Recurrence analysis and phase space reconstruction of irregular vibration in friction brakes: Signatures of chaos in steady sliding

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Abstract

Irregular friction brake vibration data have been collected with sampling rates of up to 200 kHz. The measured time series have been subjected to recurrence analysis and phase space reconstruction. The recurrence analysis indicates that irregular vibration states of friction brakes are strongly dominated by intermittency phenomena. Phase space reconstruction suggests that this intermittency is dominated by low-dimensional irregular deterministic dynamics rather than by high-dimensional stochastic processes.

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1. Introduction

The design of friction brakes with respect to acceptable noise and vibration levels poses one of the major challenges in vehicle design. Accordingly, the topic has been studied intensely over the years and considerable understanding and knowledge have been accumulated and documented. At present one could summarize the state-of-the-art in the field by saying that the overall large-scale structural dynamics of the phenomena under consideration, i.e. brake squeal, groan, judder, etc., seem to be largely understood on both a qualitative as well as a quantitative basis [1–3].

Nevertheless, computer-based modeling and simulation of brake vibration and noise is not yet considered to have reached predictive power in the sense that the results would correspond reasonably to testing without substantial further a posteriori model updating [4]. The reasons for this lack of predictive capability seem to be twofold: First, variability and uncertainty aspects concerning geometry, material and manufacturing do play an important role, and much work in that direction is in progress [5]. Second, there is a tremendous lack of knowledge with respect to the small-scale properties and the small-scale dynamics of the contact interface, although it is often thought to be highly relevant for the resulting large-scale friction affected or friction-induced dynamics [6,7]. E.g., there are only few experimentally validated measurements with respect to the normal contact and its stiffness, while knowledge with respect to normal contact damping, or even tangential or more generalized contact properties do not seem to be available at all [8–11].

Although studies on large-scale properties of friction-induced vibration have gathered a plethora of results, corresponding studies with respect to dynamics-related small-scale interface properties and interfacial dynamics seem just in their early days in friction brakes. The present work is thus striving for a better understanding of dynamical interface processes taking place on small length and short time scales, including the role they play in the overall dynamics. Of course, the discussion of subscale effects on friction-induced vibration is in no sense new, see e.g. [12–18], but most of the previous work was either theoretical, based on computer modeling, or focused on materials sciences oriented...
approaches. To complement these existing numerical and theoretical approaches, vibration data based techniques have been chosen for the present study.

The investigation is carried out with a standard commercial vehicle friction brake that under adjusted conditions does not show any vibration excitation or noise emission like squealing, moaning or groaning, etc. Nevertheless, the brake under operation emits an irregular, seemingly noisy sound, which is sometimes called roughness in the industry community. It is primarily the underlying typical irregular vibration state of the brake that is the object of the present study. In general this irregular vibration is thought to be caused by some, or many, interfacial dynamical phenomena, which are largely unknown in detail, and therefore are often characterized by statistical means.

Since direct measurement or observation of the detailed dynamics in the friction interface is difficult to achieve without perturbing the overall system, the present study attempts to use techniques known from data analysis of nonlinear dynamical systems to extract properties of the system directly from measured time series data. Although such analysis techniques have been known and applied successfully in a number of scientific and engineering disciplines [19,20], it seems that their application to friction affected and friction-induced vibration has been rather limited. Interestingly, one of the few exceptions seems to be the recent work by Oberst and Lai [21,22], which have convincingly shown that brake squeal may also appear in the form of chaotic dynamics, and not only in the form of limit-cycles. While their work is focused on the large-scale dynamics of friction-induced brake squeal vibration and noise, the present study is focusing on the small-length and short-time scale dynamics induced by the friction interface itself. For that purpose vibration time series data have been collected with sampling rates of up to 200 kHz and the time series have then been subjected to recurrence and embedding analysis. The results are described in the following.

