Abstract: Functional electrical stimulation (FES) is a method of applying low-level electrical currents to restore or improve body functions lost through nervous system impairment. FES is applied to peripheral nerves that control specific muscles or muscle groups.

Application of advanced signal computing techniques to the medical field has helped to achieve practical solutions to the health care problems accurately. The physiological signals are essentially non-stationary and may contain indicators of current disease, or even warnings about impending diseases. These indicators may be present at all times or may occur at random on the timescale. However, to study and pinpoint these subtle changes in the voluminous data collected over several hours is tedious. These signals, e.g. walking-related accelerometer signals, are not simply linear and involve non-linear contributions. Hence, non-linear signal-processing methods may be useful to extract the hidden complexities of the signal and to aid physicians in their diagnosis.

In this work, a young female subject with major neuromuscular dysfunction of the left lower limb, which resulted in an asymmetric hemiplegic gait, participated in a series of FES-assisted walking experiments. Two three-axis accelerometers were attached to her left and right ankles and their corresponding signals were recorded during FES-assisted walking.

The accelerometer signals were studied in three directions using the Hurst exponent $H$, the fractal dimension (FD), the phase space plot, and recurrence plots (RPs). The results showed that the $H$ and FD values increase with increasing FES, indicating more synchronized variability due to FES for the left leg (paralysed leg). However, the variation in the normal right leg is more chaotic on FES.

Keywords: Hurst exponent, sensor, muscle, fractal dimension, functional electrical stimulation, phase space
include motion-analysis-based asymmetry [7], walking speed [8], surface electromyography (EMG)-based muscle activity linear estimates [9], and muscular or joint pathological evaluation [10]. However, recently, researchers showed that the human gait changes with long-range fluctuation [11], and unexpected correlations of the stride interval [12] were reported. Furthermore, the impaired gait and effect of therapeutic intervention were more complex on evaluation. Therefore it may not be possible for the conventional evaluation criteria to reflect the non-linear nature of the human gait and FES-induced gait restoration process.

Non-linear time series analysis techniques [13–15] have been developed to analyse and characterize the irregular behaviour of physiological signals. Non-linear techniques mainly involve the correlation dimension, Hurst exponent, phase space plots, fractal dimension (FD), entropy-related measures, Lyapunov exponents, measures for determinism, self-similarity, interdependences, recurrence quantification, and tests for non-linearity [16].

The importance of the biological time series analysis which exhibit typically complex dynamics, has been recognized in the area of non-linear analysis. Several approaches have been proposed to detect the hidden important dynamic properties of the physiological phenomenon. The statistical characteristics of biological signals change with time and are typically highly irregular and non-stationary in many cases. Hence, non-linear dynamic techniques based on the concept of chaos may be useful in many areas including medicine and biology. The theory of chaos has been used to detect some cardiac arrhythmia such as ventricular fibrillation [17]. Methods based on chaos theory have been applied in tracking heart rate variability signals and predicting the onset events such as ventricular tachycardia [18], sick sinus syndrome, atrial fibrillation, complete heart block, pre-ventricular contraction, and bundle branch block [19]. A novel method based on the phase space technique was used to distinguish between normal and abnormal cases [19]. The effect of body responses to the accelerating platform was studied using the largest Lyapunov exponent (LLE) with the ankle front–back acceleration and ankle pitch angular velocity sensor data [20]. The results suggest that the LLE for the ankle front–back and ankle pitch rate decreases with increase in the balance platform acceleration.

This work attempts to analyse the effect of FES on gait restoration, using non-linear methods of walking-function-related accelerometers, seeking to catch the non-linear characteristics of ambulation and, in particular, the characteristics of impaired ambulation and FES-assisted walking.

2 METHODS

A 38-year-old female, who had a major neuromuscular dysfunction of the left lower limb, resulting in an asymmetric hemiplegic gait, participated in this study. Without FES support, on a treadmill she was able to walk with a highest speed of 0.4 km/h with left cane support. Because of the poor activation effect of stimulation to the tibialis anterior muscle for ankle flexion, and hamstring for knee flexion, a flexor reflex [21] was activated by stimulating the peroneal common nerve at an empirically determined moment during walking. The stimulation timing first can be applied by the subject through a button and then can be automatically detected by a gait phase detection system [22, 23]. Figure 1 shows the subject trying to walk with FES, with accelerometers fixed on her ankles.

