Analysis of Combustion Oscillations in a Staged MLDI Burner using Decomposition Methods and Recurrence Analysis

Conference Paper · January 2016
DOI: 10.2514/6.2016-1156

CITATION
1

READS
149

6 authors, including:

Brian Dolan
University of Cincinnati
20 PUBLICATIONS 32 CITATIONS

Lipika Kabiraj
Technische Universität Berlin
26 PUBLICATIONS 134 CITATIONS

Christian Oliver Paschereit
Technische Universität Berlin
479 PUBLICATIONS 2,706 CITATIONS

Effie J Gutmark
University of Cincinnati
635 PUBLICATIONS 5,938 CITATIONS

Some of the authors of this publication are also working on these related projects:

Thermoacoustic instabilities at elevated pressures; bluff body combustion; rotating detonation; Pulsed combustors. View project

AHEAD Project View project

All content following this page was uploaded by Brian Dolan on 03 December 2015.
The user has requested enhancement of the downloaded file. All in-text references underlined in blue are added to the original document and are linked to publications on ResearchGate, letting you access and read them immediately.
Analysis of Combustion Oscillations in a Staged MLDI Burner using Decomposition Methods and Recurrence Analysis

Jarred Wilhite*, Brian Dolan† and Rodrigo Villalva Gomez‡, University of Cincinnati, Cincinnati, OH, 45221-2872, USA
Lipika Kabiraj§, Technical University of Berlin, 10623 Berlin, Germany
Ephraim Gutmark¶, University of Cincinnati, Cincinnati, OH, 45221-2872, USA

Thermoacoustic oscillations in a staged multi-nozzle lean direct injection combustor are studied using dynamic pressure measurements and time-resolved OH* chemiluminescence imaging. Two conditions, one with periodic oscillations and one with intermittent bursts of high-amplitude pressure waves are investigated. Proper orthogonal decomposition and dynamic mode decomposition are used to analyze the results from the flame imaging. Both decomposition techniques correctly extract the dominant fluctuations, related to the fundamental oscillation frequency, in the OH* emission from the flames of the low-emission fuel stages. The flame response to higher order harmonics are also extracted by both decomposition techniques during limit cycle oscillations but in the case of quasi-periodic intermittent oscillations the modal decomposition methods are disrupted by the increasingly complex flame response. In addition to these decomposition techniques, recurrence analysis of phase space trajectory reconstructed from pressure time trace was performed in order to understand the temporal evolution of the oscillations. It has been identified that the two oscillatory states correspond to deterministic dynamical states. The bursting behavior observed during the intermittent combustion oscillations was identified to correspond to the previously reported dynamical state of intermittency, using recurrence analysis.

I. Introduction

Modern gas turbine combustion systems are often plagued by destructive combustion driven pressure oscillations due to increasingly common design strategies to lessen NOx production. NOx reduction is usually accomplished by lowering the products formed by the thermal NOx mechanism.1, 2 When the temperature is elevated during and immediately after the combustion reaction, molecular nitrogen and oxygen may disassociate and participate in one of several reactions that lead to NO or NOx molecules. So, a reduction in the temperature or time at peak temperatures results in lowering the concentration of NOx in the combustion products. Modern gas turbine combustors leverage lean burning flames with high degrees of mixing to lower peak temperatures that result from burning near stoichiometric conditions.3 These lean burning systems do not include dilution air injection which provides damping of acoustic energy. Combined with the fact that a reactant mixture well below stoichiometric leads to higher fluctuations in reaction rate due to equivalence ratio oscillations,4 these lean burning combustors are particularly susceptible to self-sustained coupling

*Graduate Researcher, Aerospace Engineering, ML0070, AIAA Student Member.
†Ph.D. Candidate, Aerospace Engineering, ML0070, AIAA Student Member.
‡Senior Researcher Assistant, Aerospace Engineering, ML0070, AIAA Member.
§Postdoctoral Researcher, ISTA, HF-303, Miller-Breslau Straße 8.
¶Distinguished Professor and Ohio Eminent Scholar, Aerospace Engineering, ML0070, AIAA Fellow.
between unsteady heat release and acoustics which leads to damaging pressure waves.\textsuperscript{5,6} This behavior has been a significant and persistent challenge to the development of lean burning combustors.\textsuperscript{7} Designers are faced with the challenge of lowering combustion temperatures without inflicting unacceptable penalties to other pollutant concentrations or combustion stability. This paper characterizes and examines dynamic combustion behavior in a new combustor concept which achieves the goals of reducing temperature and residence time to reach future NO\textsubscript{x} emissions goals.

Lean burning combustors are often classified into two categories: lean, premixed, prevaporized (LPP) and lean direct injection (LDI). Many practical designs are between these two extremes. LPP combustors have a region where the reactants mix and the fuel evaporates. Careful tuning of the mixing and ignition processes are required to ensure that the flame does not form or flashback in the mixing region. Due to the high pressures and wide range of power demands, this is a difficult challenge that has caused a long road to the commercial implementation of LPP combustors in the aerospace industry. Many engine manufacturers continue to focus on designs with a rich burning primary zone and dilution air. In an LDI system, the fuel is injected into the combustion region without any use of a mixing tube. Auto-ignition and flashback, a hurdle for the application of LPP combustors, is impossible in an LDI concept.