2. Experimental setup and measurement approach

For the study a vehicle friction brake has been assembled on an industrial noise dynamometer with an entire vehicle corner. The automation unit of the dynamometer was arbitrarily controllable and the measurement unit provided the operating conditions, e.g. temperature near the friction interface, sliding speed, brake pressure and load, friction coefficient, angle of rotation and environmental conditions. The vibration has been assessed by a piezoelectric accelerometer that has been mounted on the backing plate of the outer brake pad. A data acquisition system has been tuned to allow sampling rates of up to 200 kHz. The sampling rate has been motivated by the widespread ideas of asperity or plateau destruction and formation in the contact interface [23,24]: assuming, e.g., an asperity or plateau of about 1 μm in size, and assuming macroscopic sliding speeds of about 1 km/h, corresponding frequencies should be taken into account by measurements, which should become possible only with sampling rates beyond about 100 kHz. The analyzed lining materials have derived from the NAO and Low-Met families with their typical spread in terms of friction coefficient and wear performance. Each test procedure contained roughly 2000 brake applications with a widespread of parameter conditions.

Of course, since at present there seems to be no truly well-based idea on the highest relevant frequencies in the interface dynamics under consideration, the present limit of 200 kHz is definitely somewhat ad hoc. However, for all results presented in the following tests have been conducted with respect to the influence of the sampling rate: In many cases reasonable results have been obtained only for high-enough sampling rates, typically beyond the audible range. A further increase in sampling rate then did not change the qualitative and quantitative results any more, which suggests that the relevant dynamical processes in the frequency range considered have been resolved. In addition, a number of different sensor mounting configurations have been tested, and only results that are independent of the sensor mounting are presented in the following.

Fig. 1 shows a typical spectrogram of a characteristic acceleration signal for a typical brake application leading to a stop of the vehicle. One may first note that there is considerable frequency content beyond the audible range. Moreover, it is interesting to see that during a single brake application, lasting for a few seconds, the spectral distribution seems rather stationary. In contrast, the spectrogram may change substantially after longer braking, a number of brake applications, or when the braking parameters (temperature, velocity, pressure, etc.) are strongly changed.

3. Recurrence analysis and intermittency in irregular brake vibration

Recurrence, i.e. the state of a system coming arbitrarily close to a previous state after some time, is a very widespread phenomenon of many complex systems, and is especially well known from natural systems [25,26]. To visualize recurrence phenomena, a usually high-dimensional state-space has to be taken into consideration [15,16], which basically means that low-order models do fail in capturing essential properties of the arising system dynamics. Eckman et al. [27], however, first introduced a technique that overcomes the difficulty of spanning the high-dimensional state-space by introducing the approach of building up what is now typically called a recurrence plot: A square matrix is considered,

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1 Controlled parameters are in the following ranges: temperature $50^\circ \leq T \leq 300^\circ$, velocity $0.2 \leq v \leq 3.5$ m/s, brake pressure $0$ bar $< p \leq 52$ bar.
where each axis corresponds to a time-axis; if a state recurs after a certain time-interval, the corresponding point in the matrix is marked, such that formally the components $R_{ij}$ of the recurrence matrix can be specified by

$$R_{ij} = \theta(\epsilon - \|x_i - x_j\|),$$

where a (possibly high-dimensional) state at a certain time $t_i$ is denoted as $x_i$, $\epsilon$ is a threshold parameter, and $\theta$ stands for the Heaviside step function. Instead of using a threshold value, also the distance between each state at given time and the state at a later time can be plotted directly, $D_{ij} = \|x_i - x_j\|$. The resulting plot displays the distances between the states and in this way it also characterizes the recurrence behavior. Sometimes this representation is also called a global recurrence plot [28].

Fig. 2 shows typical recurrence plots for measured time-series during drag brake application, i.e. brake application with constant sliding speed. During braking the system does not exhibit squealing noise or any other low-frequency vibration, such that the vibration signal is the broad-band background signature of steady sliding. Fig. 2(a) represents the recurrence plot of a brake application at the beginning of a dynamometer test while Fig. 2(b) contains the data of a brake application after heating up the friction interface beyond 500 °C, which is a widespread procedure to explore the behavior of a friction brake after thermal load. Fig. 2(c, d) expands the first 200 samples on the basis of distance plots.

