Multiscale synchrony behaviors of paired financial time series by 3D multi-continuum percolation

M. Wang *, J. Wang, B.T. Wang
Institute of Financial Mathematics and Financial Engineering, School of Science, Beijing Jiaotong University, Beijing 100044, PR China

Highlights

• A novel financial price model is developed on 3D multi-continuum percolation.
• The model explores nonlinear dynamics and multiscale synchrony of stock markets.
• Synchrony of paired returns (volatilities) and paired intrinsic modes is investigated.
• Cross recurrence quantification is applied to measure multiscale synchrony.
• Multiscale cross-sample entropy is utilized to study synchrony behaviors.

Abstract

Multiscale synchrony behaviors and nonlinear dynamics of paired financial time series are investigated, in an attempt to study the cross correlation relationships between two stock markets. A random stock price model is developed by a new system called three-dimensional (3D) multi-continuum percolation system, which is utilized to imitate the formation mechanism of price dynamics and explain the nonlinear behaviors found in financial time series. We assume that the price fluctuations are caused by the spread of investment information. The cluster of 3D multi-continuum percolation represents the cluster of investors who share the same investment attitude. In this paper, we focus on the paired return series, the paired volatility series, and the paired intrinsic mode functions which are decomposed by empirical mode decomposition. A new cross recurrence quantification analysis is put forward, combining with multiscale cross-sample entropy, to investigate the multiscale synchrony of these paired series from the proposed model. The corresponding research is also carried out for two China stock markets as comparison.

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1. Introduction

Financial market is known as a complex and nonlinear system. It is a big challenge to model financial fluctuation dynamics, since it is influenced by many direct and indirect factors, for instance, policy, economics, technology and environment. As the increasing globalization of financial markets, understanding the mechanisms of price fluctuations becomes a key problem. It is found that price fluctuations exhibit many interesting statistical behaviors such as volatility clustering, fat-tail phenomenon, power law behaviors, multifractality and complexity [1–13]. Econophysics is developed to better understand the nonlinear financial market dynamics. There have been some statistical physics systems to model the financial market, including various agent-based model, percolation model, Ising model, and so on [14–23]. The basic assumption of these
practical systems is that the price variation is mainly influenced by the interaction of investors. Stauffer and Penna developed a price model based on lattice percolation system [24]. Continuum percolation system is also utilized to investigate the multifractality and Zipf distributions of financial markets [25,26]. Lux and Marchesi introduced chartist agents competing with fundamentalists, leading to power law distributed returns as observed in real markets, contradicting the popular efficient market hypothesis [27]. Zhang et al. study the convergence for the proportions of buyers and sellers choosing different trading strategies through an agent-based simulation model called social network artificial stock market model, which is meaningful to investigate the financial market [28].

Recently, cross correlation behaviors of financial markets have sparked considerable attentions [29–32]. Classical cross correlation functions fail in detecting some significant correlations between two nonlinear systems [33]. However, mutual information to analyzing data of complex systems need long data series. Recurrence plots are developed by Eckmann et al. [34] to overcome the difficulties with nonstationary and short data series. The patterns of recurrence plots are directly linked with the original time series. Recurrence plots quantification analysis is developed in order to quantify the nonlinear characteristics of recurrence plots [35–38]. In this paper, cross recurrence plots are introduced to the financial markets to evaluate the synchrony of two nonlinear systems. Further the cross recurrence quantification analysis is employed to quantify the corresponding multiscale synchrony behaviors.

A new random stock price model based on the 3D multi-continuum percolation system is established here to investigate the nonlinear dynamics of stock markets. In this model, the cluster of 3D continuum percolation is utilized to describe a group of investors with the same investment attitude, and we assume that the volatility of stock prices is caused by dispersal of investing information, which refers to the local interactions of neighbor investors. We focus on two different radii of spheres for the continuum percolation price model, which represent different local influences among investors. We also give a ratio parameter to denote the proportion relation between these two different influences, which is different from the ordinary continuum percolation model [39–41]. Then we comparatively study the cross recurrence behaviors of two return series for the proposed model and the real stock markets. Moreover, the multiscale cross-sample entropy is introduced to investigate the synchrony behaviors with different time scales. The real data are selected from two China stock markets, Shanghai Stock Exchange (SSE) Index and Shenzhen Stock Exchange (SZSE) Index, including the data of daily closing prices from January 4th 2000 to April 7th 2008.

