (VHL):
\[ VR = \frac{1}{N^2} \sum_{i,j}^N V(i,j) \]  

\[ DC = \frac{\sum_{i,j}^N dP(d)}{\sum_{i,j}^N V(i,j)} \]  

\[ DL = \frac{\sum_{d_{\text{min}}}^d dP(d)}{\sum_{d_{\text{min}}}^d P(d)} \]  

\[ VE = -\sum_{d_{\text{min}}}^d P(d) \ln P(d) \]  

\[ LAY = \frac{\sum_{i,j}^N hP(h)}{\sum_{i,j}^N V(i,j)} \]  

\[ VHL = \frac{\sum_{i,j}^N hP(h)}{\sum_{i,j}^N P(h)} \]

Where \( N \) is the length of the time series, \( V(i,j) \) is the \( i \)th row and \( j \)th column element in the visibility matrix, \( d_{\text{min}} \) is the minimum diagonal length in the two-dimensional visibility state plot, \( dP(d) \) is the probability density of diagonal whose length is \( d \); \( P(h) \) is the probability density of vertical/horizontal line segments whose length is \( h \), \( h_{\text{min}} \) is the minimum analysis length, usually set to 2; \( N_i \) is the number of vertical/horizontal structure length.

**D. The procedures of the VSP algorithm**

(a) Firstly, the normalized time series \( \{y_1, y_2, \ldots, y_n\} \) is obtained by standardizing the original time series \( \{x_1, x_2, \ldots, x_n\} \) using the Z-score standardization method. The processed time series satisfies a standard normal distribution with a mean of 0 and a variance of 1, which is calculated as:

\[ \bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \]

\[ \alpha = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2} \]

\[ y_i = \frac{x_i - \bar{x}}{\alpha} \]

Where \( \bar{x} \) is the mean of the original time series and \( \alpha \) is the standard deviation of the original time series.

(b) The data points in the time series are plotted equidistantly in the form of a bar on the coordinate axis, and the height of the bar represents the numerical value of the data points in the time series.

(c) If the ends of two bar are connected and are not blocked by other bars, the two data points are visible, vice versa, the two data points are not visible. By processing all the data points in a similar way, the visual state between the data points is determined.

(d) If two points \( (i, y_i) \) and \( (j, y_j) \) are visible, the visual matrix \( V(i,j) = 1 \) described with black points, on the contrary, \( V(i,j) = 0 \), described with white points. So you can draw a two-dimensional visibility state plot mapped by time-series as shown in Fig.2.

(e) Calculating the quantification measures of visibility state plot, the visibility rate (VR), the degree of certainty (DC), the mean diagonal length (DL), the visibility entropy (VE), the layered (LAY), the vertical/horizontal line length (VHL).

(f) Analyzing the quantification measures of visibility state plot.

**III. EXPERIMENTS AND DATA ACQUISITION**

In order to study the internal mechanism of indoor and outdoor wind field near the ground, this paper designed the corresponding experiment to collect indoor and outdoor wind field signals with the characteristics of spatial distribution. We choose an indoor experiment with less mobility and an empty outdoor experiment, avoiding human interference to the great extent. Anemometers are arranged in the indoor and outdoor space as shown in Fig.4, the anemometers’ height from the ground are 0.6m. In order to ensure the reliability and authenticity of the analysis results, we recorded the time series of wind speed many times, and each signal acquisition time is 1 hour and the sampling frequency is 4Hz. We select five of the high-quality signal to analysis in the subject, with a group of signals as an example, taking 1000 as the length of the time series, and draw the wind time series as shown in Fig.5.
In this section, we select 9 groups of the indoor and outdoor signals. Then we number the samples, indoor wind speed signal and outdoor wind speed signals are numbered 1 to 18, that is, indoor signals U1 ~ U9 numbered 1 ~ 9, outdoor signals U1 ~ U9 numbered 10 ~ 18. The recurrence plot and the visibility state plot are respectively quantitatively analyzed, wherein the recurrence quantification analysis measures are shown in Fig. 6, and the quantification analysis values of visibility state plot are shown in Fig. 7.