The recurrence plots show a number of marked features. First, in Fig. 2(c, d) there is a fine-structure of parallel, diagonally oriented lines. They basically indicate that after a state has recurred to a certain region in state space, both the original, as well as the recurring state evolve pretty much in parallel for a while, and also resemble each other again after a certain interval in time, which is a sign for (stable or unstable) periodic solutions underlying the overall dynamics. The diagonal fine-structure also shows that for most times, there does not seem to be a truly strong stochastic influence, which would lead to quick random separation of trajectories after recurrence. Interestingly, the fine-structure seems much weaker after thermal loading, which suggests that the thermal loading sort of randomizes the resulting dynamics. However, and second, in Fig. 2(a, b) there is also a checkerboard structure in the plot: there are darker regions indicating that the system is somehow trapped in the momentary state, and there are brighter regions where trapping or recurrence is less pronounced. In analogy to fluid dynamics the states within the darker regions are sometimes called ‘laminar states’ [29], while the states in brighter regions correspondingly might be called ‘turbulent states’.

Interestingly, the present visualization suggests that after about 1 ms (corresponding to about 200 samples) darker regions are disrupted, which means that on this time-scale the underlying evolution of the states deviates strongly from the previous ‘laminar’ evolution. It seems plausible that these disruptions should be caused by processes in the contact interface, like e.g. break away of asperities or plateaus. Although the analysis cannot give mechanistic explanations on the specifics of the underlying processes, it does allow to extract quantitative measures characterizing them.

To summarize the findings, one may say that the vibration dynamics appear to be strongly deterministic and dominated by periodic processes on short time-scales and random influences do not seem to be marked (‘laminar’). On longer time-scales of about 10 ms the system shows, however, strong intermittency between regular (‘laminar’) and disruptive (‘turbulence-like’) behavior.
4. Phase space reconstruction, embedding dimensions, Lyapunov exponents

Since the recurrence analysis suggests that the resulting dynamics is largely deterministic in the sense that remaining stochastic components seem weak, in the following data analysis techniques developed for nonlinear deterministic dynamical systems will be applied. First, a phase space reconstruction is carried out. From that an estimate on embedding dimensions is developed and maximum Lyapunov exponents are calculated for different conditions during braking.

To reconstruct the full phase space from the data, delay coordinates are used to increase the dimensionality of the trial system until uniqueness requirements of the reconstructed system’s flow are fulfilled [30]. To obtain a successful embedding an appropriate time delay has to be found, e.g. from time-scales inherent in the data, and a sufficiently large embedding dimension has to be determined [31]. To accomplish this we follow the method of ‘false nearest neighbor counting’ introduced by Kennel et al. [32]. It permits the choice of an embedding dimension which is based on the computation of the distances between each point in the time series in an $m$-dimensional phase space and its nearest neighbor. As the embedding dimension increases, the distance should not change if the points are truly nearest neighbors in an unfolded phase space. The criterion for a point to be a false neighbor – due to an embedding dimension chosen too low – is then

$$\frac{|x_{i+1} - x_{i+1}|}{\|x_i - x_j\|} > D_{th},$$  \hspace{1cm} (2)$$

where $D_{th}$ is a heuristically determined threshold value and $\|x_i - x_j\|$ denotes the Euclidean distance in the $m$-dimensional embedding space under consideration. When the fraction of points being false nearest neighbors has become sufficiently small, the embedding is considered successful.

One should note that for systems subjected to a small amount of noise – i.e. basically for all experimental setups due to measurement noise – the system dynamics can often still be unfolded to a large extent, as long as the effects of noise are
rather weak in comparison to the deterministic component of the dynamics. However, if very large embedding dimensions are needed, noise may also strongly shadow the deterministic dynamics of the system. In the present study effects of noise are not considered explicitly; it will be seen from the results that this purely deterministic approach works surprisingly well. Apparently, stochastic aspects seem to play a secondary role for the present analysis, only. For that reason the inclusion of stochasticity in the embedding approach is left to future studies; then the limitations of the purely deterministic reasoning might be explored by identifying Langevin type or Fokker–Planck type equations [33]. For completeness one should note that studies analogous to the present one, but based on computer simulation data, have also been conducted by Feeny and Liang [34] and Yuan and Feeny [35]. Since computer modeling does typically, however, not include sub-scale dynamics and corresponding complex vibration components, the specific questions related to the role of noise did not appear there.