A carefully designed LDI system can avoid near-stoichiometric diffusion flames around fuel droplets and the NO\textsubscript{x} emission can approximate that of an LPP combustor.\textsuperscript{3,8,9} However, mixing the fuel and air before combustion is a challenge, especially at elevated pressures where the Damkohler number (the reaction rate divided by the mixing rate) increases.\textsuperscript{3} Through the use of smaller fuel injectors, a fuel spray with lower mean diameter droplets can be created.\textsuperscript{10–12} To create smaller fuel droplets which atomize more rapidly, multiple-nozzle LDI concepts (where many small fuel/air mixers replace one conventional nozzle) have been proposed.\textsuperscript{8,13–15} The use of multiple nozzles supplying the same fuel flow as one conventional injector greatly decreases the length of time required to vaporize and mix the fuel with the air.\textsuperscript{16,17}

Tacina et al.\textsuperscript{13} performed early-stage operability and emissions testing of a multi-nozzle sector using a rectangular grid of uniform pressure-atomizing swirl nozzles. During testing at high power conditions, the MLDI combustor achieved very low production of NO\textsubscript{x}. Several later papers presented emissions measurements from 25-point\textsuperscript{14} and 9-point\textsuperscript{8} uniformly fueled MLDI arrays. At conditions simulating takeoff, NO\textsubscript{x} emissions index (EINO\textsubscript{x}) measurements were as low as a 70\% reduction relative to 1996 International Civil Aviation Organization standards. However, at low power conditions, the fuel flowrate per nozzle becomes low enough to cause problems in achieving an effective fuel spray.\textsuperscript{15} The combustor presented in this paper has different nozzle stages which can be independently controlled. It has demonstrated comparable EINO\textsubscript{x} levels at high power conditions but also had significantly improved flexibility at low power operation.\textsuperscript{18}

Thermoacoustic coupling has been observed in several LDI systems over the past several years.\textsuperscript{9,19,20} Thus there is incentive to increase our understanding of the characteristics of the periodic behavior in order to find the underlying causes and potential solutions to these destructive resonances.

During the nozzle research and development stages, most experimental and numerical studies focus on single nozzle geometries. Available literature analyzing combustion oscillations in multi-nozzle systems is relatively limited. Samarasinghe et al.\textsuperscript{21} imaged CH\textsuperscript{*} chemiluminescence in an acoustically forced multi-nozzle premixed combustor burning natural gas. They showed the asymmetric nature of forced multi-nozzle flames and demonstrated that single-nozzle flame response should not be used to an an indication of multi-nozzle behavior. A separate experiment by Aguilar et al.\textsuperscript{22} visualized the flow disturbances in the central nozzle of a transversely forced array of three nozzles and compared with a single nozzle. There was significant non-axisymmetry in the flowfields for the multi-nozzle case, particularly downstream of the jet merging location. The mode of the flow response to the acoustic forcing (helical or axisymmetric) was dependent on the location of the nozzle relative to the pressure wave shape. For example, if the swirl cup was located at a pressure anti-node, the flow disturbance was axisymmetric because the pressure perturbation was the same sign (i.e. positive or negative relative to local average pressure) on both sides of the nozzle.\textsuperscript{22}

Other past studies have used experiments\textsuperscript{23} or numerical simulations\textsuperscript{24} to study combustion oscillations in annular combustors with multiple nozzles. One massively parallel LES investigation of a full annular combustor simulated natural azimuthal modes, but due to the relatively large distance between swirlers did not identify any mechanism of flame interactions between burners.\textsuperscript{23} On the other hand, an experimental investigation of an annular combustor with different numbers of nozzles (and therefore varying nozzle spacing) showed that the flame shape, frequency of acoustics, and flame disturbance mode was dependent on the nozzle spacing.\textsuperscript{24} There was a breakdown of the axisymmetric flame structure as the nozzle interaction increases when the spacing is reduced. In both cases, phase averaged chemiluminescence showed a spinning transverse
pressure wave. However at higher flame spacings, the nozzle flow instabilities were independent helical modes about each nozzle whereas with smaller spacing, flame merging caused the disturbances to change to large-scale bulk heat release structures corresponding to the transverse acoustic mode. These studies, while not directly applicable to an MLDI combustor, demonstrate that the presence of multiple nozzles can have dramatic effects on the local and global response of the combustor to acoustics.

The present experiment and analysis is different from past work due to the LDI design of the combustor, the use of liquid fuel, the presence of fuel staging, the use of different nozzle designs across stages, and the fact that nozzles are intended to replace one conventional nozzle and are therefore of comparatively small in scale. Due to the large number of nozzles and the fact that they are located in close proximity, it is expected that interaction between adjacent nozzles will be a powerful effect and will influence the nature of the periodic reaction behavior. The designs of the nozzles differ between fuel stages, and understanding the thermoacoustic interactions between heterogeneous arrays of swirl cups is important to enable similar low NOx concepts. This publication includes an extensive characterization of the stability levels throughout the operating envelope and high-speed chemiluminescence imaging which reveals further information on the nature of the reaction zone oscillations.

The multi-nozzle burner experimentally investigated in this study can develop significant self-sustained thermoacoustic coupling. Effects of operating conditions, particularly equivalence ratio and fuel distribution, on the presence and amplitude of the thermoacoustic oscillations is quantified. Changes in the reaction zone during an oscillation is visualized using 10 kHz OH* chemiluminescence imaging. Theories on the causes and driving mechanisms of coupling can be derived from these observations and will be discussed. The pressure waves at one test condition have a quasi-periodic nature where the oscillation amplitude intermittently transitions from relatively low (but not zero) to high. The intermittent oscillation amplitude exists for an extended period of time and is therefore a stable dynamical state rather than a transient condition. This case is described in more depth using pressure data and a POD analysis. Better characterization and understanding of the thermoacoustics in this and other modern combustion systems is important for the future development of low-emission combustor concepts.