Fig. 6 shows the results of recurrence quantification analysis of wind speed signals at different locations. Among them, 1 to 9 represent the positions of 9 anemometers in different indoor positions, and 10 to 18 represent the positions of 9 anemometers in different outdoor positions. It can be seen from Fig. 6 that the quantification measures of recurrence quantification analysis of indoor and outdoor wind speed signals generally show the same transition rule. The range of quantification measures of indoor wind speed is obviously larger than that of outdoor wind speed, which indicates that the indoor local variation is obviously larger than the outdoor in the same indoor and outdoor area. That is to say, the indoor local characteristic is more obvious. And the range of quantification measures of outdoor wind speed is within the range of indoor measure transition. Therefore, the quantification measures of recurrence quantification analysis can not distinguish indoor and outdoor wind speed signals.

Fig. 7 shows the quantification analysis results of visibility state plot proposed in this paper. The analysis results of the six quantification measures are basically the same, that is, the range of quantification measures of outdoor wind speed is not coincident with that of indoor speed of wind speed, and the quantification values of outdoor are less than those of indoor ones. In addition, the same as the recurrence quantification analysis, the proposed method can also reflect that local variation of indoor wind speed is obviously stronger than that of outdoors. Furthermore, the complexity of the indoor wind speed time series is generally less than the outdoor wind speed time series. Therefore, the proposed method can not only distinguish indoor and outdoor wind speed signals, but also characterize the local characteristics of indoor wind field signals. Our results indicate that quantification analysis of visibility state plot has broad application prospects in wind energy prediction, wind field pattern classification and wind farm zoning.

### IV. RESULTS AND DISCUSSIONS

#### A. Quantification analysis of visibility state plot

In this section, we use the newly proposed quantification analysis of visibility state plot to distinguish the indoor and outdoor wind field signals, comparing them with the recurrence quantification analysis. Then we analyze whether the two methods have significant differences in the processing and discrimination of complex natural signal and whether the proposed method has certain advantages.

In order to quantify the visibility state plot of the acquired data. First the acquired data is normalized, taking the time series in Fig. 5 as an example to draw a normalized time series graph. It is noteworthy that the normalization does not change the statistical characteristics of time series.
B. Advantages of quantification analysis of visibility state plot

Compared with the traditional time series analysis method, the proposed method is suitable for analyzing more signal types. The traditional time-domain method is mainly used for the analysis of linear and stationary signals. However, the time-domain indexes of nonlinear and non-stationary signals are usually time-varying, so the inherent characteristics of the signal cannot be expressed in detail and the method is susceptible to noise. Furthermore, the robustness of these methods is poor. For the frequency-domain analysis method, due to the limitation of orthonormal basis function, when the method is applied to non-linear and non-stationary signal analysis, the corresponding spectral information cannot accurately reflect the intrinsic composition of the signal and thus cannot obtain valuable analysis of the results. However, the two-dimensional visibility state plot method proposed in this paper can be mapped to a more regular graph for linear and stationary signals and can be mapped to a complex and diverse image modality for complex non-linear and non-stationary signals. Therefore, it is convenient to comprehensively mine different types of time series kinetic characteristics from the macro and micro perspectives.

The proposed method greatly expands the acquisition way of quantification analysis of time series. Due to the limitation of the analysis dimension, traditional analysis methods based on one-dimensional time series are very difficult to propose new quantification analysis measures. However, the method in this paper innovatively proposes to map one-dimensional time series into a two-dimensional visibility state plot, and obtain rich quantification analysis indexes with powerful image analysis technology. With the continuous improvement and refinement of image analysis technology, it provides reference for proposing more effective quantification analysis indexes in the future.

The two-dimensional visibility state plot of time series and quantification analysis of visibility state plot proposed by this paper is a universal method. Therefore, it has broad application prospects in many application fields (such as weather, finance, medical treatment, etc.).

V. CONCLUSION

In this study, for complex nonlinear and non-stationary signals in nature, we use visibility state plot which maps an one-dimensional time series into a two-dimensional visibility state plot and propose some quantification analysis measures based on it. Visibility state plot can not only display intrinsic structure of the original time series concisely and intuitively, but also can take full advantage of powerful image analysis techniques to extract a large number of valuable quantification measures from the image level. So it can reveal the inherent regularity of the system operation more completely and accurately. Compared with the recurrence quantification analysis method, it is proved that the proposed method is more superior and effective.

REFERENCES