Fig. 3 illustrates the results of calculations applying the false nearest neighbor method to typical signals. Three different brake applications are divided each into six equally spaced sections and the first 5000 samples are exploited in Fig. 3(a–c). Evidently, the fraction of the false nearest neighbors decreases rapidly for embedding dimensions up to about 10–15,
see Fig. 3(a). Then either the number of data-points becomes too small for the analysis, and the corresponding data points have been set to zero for practical purposes, or the false nearest neighbor count increases again, due to the influence of measurement noise. Fig. 3(b) depicts six different data sections from a friction couple that tends to generate more squeal in general. In this case the necessary embedding dimensions are even lower, roughly in the span of 6–10.

According to the previous considerations the present results suggest that the deterministic part of the system’s dynamics could be described in a phase space with only about 6–15 dimensions. Obviously, noise is superimposed on the deterministic dynamics, nevertheless analyzes with several data records have shown that even with varying conditions in terms of brake system parameters, the aspect of a rather low embedding dimension persists.

In addition one may note that in case of too low sampling rates the estimation of the dimensionality fails. E.g. Fig. 3(c) shows a sampling rate of only about 40 kHz, which corresponds to the beginning of the ultrasonic range in terms of the frequency resolution; the fraction of false nearest neighbors remains still rather high, between 0.3 and 0.4, and does therefore not yet allow a reasonable estimation of the embedding dimension. In turn this suggests that sampling with 200 kHz in fact seems to capture the relevant dynamics for reaching a successful embedding. For completeness Fig. 3(d) shows the results of the approach at hand when a synthetic white noise time-signal is analyzed: then the ratio of false nearest neighbors does not fall below a value of about 0.5. From this one may understand that false-nearest neighbor ratios below values of 0.1 or 0.2, as they result for the presently measured data, strongly indicate a dominant deterministic dynamics.

A large number of dynamical processes is active at the interface of a brake pad sliding over a brake disk: wear forms a third body, surface waves may be excited, bulk waves are involved, relaxation processes take place, possibly synchronization appears, to name just a few. Thus, to capture all these dynamically active and relevant effects, one should think that a large number of degrees of freedom have to be considered. In view of this and the overall complexity of a complete braking system, the apparently low embedding dimension during non-squealing conditions is quite a surprising result. It suggests that the resulting irregular dynamics of the friction excited system during steady squeal-free sliding might in fact be due to a more or less low-dimensional strange attractor which is only slightly affected by superimposed noise. This result is in sharp contrast to the widespread view, assumed e.g. also in state-of-the-art modeling, that steady sliding corresponds to a simple steady state, or a fixed point of the system. The present analysis rather suggests that the steadily sliding non-squealing system is in fact in a state of irregular chaos-like dynamics; of course it will – on long time and large length scales – appear stationary. But when taking into account the faster time and shorter length-scales, this seems not to be an adequate picture. Also, the system does not seem to be in a state that is just randomly excited by interface processes: the embedding analysis suggests that the nonlinear structural dynamics seems to select a rather small number of active degrees of freedom responding to the processes at the friction interface.

To gain additional confidence in the idea of a rather low-dimensional strange attractor hiding behind steady sliding in friction brakes, and to possible further characterize it, Lyapunov exponents have been calculated from the data: The Lyapunov exponents of a dynamical system characterize the temporal change in separation between adjacent points in phase space. They jointly form the Lyapunov spectrum [36]. A positive exponent corresponds to a direction with phase space expansion and indicates a strange attractor. Negative exponents are related to phase space contraction in the corresponding directions. In most cases, however, only the largest exponent is considered, since in deterministic systems a single positive Lyapunov exponent is already a sufficient criterion for chaos. In the following we will correspondingly always mean the largest exponent when we speak about ‘the’ Lyapunov exponent.

We determined the largest Lyapunov exponents after embedding by the following well-established approach [19]: for a given reference point of the embedded time series the average of all distances to neighboring points within a neighborhood of radius $\epsilon$ is computed. Then all the so selected points are followed forward in time and the time-evolution of the averaged distances is recorded. The slope of the natural logarithm of the resulting function gives the maximum Lyapunov coefficient. To obtain a global measure, the procedure is applied to a large number of reference points, and the local results are averaged. Formally the procedure can then be denoted as

$$S(\epsilon, m, n) = \frac{1}{N} \sum_{n_0 = 1}^{N} \ln \left( \frac{1}{|U(X_{n_0})|} \sum_{x_{n_0} \in U(X_{n_0})} |X_{n_0 + \Delta n} - X_{n_0}| \right),$$

where $x_{n_0}$ are the N reference points, $U(x_{n_0})$ stands for the neighborhood of $x_{n_0}$ with neighborhood size $\epsilon$ and $\Delta n$ denotes the discrete time variable (due to data acquisition).