II. Combustion System

The combustion system investigated in this study is based on a multiple-nozzle array of swirlers with lean direct injection (LDI) of jet A fuel. United Technologies Aerospace Systems (UTAS) designed this burner under contract of NASA’s Environmentally Responsible Aircraft Program. A primary goal of this NASA program is to reduce landing and take-off cycle NOx emissions by 75 percent of International Civil Aviation Organization standards by 2020. Previous publications have focused on the emissions and operability of this combustion system.25, 26

The multiple-nozzle concept replaces a single traditional nozzle with a number of small-scale injectors. For the experimental tests presented here the array uses 13 injectors with an estimated total air effective area ($A_{C_d}$) of 12.9 cm$^2$. This value was estimated via CFD and confirmed by test data. The array consists of three pressure atomizing pilot fuel injectors, four intermediate air blast fuel injectors, and six outer air blast fuel injectors as shown in Figure 1.

The thirteen nozzles are grouped into three stages (pilot, intermediate, and outer stages) which can be independently controlled. Details of the design intent of the different types of nozzles can be found in a publication by the group at United Technologies.15 A more brief summary is given here.

The center row of pilot swirlers uses pressure atomizing injectors and a high level of swirl. This row

![Figure 1. Axial view of combustor nozzles and domeplate.](image-url)
supplies 12 percent of the total airflow. The high level of swirl is intended to create a wide and strong recirculation zone which encourages flame stability. Two rows of intermediate nozzles are above and below the pilot swirlers. These are airblast injectors with a much lower level of swirl. Six outer fuel nozzles are located at the top and bottom of the array and are built similarly to the intermediate nozzles. The lower level of swirl is intended to reduce the residence time and result in high levels of mixing. The intermediate row supplies 39 percent while the outer nozzles supply 49 percent of the air for combustion. All three fuel stages are used for high-power operation such as take-off while the pilot array alone would be used during a low-power idle.

Figure 2 shows a cross-section of the combustor. Directly downstream of the domeplate there is a convergence which extends the length of the burner. The nozzles are angled towards the center of the combustion chamber, and spaced so that recirculation zones could form between the different swirlers. These traits encourage interaction between the nozzles to result in improved mixing and increased stability for the airblast nozzles. Uncooled ceramic liners were used to form the convergence. All air for combustion enters through the combustor dome (i.e. there are no dilution or film cooling holes).

The reaction zone changes dramatically as different fuel stages have fuel supplied to them. Figure 3 shows OH* chemiluminescence images of the pilot nozzles burning alone, the pilot and intermediate fuel stages, and all three fuel stages. The trapezoids on the right indicate the location of the pilot nozzles (tan), intermediate nozzles (green), and outer nozzles (blue). These ensemble averaged images were taken at characteristic design conditions for each staging mode. The fuel split (FS) describes the fraction of total fuel flow which is sent to each stage. Images in Figure 3 have FS values of 1/0/0 for the pilot only, 0.25/0.75/0 for the pilot and intermediate stage, and 0.12/0.39/0.49 for all three fuel stages.

In Figure 3a, the reaction zone of the pilot nozzle array is shown. Because even at very lean overall equivalence ratios the local equivalence ratio behind the pilot is above one, there is also significant heat release outside of the V-shaped primary flame. This is similar to a rich-burn, quick-quench, lean-burn (RQL) system; however the excess air is injected from the other nozzle arrays as opposed to dedicated dilution holes. Adding fuel to the intermediate fuel stage (Figure 3b) causes the reaction zone to become U-shaped with the region furthest upstream dominated by the pilot and a reaction zone from the intermediate nozzles penetrating more deeply into the burner. When all three fuel stages have fuel flow supplied, there is OH* emission throughout the combustor (Figure 3c). When the fuel distribution is relatively even, such as shown in Figure 3, the intensity of the chemiluminescence drops dramatically. This indicates that localized regions of high heat release and high temperatures (correlated with NOx production) are not prevalent. Villalva et al. provides a detailed description of the reaction zone characteristics and emissions performance of this combustor.
III. Experiment Description

All testing was conducted at the University of Cincinnati pressurized combustion facility. In this facility a range of fuel/air nozzle designs and combustor prototypes can be installed in a cylindrical pressure vessel. Dynamic pressure measurements and time-resolved OH* chemiluminescence constitute the primary diagnostics in this paper.

A. Facility and Instrumentation

Figure 4 shows the test combustor mounted and enclosed within a cylindrical stainless steel pressure vessel. In this arrangement the outer vessel handles the pressure stresses. A control valve in the exhaust duct is used to regulate the combustion pressure and a 192 kW heater in the inlet duct preheats the air. Five inch diameter silica windows (with very good transmittance of UV wavelength light) provide optical access to the test article through the pressure vessel. One wall of the test combustor is made of fused silica to provide a full view across all fuel stage arrays. The facility is capable of the following maximum conditions: combustor inlet pressure ($P_3$) of 7 bar, inlet air temperature ($T_3$) of 700 K, and combustion airflow ($W_3$) of 0.802 kg/s. For further information on the facility, please see Ref. 27. The facility was fully instrumented to monitor and control the process. Type K thermocouples monitor the inlet air ($T_3$), combustor domeplate, and pressure vessel exhaust. Druck static pressure sensors measure the inlet pressure ($P_3$) and combustion pressure ($P_4$). Test airflow is measured by a thermal mass flowmeter and the fuel flows for the three fuel stages are measured by coriolis mass flowmeters. The main parameters which define a test condition are the inlet air temperature, the combustion pressure, the pressure drop across the nozzles ($\Delta P = (P_3 - P_4)/P_3$) which is driven by the preheat temperature and the airflow rate ($W_3$), the equivalence ratio, and the fuel distribution between stages (fuel split). Process information is measured continuously while the high-speed and imaging measurements are only acquired when appropriate. Jet A is used as fuel for all results in this paper.

High-sensitivity charge output dynamic pressure sensors (Kistler 7061 B) are located in the plenum ($P_3'$) and combustor ($P_4'$). At each test condition, these sensors are sampled at 20 kHz for approximately 8 seconds. A stand-off tube is used to protect the $P_4'$ transducer from the combustion temperatures. Because of this, there is an unknown phase delay between the true combustor pressure oscillation and the measurement at the sensor. So, the acoustic signal and flame imaging will not be directly linked to each other.