After the experiment goal and procedure were explained to the subject, written consent was obtained from the subject before participating in this experiment. The entire experiment was monitored by an experienced medical doctor.

2.1 Stimulation and devices

A premodulated stimulation was employed and a typical stimulation waveform is shown in Fig. 2. The carrier frequency can be adjusted from 1 kHz to 10 kHz. The skin impedance is reduced during...
stimulation and hence less energy is needed to achieve a desired movement. A burst frequency (10–100 Hz), whose amplitude can be adjusted up to 60 V peak was used to generate enough current to move the lower limbs. This premodulated stimulation method lowers the skin impedance and causes less discomfort [24]. The output signal is a square wave with six variables to be controlled: the carrier frequency, amplitude, inter-pulse, burst frequency, burst width, and amount of burst (Fig. 2). The inter-pulse duration is important, because it reduces the phase charge necessary to activate the peripheral nerve [25]. The electric stimulation device used was developed by NihonMedix Corp.

The accelerometer used was an O-Navi GYROCU-BE3A three-axis accelerometer, with a three-axis gyro. Two such sensors were fixed at the left and right ankles, such that the Y axis was along the proximal longitudinal direction of lower legs, and the Z axis perpendicular to the sagittal plane.

Data were sampled using an analogue-to-digital converter (National Instruments NI USB-6218) at 1600 Hz. These digital data were recorded on a notebook computer by using LabVIEW 7.1 software (National Instruments). Figure 3 shows typical accelerometer signals without and with FES for the left- and right-axis sides.

The subject was asked to walk for about ten steps at around 0.6 km/h and 0.4 k/h with and without FES support respectively.

3 METHODS

The accelerometer signals collected in all the three directions (X, Y, and Z) from both legs were analysed using the Hurst exponent H, the FD, and recurrence plots (RPs). They are briefly explained below.

![Typical stimulation waveform](image)

**Fig. 2** Typical stimulation waveform

**Fig. 3** Accelerometer signals: (a) left-side X axis without FES; (b) left-side X axis with FES; (c) right-side X axis without FES; (d) right-side X axis with FES
3.1 Hurst exponent H

The Hurst exponent H is used to evaluate the self-similarity and roughness in a time series. Therefore, it can evaluate the presence or absence of a long-range dependence and its degree in a time series. H provides a means of classifying time series in terms of predictability.

It is a measure of the smoothness of a fractal time series based on the asymptotic behaviour of the rescaled range of the process. H is defined as

\[
H = \frac{\log(R/S)}{\log(T)}
\]

(1)

where \( T \) is the duration of the sample of data and \( R/S \) is the corresponding value of the rescaled range.

If \( H = 0.5 \), then the time series indicates a random walk (Brownian time series). There will be no correlation between the present and future samples and there is 50 per cent probability that future return values will go either up or down. Hence, it is difficult to predict such signals [26].

However, if \( H > 0.5 \), then it indicates ‘persistent behaviour’, exhibiting a trend in the series. If there is an increase from \( t-1 \) to \( t \), then it is probable that there may be an increase from \( t \) to \( t+1 \). Therefore a larger value of \( H \) indicates a higher predictability and vice versa.

If \( H < 0.5 \), the time series exhibits ‘anti-persistent behaviour’. This means that, if there is an increase from \( t-1 \) to \( t \), then there may be a decrease from \( t \) to \( t+1 \) and vice versa. In this work, \( H \) was used to evaluate the variability of accelerometer signals with and without FES.

3.2 Fractal dimension (FD)

Fractals exhibit exact self-similarity across all spatial or temporal scales, such that successive magnifications reveal an identical structure. The term ‘fractal’ was first introduced by Mandelbrot [27].

The FD measures the degree of complexity by evaluating how fast the measurements increase or decrease as the scale becomes larger or smaller. The FD of a waveform helps in transient detection. The FD has been used to quantify the structures of a wide range of objects in biology and medicine. This feature can be used for the analysis of an electrocardiogram (ECG) or an electroencephalogram (EEG) to identify and distinguish specific states of physiological function [16, 28]. The Higuchi and the Katz algorithms are available to evaluate the FD of the physiological signals, but in this work the Higuchi algorithm was used to calculate the FD.