The procedure can be applied successfully if the resulting function $S$ shows linear scaling behavior for a substantial range of time. Of course scaling cannot be expected for too short times, since then the exponentially growing part of the flow does not dominate the other components, yet. And also for too large times scaling will break down, since then the idea of a Lyapunov exponent as a local measure does not apply any more. Hence, only if $S$ shows reasonably linear scaling for a substantial interval, the maximum Lyapunov exponent can be determined from the slope in that interval.

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2 The velocity within the break application in Fig. 3(a) decreases from $v = 80$ km/h to $v = 30$ km/h while the temperature increases from $T = 100$ °C to $T = 160$ °C. Fig. 3(b) shows the result for a drag brake application with a constant velocity $v = 3$ km/h and temperature $T = 60$ °C, while the friction coefficient increases from $\mu = 0.38$ to 0.48. Similar to the data in Fig. 3(d) but with a temperature of $T = 100$ °C the estimation of dimension is depicted in Fig. 3(c). The delay in these examples is $\tau = 4$. 

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Before the approach can be applied, one has to make sure that the sampling rate is high enough to catch the relevant dynamic phenomena. Fig. 4(a) shows $S$ for different sampling rates. One may see that for too low sampling rates, i.e. even for rates at about 100 kHz, a scaling range cannot be identified. For larger sampling rates a scaling range shows up. This suggests that (1) the sampling rate is then high enough to include the significant interface processes and (2) that the calculation approach for the largest Lyapunov exponent has then become self-consistent. As a consequence, all subsequent analysis has been based on data acquired with a sampling rate of 200 kHz.

Another important issue is the estimation of the neighborhood size $E$ (where a Euclidean norm is assumed). Obviously too small values will render the analysis highly erroneous, since then too small a population of states is followed to evaluate the Lyapunov exponent and measurement noise plays a too large role. Contrariwise, when $E$ is too large, nonlocal effects grow in importance and the approach should also fail [19]. Fig. 4(b) gives the actual behavior of $S$ for different $E$. It shows that for $0.003 < \varepsilon < 0.012$ the results yield quite about the same slope for $8 < n < 18$. This suggests that the approach yields self-consistent results for appropriately chosen environment size.

From the data used to determine the neighborhood size $E$, Lyapunov exponents can directly be read of, as they are just given by the slope resulting within the linear scaling range of $S$. In the present analysis of Fig. 4(b) this estimation of the largest Lyapunov exponent yields $\lambda = 0.021 \pm 0.004$. In this result the error has been estimated from the linear regression used to obtain the slopes and is given as a measure of uncertainty or error due to the arbitrariness of selecting $E$ within the described range. The calculation was accomplished for a series of $n=1000$ samples in non-squealing condition with a dimension $m=8$ and a delay $\tau = 4$.

Since the analysis described until now was based on a data section selected somewhat arbitrarily from a longer data set, Fig. 5(a) gives results for a number of different sections of data sets during the same test but with a different characteristic load history, where the environment size $E$ has been set to the fixed value of $\varepsilon = 0.005$. Again a reasonable scaling region shows up, from which an average Lyapunov exponent of $\lambda = 0.032 \pm 0.003$ can be estimated.

In carrying out the data acquisition, the brake was running in a non-squealing state for roughly a minute under the same external conditions. All analyzes presented by now have been based on data sections within early phases of this total measurement. Apparently, the resulting time series may be considered as reasonably stationary on the time-scale of a few seconds, since otherwise the analyzes would not have given consistent results. After about 30 s the brake did, however, start to squeal. The phenomenon of a brake system changing from a non-squealing to a squealing state, or vice versa, is actually quite well known in the field and typically attributed to the change of some inner or underlying variables, like e.g. interface temperature or wear state. Within the present context we thus repeated the determination of the largest Lyapunov exponent when the brake was in non-squealing state, just before squeal and after the squeal had set in (see also Fig. 5).