B. OH* Imaging System

During combustion, some atomic and molecular species in the reacting flow are naturally excited to a higher energy level by the high temperatures and then spontaneously decay back to the original state. During this event, a photon at a specific wavelength (related to the discrete difference in energy levels) is released in a process called chemiluminescence. Since the change in energy is dependent on the atom or molecule, filtering the light emission from the flame can allow imaging of one particular species. The filtered chemiluminescence intensity is related to the concentration of the molecular species being imaged.
The hydroxyl radical (OH*) is created within the heat release zone. OH* is useful because it marks the location of the reaction and is related to the local temperature and heat release rate. Since it is a very short-lived radical, it can show dynamic behavior such as changes in the reaction zone due to thermoacoustic coupling. Multiple experimental and numerical studies have shown that OH* exists within the heat release region of the reaction. However, using chemiluminescence as a measure of heat release rate is complicated by the fact that OH* emission is sensitive to equivalence ratio. The OH* concentration is linearly related to the heat release rate (i.e., the rate of fuel being burnt) and exponentially to the temperature (strongly related to the equivalence ratio). The overall chemiluminescence emission (such as found by a summation of the chemiluminescence images) is also affected by changes in the area of the flame. Due to these complicating factors, care must be taken when interpreting the images. However, periodic changes in local equivalence ratio and flame area have both been shown to be related to thermoacoustic oscillations in lean systems.

Due to the presence of multiple nozzles with varying local fuel/air ratios, this burner will have significant spatial differences in equivalence ratio. Additionally, in any combustor where the nozzles can be affected by the pressure fluctuations, including the LDI burner presented here, there is feed/system coupling which will cause temporal variations in the fuel to air ratio. Due to changes in the local flame temperature, the OH* chemiluminescence intensity cannot be viewed as being simply related to the heat release rate. For this reason, in many combustors it is common for the chemiluminescence emission to not be quantitatively correlated to the heat release rate.

However, despite this limitation, imaging the OH* distribution can still provide extremely valuable information on the location and fluctuations of the reaction. This is particularly true in a burner such as...
this where the phase relationship between the oscillations in the different stages is of particular interest and where the OH* signal strength goes from very strong to almost non-existent in large regions of the combustor. Thus, the qualitative reaction strength within and between stages, if not the exact levels of heat release, can be viewed with confidence. In a system such as this, the changes in OH* chemiluminescence would generally be the result of both changes in the heat release rate and the equivalence ratio.

A high-speed intensified imaging system is used to capture time-resolved images of the flame chemiluminescence. A LaVision HS-IRO provides intensification for a Phantom v16 camera. The camera and intensifier are kept in sync by a LaVision high-speed timing controller. Images are acquired at a rate of 10 kHz with a resolution of 1280x800 pixels. The gate width is set to 90 µs so that subsequent frames are independent from the previous acquisition but a high signal to noise ratio is achieved. A first surface mirror turns the image 90° before it is imaged by the camera. Because OH* chemiluminescence has a wavelength of 309 nm, the lens is fitted with a narrow bandpass filter with transmittance curve centered at 310 nm. The pressure vessel window, combustor window, and lens are all made of material with very high transmittance in the UV wavelengths. The chemiluminescence intensity is recorded as a line-of-sight integration with a depth of field larger than the depth of the combustor. The entire reaction zone is visible. For most cases, 1000 images are acquired at each test condition.

C. Proper Orthogonal Decomposition

Proper orthogonal decomposition (POD) is a method of decomposing a set of possibly correlated observations into a set of linearly uncorrelated variables. These orthogonal variables are referred to as the POD modes and form an empirical basis for a transformation of the ensemble of observations. Here the ensemble of data is the OH* intensity distribution as imaged during a prominent thermoacoustic oscillation and is represented by the scalar function \( u(x, t) \) where \( x \) is the grid point (pixel) and \( t \) refers to a specific image acquisition. The POD procedure seeks a base of orthogonal spatial eigenfunctions, referred to as modes and represented by \( \Phi_n(x) \), and time coefficients \( \{a_n\} \) such that each observation in the data ensemble can be represented as:

\[
    u(x, t) = a_0 \Phi_0(x) + \sum_{n=1}^{N} a_n(t) \Phi_n(x)
\]

The zeroth eigenfunction, Mode 0, contains the mean field whereas the subsequent modes capture the fluctuating component of the field (i.e. the combustion dynamics). The Fourier decomposition is analogous in many ways. However, in Fourier analysis the basis is given as trigonometric functions without regard to the data. In POD, the basis is entirely empirical and chosen to maximize the quantity:

\[
    \frac{< (u, \Phi)^2 >}{(\Phi, \Phi)^2}
\]

where \( <> \) denote the ensemble average and \( (a,b) \) represents the inner product. This means that the POD seeks to decompose the ensemble of realizations \( u(x, t) \) onto a base which maximizes the variance content of the first \( N \) modes. Furthermore, the modes are ordered according to their fluctuating energy content, with mode 1 as the maximum. The energy content is described by the eigenvalue \( \lambda_n \) which corresponds to each mode \( \Phi_n \). A larger eigenvalue means more of the fluctuation is described by that mode.

In the case of an oscillating flame the first modes should capture the dominant periodic behavior while subsequent modes decrease in energy until they only capture random turbulent fluctuations.

The POD technique was first proposed by Lumley to study turbulence. In this case the velocity vectors form the data ensemble and the POD procedure seeks to maximize the turbulent kinetic energy of each mode. More recently POD has been used to supply useful information for studying the dynamics of a flame undergoing thermoacoustic oscillations. When using the fluctuating OH* field as the data ensemble the POD seeks to maximize the variance energy within each mode.