3.2.1 The Higuchi algorithm

Assume that \( x(1), x(2), ..., x(N) \) are the signals to be analysed. Now, \( p \) new signals \( x_p^m \) are constructed as \( x_p^m = \{ x(l), x(l+p), x(l+2p), ..., x(l+(N-1)/p) \} \) for \( l = 1, 2, 3, ..., p \), where \( l \) indicates the discrete time intervals varying up to \( p \), and \( \lfloor a \rfloor \) means the integer part of \( a \). For each of the \( p \) time series or curves \( x_p^m \), the length \( L_p \) is computed using

\[
L_p = \sum_{i=1}^{\lfloor (N-1)/p \rfloor} |x(l+ip)-x(l+(i-1)p)|/[(N-1)]
\]

where \( N \) is the total length of the signal \( x \), \( (N-1)/p \) is a normalization factor and \( a = \lfloor (N-1)/p \rfloor \). An average length is computed as the mean of the \( p \) lengths \( L_p \) for \( l = 1, 2, 3, ..., p \). This procedure is repeated for each \( p \) ranging from 1 to \( p_{\text{max}} \), obtaining an average length for each \( p \). In the curve of \( \text{ln}(L_p) \) versus \( \text{ln}(1/p) \), the slope of the least-squares linear best fit is the estimate of the FD [29].

This feature has been used in the analysis of ECGs and EEGs to identify and distinguish specific states of physiological function [30, 31].

3.3 Time delay (\( \tau \))

The dynamics of the acceleration signal can be better understood by the construction of the phase space. The geometry of the phase space and its attractor describe the dynamics of the system. The signal can be represented in the phase space domain using delayed coordinate embedding [32]. Each observation in accelerometer signal \( x(n) \) is substituted by a vector

\[
y(i) = [x(i), x(i+\tau), ..., x(i+(M-1)\tau)]
\]

where \( \tau \) is the time delay and \( M \) is the embedding dimension. Several methods can be used for determination of the best time delay [33]. The average mutual information (AMI) method was used to estimate the time delay in this paper.

The optimum choice for \( \tau \) of the system is one, which provides maximum new information with each measurement \( x(i+\tau) \). The value of \( \tau \) is evaluated by plotting a graph of mutual information \( I(\tau) \) between \( x(i) \) and \( x(i+\tau) \) and finding the first value of \( \tau \) for which \( I(\tau) \) is a minimum [34]. Figure 4 shows the time delay plot calculated using the AMI method.
For this work, the time delay was obtained as equal to 17.

3.4 Embedding dimension \((M)\)

The false-nearest-neighbours (FNN) algorithm finds the nearest neighbour of every point in the phase space for a given dimension and then checks to see whether the points are still close neighbours in the higher dimension \([35]\). If the distance between the two points in the higher dimension divided by the distance in the present dimension exceeds a 30 per cent threshold, the point is denoted an FNN. The percentage of false neighbours should drop to zero when the appropriate \(M\) is reached. The embedding dimension is the value for which there will be no FNN or the value within the acceptable level.

Figure 5 shows the estimation of the embedding dimension for the present data using the FNN method. In this work, \(M\) was obtained as equal to 6.

3.5 Recurrence plot (RP)

RPs are used to reveal the non-stationarity of the time series \([36]\). The time series is represented as a two-dimensional plot which highlights the drift and hidden periodicities in the signal. It can be useful in the diagnosis of diseases \([16, 37]\). A brief description of the construction of RPs is described below.

![Fig. 4 Estimation of the delay using AMI](image1)

![Fig. 5 Estimation of the embedding dimension using the FNN method](image2)
Consider the acceleration signal $x$ with total number $N$ of samples given by $x_0, x_1, x_2, \ldots, x_{N-1}$. Then a vector $y_i$ is formed of length (embedding dimension) $P$ and delay $d$, i.e., $y_i = (x_p, x_{i+d}, x_{i+2d}, \ldots)$, where $P \geq 2$ and $d \geq 1$.