When the brake is still non-squealing, but a posteriori it is known that squeal will appear in a few seconds, the behavior (Fig. 5b) is rather similar to what was seen before. There still is a scaling range pointing to a positive Lyapunov exponent. However, the length of the scaling region seems to have become markedly smaller. In contrast, when squeal has set in, the scaling range has disappeared (Fig. 5c). From analyzing this ‘approach to squeal’ for a number of different measurements.
we actually come to the conclusion that the length of the scaling range continuously decreases when the system comes closer to the squealing state. Squealing and non-squealing states thus seem to differ substantially with respect to their largest Lyapunov exponents: The non-squealing state seems to have a positive exponent, which points towards underlying chaos-like dynamics, while the squealing state does not manifest positive exponents.

In part the findings correspond well with the usually accepted picture of generation of squeal: typically squeal is thought to be related to the occurrence of a limit-cycle oscillation of the brake, possibly initiated through an instability of the steady sliding state. The present finding of non-positive Lyapunov exponents in the case of squeal corresponds well to this picture, since the largest Lyapunov exponent during limit cycle oscillation should just be zero. However, the finding of positive Lyapunov exponents during steady sliding sort of contradicts the idea of steady sliding corresponding to a fixed-point, or equilibrium solution of the system. In a sense this observation agrees with other findings in the context of surface dynamics in steady sliding (e.g. [6,13,37]).

In contrast to the widespread idea of the squealing limit-cycle appearing due to an instability of a steady sliding state, i.e. a fixed point in terms of dynamical systems, the present analysis rather suggests that the state that is called steady sliding bears characteristics of chaotic dynamics, and that the transition between steady (quiet) sliding and (audible) squeal could be the generation of a limit-cycle from a state bearing strong characteristics of chaos.

Interestingly, rather recently [21] time series analysis of acoustic emission during brake squeal has shown that squealing states may be periodic or chaotic, while the analyzed steady sliding resulted as a static equilibrium solution.

![Fig. 5. Estimation of the Lyapunov exponent of brake vibration data of five different sections in non-squealing conditions (a) and in transition from non-squealing to squealing condition (b). Attempt to estimate the maximum Lyapunov exponent in squealing conditions (c).](image-url)
In contrast, in the present study irregular, chaotic squealing has not been detected, while steady sliding seems to carry signatures of chaos. Although the findings seem contradictory, they might in fact just be complementary: both from a theoretical point of view, as well as from experimental findings, it is well accepted that friction brakes may sometimes show different states for the same external parameters. So it is tempting to hypothesize that both steady sliding, as well as squeal may come in different dynamical manifestations: in the form of a fixed point or a strange attractor, when steady sliding is concerned; or as limit cycle or strange attractor when it comes to squeal. Definitely further studies are necessary to test these hypotheses on the nature of states in brake dynamics and to understand better the corresponding transitions between these states.

5. Conclusions

Friction brake vibration data have been collected with sampling rates of up to 200 kHz. From the field of nonlinear time-series analysis three techniques have been adopted to analyze the data. (1) Recurrence analysis has shown that the system behavior is strongly intermittent, comprising laminar phases as well as disruptions. (2) Phase space reconstruction has been used to estimate the dimensionality of the dynamics in non-squealing conditions. It turns out that the resulting dynamics is not dominated by stochastic processes, but seems to be best characterized through a rather low-dimensional deterministic strange attractor, with some additional random components superimposed. (3) Extraction of Lyapunov exponents supports the hypothesis that behind the seemingly steady sliding of the non-squealing brake, a strange attractor, i.e. chaotic dynamics, is hidden.

From the findings one may conclude that analyzing measured vibration data of friction affected systems with techniques from nonlinear time-series analysis generates a number of new and interesting perspectives and results. The present analysis already allowed to estimate the dimensionality of the system’s deterministic kernel during steady sliding and to extract Lyapunov exponents. In consequence it seems that during steady sliding the system is in a chaotic state with some weak additional random components. Moreover, the analysis suggests that the appearance of squeal can be understood as the generation of a limit-cycle oscillation from this chaos-dominated state of steady sliding.

References