To reduce the problem to a computationally possible scale, it is reformulated using Sirovich’s method of snapshots. This common method results in the same number of modes as the number of individual image acquisitions (\( N = 500 \) in this case). However, it is expected that only a small number of modes will be required to capture the dominant periodic fluctuations due to the combustion oscillation.

POD provides the ability to reduce complexity in the analysis of difficult periodic behavior. The POD modes \( \{\Phi_n(x)\} \) provide a visualization of the regions where fluctuations from the average field (in this case
an image of the OH\textsuperscript{*} concentration) generally occur. Because the modes are ordered according to their energy content, the most dominant oscillations can be separated from more minor fluctuations. The time coefficients \( a_n(x) \) are weighting functions which give the influence of each mode on an individual image acquisition from the experiment. Analysis of the temporal coefficients can give insight into how repeatable the fluctuations are and the phase location of each image.

D. Dynamic Mode Decomposition

Because the POD modes are usually not restricted to one specific frequency, the temporal description of POD modes is not preferential for some cases. For more accurate temporal description, it is better to use dynamic mode decomposition, or DMD. DMD is a method that is used to decompose the flow field into a set of modes that correspond to unique, characteristic frequencies.\textsuperscript{41} While POD arranges modes based on their energy content, DMD orders the modes based on their respective frequencies. The dominant DMD mode will correspond to the fundamental frequency of a particular dataset. Schmid proposed the DMD method, which can also be used to provide the growth rate for modes at a specific frequency.\textsuperscript{42}

Since it is difficult to analyze a nonlinear system, it is assumed that a linear operator can be used to approximate a nonlinear system with the DMD method.\textsuperscript{42} This linear operator, or Koopman operator, operates on a function evaluating a parameter such as velocity or pressure. However, in this study, the OH\textsuperscript{*} intensity distribution is evaluated instead of a velocity field. The linear operator is studied since it represents the relevant information of the entire nonlinear system. The eigenvalues \( \lambda_j \) and amplitude factors \( \eta_j \) can be calculated from the linear operator.\textsuperscript{43}

For instance, the OH\textsuperscript{*} intensity distribution \( u(x, t) \) for various grid points, \( x \) and various image acquisitions of thermoacoustic oscillations, \( t \) can be expressed by a series expansion with eigenfunctions:\textsuperscript{41}

\[
  u(x, t) = \sum_{j=1}^{N} \eta_j \lambda_j \Phi_j(x) \tag{3}
\]

where \( \eta_j \) represents the amplitude factor of the \( j \)th mode and \( \phi_j \) represents the dynamic modes. The eigenvalues \( \lambda_j \) of the linear operator can be used to calculate the frequency and the growth rate for each mode. Thus, the eigenvalues govern the temporal evolution of the system. Since the DMD modes are ordered based on their frequencies, frequency is an essential parameter to this study and it is defined as:\textsuperscript{43}

\[
  \omega = \frac{\arctan \left( \frac{\text{Im}(\lambda_j)}{\text{Re}(\lambda_j)} \right)}{\Delta t} \tag{4}
\]

where \( \Delta t \) is the change in time between each image acquisition, which was 1/10000 seconds for this study. The \( \text{Im} \) and \( \text{Re} \) denote the imaginary and real parts of the complex eigenvalues. Similar to POD, the DMD modes also show where the fluctuations from the average field or OH\textsuperscript{*} concentration images occur, which makes it easier to determine which modes are dominant and which modes have minor variations. Further mathematical details of DMD have been discussed in many other sources,\textsuperscript{42, 43} but the ones discussed here are the most essential to the current study.

E. Recurrence Plots

Recurrence analysis\textsuperscript{44} is a technique to study the trajectories representing a dynamical system in the phase space. A system, defined by a set of variables, evolving in time in accordance with rules governing the relationship among the system variables, can be represented in a multi-dimensional space that is formed by its variables: the phase space. The line connecting the states of the dynamical system in the phase space forms its trajectory. As the system reaches an asymptotic state, the trajectory forms a geometry—the attractor—in the phase space. A fundamental property of the trajectory, through which the attractor can be characterized, is recurrence: Recurrence analysis is a powerful technique and can be used to identify and classify the asymptotic state of the system, distinguish dynamics from noise and track dynamical transitions.\textsuperscript{45–47}

Measured data from experiments can be employed into a suitable reconstruction method to reconstruct the phase space. The phase space, thus obtained, preserves the dynamical features of the original phase space. The method of delays is one of the phase space reconstruction methods, where, a multidimensional space is created from time-delayed vectors extracted from the experimentally acquired time series; Scalar measurement is sufficient. Given a scalar measurement, \( s_i = [s_1, s_2, \ldots, s_N] \), where measurements are acquired at a
constant rate (sampling frequency), an m-dimensional phase space constructed: \( y_i = [s_i, s_{i+\tau}, \ldots, s_{i+(m-1)\tau}] \). Optimum embedding where the reconstructed phase space is a one-to-one mapping of the original phase space and that which preserves dynamical properties of the original attractor requires m and \( \tau \) to be chosen appropriately. As a result of this reconstruction, the evolution of the system, in terms of points in the phase space and the corresponding trajectory is obtained.