The RP is formed by comparing all embedded vectors with each other and drawing points when the distance between two vectors is below some threshold $r$. Therefore, there will be a point at the coordinates $(i, j)$ if the $i$th and $j$th embedded vectors are less than some distance $r$ apart, e.g., a point is drawn if $||y_i - y_j|| < r$. $i$ is the horizontal axis and $j$ is the vertical axis. If the points in the RP are homogeneous but irregular points in the distribution, then this implies a stochastic series. Long parallel straight lines on the RP indicate inherent periodicity in the series, and short scattered diagonal strips and points indicate chaotic series [38].

**Fig. 6** Results of the RPs for the accelerometer signal on the X axis: (a) left leg without FES; (b) left leg with FES; (c) right leg without FES; (d) right leg with FES
Figure 6 shows the RP of an accelerometer sign on the X axis with and without FES. In this work, the colour of the point indicates the distance between two vectors.

3.6 Non-linearity

Before analysing the accelerometer signal using non-linear methods, it is necessary to test for non-linearity of the data. One of the tests available to test this is the surrogate data test.

3.6.1 Test for non-linearity

The surrogate data technique can be used to distinguish the chaotic systems from the linearly correlated noise signal. The surrogate data sets can be generated from the original data. The surrogate data are completely stochastic but contain exactly the same linear correlations as the original accelerometer signal. In practice, it is implemented taking the Fourier transform of the original accelerometer signal, then randomizing the phases without changing the magnitudes, and later using the inverse Fourier transform to obtain the surrogate accelerometer signal. This resulting signal will have the same power spectrum as the initial accelerometer signal, but they are random in all other respects. This was obtained using the chaos data analyser. Two sets of surrogate data were generated for each person. H was obtained for both the original and the surrogate data sets. It was found that H for the surrogate data and for the original data H are different from each other by more than 62 per cent. This rejects the null hypothesis and hence the original data contain non-linear features.

4 RESULTS

The variation in H and the FD (mean ± standard deviation (SD)) without and with FES for the right leg on the X axis, Y axis, and Z axis are shown in Tables 1, 2, and 3 respectively. Similarly Tables 4, 5, and 6 show the variation in H and the FD without and with FES for the left leg on the X axis, Y axis, and Z axis respectively.

It can be seen from these tables that FES assist caused a clear difference in the parameter H on the left side. The differences (H_withFES - H_withoutFES) between the averages of the with-FES and without-FES cases are +0.11, +0.06, and +0.11 for the X, Y, and Z axes respectively. The differences are statistically significant (although, for the Y axis, p = 0.06). On the right side, the same parameter is reduced for all three axes; the differences (H_withFES - H_withoutFES) between the with-FES and without-FES cases are −0.05, −0.07, and −0.00 for the X, Y, and Z axes respectively. However, the differences are not statistically significant.

On the other hand, for the FD, FES assist increases for both sides. On the left side, the differences

### Table 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Without FES (mean ± SD)</th>
<th>With FES (mean ± SD)</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>0.6823 ± 0.150</td>
<td>0.6169 ± 0.180</td>
<td>0.16</td>
</tr>
<tr>
<td>FD</td>
<td>−1.3386 ± 0.184</td>
<td>−1.2960 ± 0.210</td>
<td>0.023</td>
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</table>

### Table 2

<table>
<thead>
<tr>
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<th>With FES (mean ± SD)</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>0.6656 ± 0.158</td>
<td>0.6081 ± 0.177</td>
<td>0.29</td>
</tr>
<tr>
<td>FD</td>
<td>−1.3483 ± 0.168</td>
<td>−1.2570 ± 0.429</td>
<td>0.0011</td>
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</table>

### Table 3

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Without FES (mean ± SD)</th>
<th>With FES (mean ± SD)</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>0.73087 ± 0.143</td>
<td>0.7276 ± 0.164</td>
<td>0.58</td>
</tr>
<tr>
<td>FD</td>
<td>−1.2978 ± 0.162</td>
<td>−1.2374 ± 0.429</td>
<td>0.23</td>
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</table>

### Table 4

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Without FES (mean ± SD)</th>
<th>With FES (mean ± SD)</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>0.62934 ± 0.193</td>
<td>0.73966 ± 0.215</td>
<td>0.0009</td>
</tr>
<tr>
<td>FD</td>
<td>−1.3814 ± 0.188</td>
<td>−1.2333 ± 0.194</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

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(FD_{withFES} - FD_{withoutFES}) between the with-FES and without-FES cases are +0.15 and +0.09 for the X and Y axes respectively and, on the right side, they are +0.09 and +0.07 respectively. The differences are statistically significant (although, for the left Y axis, $p = 0.088$). However, the Z axis did not show a significant difference.