Two close phase space points, which are not temporally correlated, suggest a recurrence of the phase space trajectory. A binary recurrence matrix can be obtained by checking all possible point pairs in the phase space for recurrence:

\[
R_{ij} = \Theta(\delta - \|y_i - y_j\|) \tag{5}
\]

\( \Theta \) is the Heaviside function and \( y_i \) is a vector that defines the \( i^{th} \) point on the phase space. This binary square matrix can then be plotted to obtain a visualization of the recurrence behavior of a phase space trajectory. Patterns and features of this plot are directly associated with the dynamical features of the system under consideration. The most important features of the plot include (see Ref.\(^{45}\) for details and Refs.\(^{48, 49}\) for typical RP obtained from measurements on combustion noise and thermoacoustic instability):

- Line of identity (LOI): The main diagonal formed when \( i = j \).
- Line segments parallel to the LOI indicating periodicity. The horizontal (or vertical) separation between line segments corresponds to the period of oscillation: The RP of a sinusoid would correspond to parallel lines with a constant separation.
- Noise, transitions and chaotic behavior leads to isolated points, white patches and broken diagonal lines of varying lengths.

The threshold that defines the Heaviside function, \( \Theta \), is typically chosen such that the percentage of point pairs with positive recurrence is close to 1%.

**IV. Results and Discussion**

One purpose of this paper is to highlight the efficacy of different decomposition and analysis techniques on extracting physically relevant information from a complicated combustion system. To do this, two test cases are contrasted. The first is a condition with relatively periodic pressure waves. This case approaches an ideal limit cycle but it is complicated by chaotic turbulent variations inherent to this complex combustor design. The second case displays combustion dynamics which are intermittent; there are repetitive low-frequency pulses of high amplitude pressure waves.

Table 1 shows the operating conditions for these two cases. Both the limit cycle and intermittent dynamics are encountered during three-stage operation (i.e. the pilot, intermediate, and outer fuel stages are all supplied fuel). The combustion pressure (\( P_4 \)), inlet air temperature (\( T_3 \)), and pressure drop (\( \Delta P \)) remain constant. There is a small change in overall equivalence ratio (\( \phi \)) and a larger shift in fuel split (FS). The FS is the percentage of fuel sent to each fuel stage (pilot/intermediate/outer).

Figure 5 shows pressure measurements and corresponding FFT spectra for the limit cycle and intermittent cases. Although turbulence causes the pressure trace in the limit cycle case to not be perfectly periodic, it is clear that the pressure waves are fairly similar in amplitude. The FFT spectra shows a high amplitude peak at the fundamental frequency of 540 Hz. There are also fairly well-defined peaks at the frequencies of the first, second, and third harmonics. For the case with intermittent combustion dynamics, the pressure measurement shows repetitive bursts of high-amplitude oscillations followed by a decrease in dynamic pressure amplitude and a region of low-amplitude oscillations. This cycle occurs at a frequency of about 5.5 Hz. The FFT

<table>
<thead>
<tr>
<th>Dynamic Condition</th>
<th>( P_4 ) (bar)</th>
<th>( T_3 ) (K)</th>
<th>( \Delta P ) (%)</th>
<th>( \phi )</th>
<th>Fuel Split</th>
</tr>
</thead>
<tbody>
<tr>
<td>Limit Cycle</td>
<td>3.6</td>
<td>560</td>
<td>4</td>
<td>0.55</td>
<td>0.25/0.37/0.38</td>
</tr>
<tr>
<td>Intermittent</td>
<td>3.6</td>
<td>560</td>
<td>4</td>
<td>0.575</td>
<td>0.18/0.38/0.44</td>
</tr>
</tbody>
</table>
corresponding to this intermittent condition is much less defined. The frequency content of the fundamental peak as well as that of the harmonics is found at a wider range of frequencies. This is an indication of the increased chaotic nature of the pressure oscillations, which do not consistently occur at a small set of frequencies. The intermittent dynamics observed are the result of non-linear interaction leading to quasi-periodic behavior.

Figure 6 shows instantaneous OH* images at stable and thermoacoustically coupled conditions. At a lower equivalence ratio, $\phi = 0.5$, there are almost no pressure oscillations. These images provide a look at how a stable flame with three fuel circuits appears. The second image set is at a higher equivalence ratio, $\phi = 0.6$, which is experiencing very strong thermoacoustic coupling. There are significant variations in the reaction zone including large regions downstream of the intermediate and outer fuel stages which appear to completely extinguish and reignite during an oscillation.

The instantaneous flame imaging measurements greatly resemble previously published phase-averaged OH* chemiluminescence images.\textsuperscript{50, 51} Those phase averaged images, and POD processing, showed how the periodic changes in the OH* emission were dominated by repeated quenching and strengthening of the reaction zone of the low-stability nozzles. There is also a weaker oscillation in the pilot flame which precedes the rest of the reaction zone by 90° phase. The instantaneous high-speed imaging shows that the OH* emission from the intermediate and outer stages often completely disappears indicating that the flame here is either entirely or almost entirely extinguished during part of the oscillation. During cases with intermittent combustion dynamics, the fluctuations in the OH* emission appears and disappears with the pressure oscillations.\textsuperscript{52}

A. Proper Orthogonal Decomposition

POD seeks to decompose an dataset into a minimum number of spatial modes which can best represent the unsteadiness in the measurements. In the case of strong periodic phenomenon, which dominates the unsteady component, POD effectively isolates the shape and temporal dynamics of the repetitive process. Here it is applied to the time-resolved OH* chemiluminescence images. Because the images are acquired...
Figure 6. OH* images during three-stage operation for (a) stable and (b) coupled conditions.

(a) $\phi = 0.5$, FS = 0.18/0.39/0.42.  
(b) $\phi = 0.6$, FS = 0.13/0.4/0.47.

Figure 7. First eight modes from POD analysis of limit cycle condition. Mode spatial distribution with FFT of corresponding time coefficients.

Figure 7 shows the POD modes and time coefficient FFTs for the four most significant modes. Mode 1, the most energetic mode by a significant margin, is dominated by fluctuations in the low-emission fuel stages (intermediate and outer). So, the periodic changes to the OH* images are primarily influenced by fluctuations in these regions. Mode 2 shows fluctuations in the top and bottom of the combustor which are out-of-phase with fluctuations in the center. These two modes, as shown by the FFTs, dominate the fluctuations in the OH* emission which are at the frequency of the fundamental. Modes 3 and 4 appear more complicated with smaller-scale structures. There are bands of out-of-phase fluctuations stretching from the top to the bottom of the combustor. The FFT spectra show that these modes have frequency content at the harmonics which are stronger than that of the fundamental. The shorter wavelength to the pressure oscillations is consistent with the smaller scale structures in the POD spatial mode.

Figure 8 gives similar POD plots for the first four modes relevant to the case with intermittent combustion dynamics. The bottom range of the FFT spectra are extended to 1 Hz so that low frequency content, corresponding to the roughly 6 Hz pulsations, can be seen. Modes 1 and 2 are largely similar in shape to that of the limit cycle case. Because there is more fuel flow to the outer nozzles in the intermittent case, there is also more variation in the OH* emission at that location and therefore that region is represented more strongly in the POD mode. A peak in the frequency spectra is seen at 6 Hz which corresponds to the intermittent combustion dynamics. Modes 3 and 4 are significantly different than what was seen in the
Figure 8. First eight modes from POD analysis of case with intermittent combustion dynamics. Mode spatial distribution with FFT of corresponding time coefficients.

Figure 9. DMD Modes and Spectrum Plot for Limit Cycle Case.

more periodic case. In the spectra of these modes the frequency peak with the highest amplitude is the fundamental. This is different than seen in Fig. 7.

POD decomposes the dataset based on mode shapes and energy content, not frequency. In the limit cycle case, the mode shapes happen to be formed such that the first two modes mostly represent the fundamental whereas the modes 3 and 4 represent harmonics. This is an efficient and useful extraction of physically relevant data from a chaotic dataset. In the case with intermittent dynamics, this pattern is disrupted by the increased complexity added by the low frequency content. More modes are required to represent the fundamental frequency content. Although not shown, higher modes also do not clearly represent harmonic content. So, in this case, the POD is not effective at decomposing the OH* images into a basis with relevance to the frequency content.

B. Dynamic Mode Decomposition

The DMD method is used to effectively decompose the OH* images relative to their characteristic frequencies. A MATLAB code was written to run the OH* images through the DMD. Alenius\textsuperscript{41} and Grillis\textsuperscript{53} approach to the DMD method were key sources used when developing this code. The DMD code read in 1000 OH* images and outputted DMD modes for each image. This was done for both the limit cycle case and the intermittent combustion dynamics case.

The plot on the left of Fig. 9 shows the DMD mode spectrum for the limit cycle case, which shows the amplitude distribution of the DMD modes as a function of frequency. The first mode on the right, Mode 1
corresponds to the fundamental frequency (540 Hz) of the limit cycle case. The visualization shows that this is the most clearly defined mode in this case. Since this mode corresponds to the highest peak in the DMD mode spectrum plot, this shows that Mode 1 is in fact the most dominant mode in the dataset. Peaks can also be seen at frequencies of 540 Hz, 1080 Hz, 1620 Hz, and 2160 Hz, respectively. These peaks correspond to modes 2, 3 and 4, respectively which correspond to the second, third and fourth harmonic frequencies.

The shapes of first three DMD modes in this case are very similar to those in the POD limit cycle case. For instance, positive variations to the OH* images can be seen in DMD Mode 1. These variations are influenced by the intermediate and outer stages, which is expected since this mode is dominated by these stages. Variations in the top and bottom of the combustor can be seen in DMD Mode 2. Similar variations can be seen in the second POD Mode of the same case. These positive (red) fluctuations are out of phase with the negative (blue) fluctuations in the center. Out-of-phase fluctuations can also be seen in multiple regions throughout DMD mode 3 from the top to the bottom of the combustor. The spotty visualization in DMD Mode 4 has a much less defined shape than the other modes. This shows the vast distinction between the dominant DMD modes and the modes with less variation from the original OH* images.

Similar plots are shown in Fig. 10 for the intermittent combustion dynamics case. Similar to the limit cycle case, the first mode on the right, Mode 1 corresponds to the fundamental frequency (540 Hz) of the intermittent case. The visualization shows a slightly more defined mode than the others in this case. Again, Mode 1 is the most dominant in this case since it corresponds to the highest peak in the DMD mode spectrum plot. Modes 2 and 3 correspond to frequencies of 530 and 550 Hz, which are very close to the fundamental frequency. Modes 4 and 5 correspond to frequencies of 10 and 20 Hz, respectively. The FFT for the POD intermittent case showed a peak in frequency spectra at 6 Hz, which explains why there are peak amplitudes at low frequencies of 10 and 20 Hz. Mode 6 is the only mode that corresponds to a harmonic frequency in the intermittent case.

The outer regions of the combustor are shown to have more variation in DMD Modes 1-5, similar to the POD intermittent modes. However, the variation is more positive overall than in the POD modes. The DMD modes for the intermittent case are not as clearly defined as those seen in the limit cycle case. This could be a result of the intermittent case being more complex than the limit cycle case, as mentioned previously. Again, the Mode 6 is shown to be spotty and much less defined than the other modes. This case, too shows how the definition in the visualization diminishes for each subsequent case. Thus, making it easier to identify the dominant modes in a given dataset.

C. Recurrence Plots

The spectra, POD and DMD, discussed in the earlier sections provide the frequencies involved, and the spatial structure of the most dominant modes associated with the two cases of instability discussed here. All the three decomposition methods involve the processing of a sufficiently large section of the time-series/image sequence at once.