Figure 6 shows the RPs for the X axis of both legs without FES and with FES. Figures 6(a), (b), (c), and (d) show the RP of the left-leg X axis without FES and with FES and the right-leg X axis without FES and with FES respectively. It can be seen from the plots that the changes are more rhythmic and subtle for the left leg with FES as compared with the left leg without FES. Similarly, for the right leg with FES there are more abrupt changes than with the right leg without FES, indicating more variability.

There is a more symmetric pattern (which indicates rhythmic variation) present without FES for the left leg (paralysed leg). The values of H and FD decreased with FES for the right leg. This clearly indicates that the variation is more disorderly and chaotic compared with the left leg (paralysed leg).

### 5 DISCUSSION

A higher value of H indicates that the signal is more predictable. In this work, FES assist resulted in a higher H value than without-FES walking on the left side (seriously impaired and FES-assisted side), and a lower H value on the right side. This indicates that the variation in the left leg is more synchronous with the FES assist. In contrast, the motion of the right leg became more difficult to predict. This might be because, in the hemiplegic gait, the subject tends to rely more on the less impaired leg (right side) to support her weight and pushes forward her body, while using the seriously impaired side (left side) to prepare for the horizontal displacement of the (centre of gravity) and transitionally to bear the weight. Thus, without FES, the walking is ‘under the control’ of the right side. With FES assist, the left side could also play some important role in walking; thus the right side is not and need not be so dominant, which resulted in a decrease in the H value for the right side, and an increase for the left side.

Moreover, the change on the left side is higher than that on the right side and the left side is statistically more significant (lower p value) than the right side (Tables 1 to 6). This demonstrates that the improvement in predictability on the left side might be more crucial for walking function improvement, and the change on the right side might be concomitant.

A higher value of the FD indicates that there is more self-similarity in the signal. For both right and left sides, the FES assist resulted in an increase in the FD, which means that, for both sides, the gait tends to have a self-similar structure.

The FD was calculated for data containing up to 1000 strides to analyse the long-range correlations [11]. However, it is apparent that, for paralysed patients, it was difficult to collect data in an experiment.

In the present work, the accelerometer signals were analysed with FES and without FES using H, the FD, the phase space, and RPs. The findings from the results and analysis were basically in agreement with the previous studies.

### 6 CONCLUSION

FES is the electrical stimulation of nerves to produce a controlled contraction of muscles. In this work, the effect of stimulation on a hemiplegic subject was analysed. The accelerometer signals of the subject were analysed without and with FES using nonlinear parameters, namely H, the FD, the phase space plot, and RPs. The results show higher values for H and the FD, and less spread in unique pattern of the RP, indicating more rhythmic variation due to

### Table 5 Ranges of H and FD for left-leg Y axis without and with FES (channel 11K)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Without FES (mean ± SD)</th>
<th>With FES (mean ± SD)</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>0.7328 ± 0.209</td>
<td>0.7914 ± 0.226</td>
<td>0.06</td>
</tr>
<tr>
<td>FD</td>
<td>-1.2691 ± 0.204</td>
<td>-1.1760 ± 0.194</td>
<td>0.0065</td>
</tr>
</tbody>
</table>

### Table 6 Ranges of H and FD for left-leg Z axis without and with FES (channel 12L)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Without FES (mean ± SD)</th>
<th>With FES (mean ± SD)</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>0.6295 ± 0.164</td>
<td>0.7313 ± 0.208</td>
<td>0.0069</td>
</tr>
<tr>
<td>FD</td>
<td>-1.3807 ± 0.166</td>
<td>-1.2443 ± 0.184</td>
<td>0.23</td>
</tr>
</tbody>
</table>
FES for the paralysed side. However, it becomes more random for the right leg (less impaired leg). Hence, the proposed method was able to quantify the response of accelerometer signals to the FES. This finding can be utilized to test the efficacy of physiotherapy exercises and drugs.

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