RP analysis complements previous analysis by providing details on the temporal evolution of the oscillatory behavior. Figure 11 presents the RPs of the two states discussed: the one close to limit cycle behavior.
Figure 11. Recurrence plot together with the corresponding time series for the two cases in Tab. 1. The inset shows that the global pattern of the plot is formed by diagonally-aligned line segments. (left) and the second showing intermittent dynamics (right). Both the RPs contain shapes, patterns formed by an arrangement of diagonal line segments (as emphasized by the inset within the RP on the left) of different lengths and points. While according to the spectra and time series, the first case was identified as a limit cycle, the RP indicates that the state is not defined by continuous diagonally-aligned lines, and is hence, more complicated than limit cycle behavior. As observed before, noise is inherently present in the system. Noise definitely plays a role in the observed RP features. However, it is also likely that this particular state corresponds to a more complicated deterministic state. Re-examining the spectra, it can be seen that while it contains a dominant peak, smaller broadband peaks beside the dominant peak as well as beside its superharmonics. Accordingly, a quasi-periodic or chaotic state is likely. Secondary bifurcations of limit cycle oscillations to quasi-periodic and chaotic dynamics has been shown to be a features of thermoacoustic systems.54–57

The most striking feature of the RP of the second (intermittent) case are the repeating kite-like patterns. The patterns are observed to repeat at a rough interval of 3000 points, while the general shape remains the same throughout the entire measurement (only a 9000 point section shown here for emphasizing the shape). The term kite-like is of relevance because, it has been shown before that this shape in the RP corresponds to a specific dynamical state: type-II intermittency. That limit cycle thermoacoustic oscillations can undergo bifurcation to type-II intermittency has been shown in experiments on an academic, laminar premixed flame combustor configuration.48 The identification of this state in our configuration has implications: Firstly, these intermittent oscillations are not random. This state appears possibly due to interaction among the adjacent nozzles and the modes of combustion. They belong to a particular state of thermoacoustic instability—like the limit cycle. With slight variations in one of the parameters (fuel distribution, equivalence ratio), the intermittent burst are expected to either increase or decrease in the repetition rate, depending on the direction of parameter variation. In fact, in the RP of the former case, remnants of repeating patterns that are clear for the intermittent case can be identified. It can accordingly be proposed that the former case is already approaching intermittent dynamics.

V. Conclusions

The objective of the paper is to investigate the characteristics of thermoacoustic instability in a promising multi-burner combustor configuration. Thermoacoustic coupling was observed in the rig for several combinations of operating parameters. Furthermore, the features of the instability were observed to vary significantly
with the operating conditions. Two specific parameter combinations, with the same overall conditions except for slightly different equivalence ratio and fuel-split among the premixed, pilot and intermediate stages, was the focus of this report. The features of the instability, namely, mode shapes of coherent flame oscillations, frequency content of the oscillations and temporal dynamics of pressure fluctuations were extracted using mode decomposition techniques: POD & DMD, spectral analysis, and recurrence analysis respectively.

The POD method decomposes and orders modes based on their energy content, while DMD orders and decomposes modes based on their respective frequencies. Thus, the dominant POD modes are the most energetic modes, while the most dominant DMD modes correspond to the fundamental and harmonic frequencies. Despite the differences between POD and DMD, the most dominant modes looked fairly similar for both cases. Modes 1, 2 and 3 in the limit cycle case for both POD and DMD had similar shape, and fluctuations occurred in similar regions. For instance, Mode 1 was dominated by fluctuations in the intermediate and outer stages. As a result, variations to the OH* image were evident in these regions for the first mode in both the POD and DMD. Also, Mode 2 for POD and DMD both had similar mode shapes, which included fluctuations from the top to the bottom of the combustor, which were out-of-phase with the center region.

Another similarity between POD and DMD was the FFT for the POD modes and the DMD Spectra plot. For example, the FFT for Mode 1 peaks at the fundamental frequency of 540 Hz, while the DMD Mode Spectra also peaked at 540 Hz for Mode 1. FFT plots for certain subsequent POD modes showed peaks at harmonic frequencies, which correlated to peaks at the same frequencies for harmonic DMD modes. Also, the FFT for the intermittent case shows peaks at a low frequency of 6 Hz for Modes 2 and 3. The DMD Mode Spectra plot also shows peaks at low frequencies for the intermittent case corresponding to Modes 2 and 3. There is a noticeable difference between the POD and DMD mode shapes for the intermittent combustion case. The POD modes for this case were much more defined and had both positive and negative variations, while most of the DMD modes were less defined and had mostly positive variations in each. The poor definition in some of the DMD modes could be a result of the complexity of the intermittent case. However, one similarity between these POD and DMD mode shapes is that the outer regions of the combustor were shown to have the most variation for the most dominant modes. For both the limit cycle and intermittent cases, the definition and quality of the DMD modes was shown to diminish for the subsequent modes.

RP analysis indicates that both the oscillatory states investigated correspond to complex dynamical states. In accordance with previous studies, the observed complexity can be ascribed to secondary bifurcations of the limit cycle oscillations in response to operating parameter variation. It is interesting to note that the (type-II) intermittent state reported previously in a laminar premixed flame combustor, is found to be responsible for the bursting behavior observed in the presently investigated combustor configuration. The combustor is unstable for both the cases, but the oscillations correspond to different dynamical states. In addition, the strong interaction between the closely positioned flames running under different combustion stages are also expected to contribute significantly to the observed dynamics of the oscillatory states.

The similarities between the dominant POD and DMD modes shows that the results from this experiment are consistent, which proves that either of these methods can be used as an adequate modal decomposition method. Based on this study, both POD and DMD are sufficient methods to use when a complicated combustion system or complex datasets need to be analyzed. This study also shows that the FFT plots for POD modes can provide accurate fundamental frequencies, since they correlated with peaks in the DMD Spectra Plot.

Acknowledgments

The authors gratefully acknowledge the combustor design work by United Technologies Aerospace Systems and initial program funding from NASA Glenn Research Center.

References