2. Financial price model by 3D multi-continuum percolation

2.1. 3D multi-continuum percolation system

Consider a $d$-dimensional Euclidean space $\mathbb{R}^d$, $X = \{\xi_1, \xi_2, \ldots\}$, which is a homogeneous Poisson point process, has a positive intensity $\lambda$. For each point $\xi_i$ of Poisson point process, place a sphere of radius $\rho_i$ with its center located at the point $\xi_i$, denote by $D(\xi_i)$. Let $\rho = \{\rho_1, \rho_2, \ldots\}$ be i.i.d. random variables and independent of Poisson point set $X$. Next we focus on the shape of a cluster of overlapping spheres. We say that percolation occurs, if any given random sphere belongs to a part of an infinite set of random spheres, with positive probability. Let $D(\xi_i, \rho_0 + \rho) = \{\xi_i: \|\xi_i - \xi_j\| \leq \rho_0 + \rho, \xi_i, \xi_j \in X\}$, which represents the sphere $D(\xi_i)$ located at its center $\xi_i$. If $\|\xi_i' - \xi_j'\| \leq \rho_0' + \rho'$, we say that two spheres $D(\xi_i')$ and $D(\xi_j')$ are adjacent, i.e., $\xi_i' \in D(\xi_i, \rho_0')$. And we say that two spheres are in the same cluster if there exists a sequence of $\xi_1', \xi_2', \ldots, \xi_k'$ such that $\xi_1' = \xi_i$, $\xi_k' \in D(\xi_{u+1}', \rho_0' + \rho_1')$, $1 \leq u \leq k - 1$, and $\xi_u' = \xi_j'$. The size of a cluster is calculated by the number of spheres that belong to the cluster itself.

Now consider the situation of $d = 3$, that is, three-dimensional Euclidean space $\mathbb{R}^3$. In the area of $[l, l]^3$ of $\mathbb{R}^3$, where $l$ is a positive integer, let random variable $\xi_0$ denote the Poisson point which is the nearest to the origin. And let $C(\xi_0)$ be the maximum adjacent subset containing $\xi_0$, which is usually called the percolation connective cluster. $|C(\xi_0)|$ represents the size of $C(\xi_0)$, which denotes the number of spheres in the cluster $C(\xi_0)$. In $\mathbb{R}^3$, if there exists a point $\xi_0$ such that $|C(\xi_0)| = \infty$, the cluster is known as an infinite cluster. Following the theory of continuum percolation, there is a non-trivial critical value $\lambda_c$ such that (almost surely) an infinite cluster always exists only to satisfy $\lambda > \lambda_c$. Let $P_{\lambda}$ be the probability function. $\lambda_c$ is defined by Meester and Roy [39] and Roy [41] as

$$\lambda_c = \inf\{\lambda \geq 0: P_{\lambda}(|C(\xi_0)| = \infty) \geq 0\}. \quad (1)$$

In the present paper, we consider a three-dimensional multi-continuum percolation system with different radii $\rho_1$ and $\rho_2$ ($\rho_1 < \rho_2$). We randomly assign them to the spheres with $\rho_1$ and $\rho_2$. Since radius parameter $\rho_i$ of the proposed model is utilized to describe the scope of spread of investment information, traders in the spheres with $\rho_2$ have stronger local influences than those in the spheres with $\rho_1$. And we add a new ratio parameter $\beta$ to represent the proportion relation between these two kinds of spheres, we have

$$\beta = \frac{\text{Number of spheres with } \rho_1}{\text{Number of spheres with } \rho_2}. \quad (2)$$
In this way, the local influences of investors are vividly showed by spheres of different radii in three-dimensional space. Fig. 1 presents the clusters of the 3D multi-continuum percolation system with different radii.

2.2. Modeling the financial price series

In the proposed financial 3D multi-continuum percolation model, we assume that the spread of investment opinions and information in the stock market is the main factor of the fluctuation of stock prices. Each trader can decide among three kinds of investment attitudes including buying, selling and neutral attitudes. The crucial radius parameter \(\rho\) of the proposed model is utilized to describe the scope of spread of investment information. Since different investors have distinct abilities and influence in the market, we generalize this into two types of radii in the proposed financial model, say \(\rho_1\) and \(\rho_2\) \((\rho_1 < \rho_2)\). Within the distance of \(\rho_1 + \rho_2\), investors are called neighbors, whom may group into a percolation cluster sharing the same investment attitude. Only neighbors can exchange investment opinions and information from each other in the light of the theory of continuum percolation.

Suppose that the traders of the stock market are represented by the Poisson points of \(X = \{\xi_0, \xi_1, \xi_2, \ldots\}\) in \([-l, l]^3\) of \(\mathbb{R}^3\) (where \(l\) is a positive integer) with intensity \(\lambda\). For every trading day \(t \in \{1, 2, \ldots, T\}\), each trader cannot trade more than one unit number of the stock for each transaction, then we establish the price stochastic dynamics as follows: For the percolation connective investors cluster \(C(\xi_0)\) (\(\xi_0\) is the Poisson point nearest to the origin), let \(\psi_t(\xi_0)\) represent the investment attitude of the trader at \(\xi_0\) who takes buying \((\psi_t(\xi_0) = 1)\), selling \((\psi_t(\xi_0) = -1)\) and neutral attitude \((\psi_t(\xi_0) = 0)\) with probability \(\omega_t, \nu_t\) and \(1 - \omega_t - \nu_t\), respectively (in the following, we let \(\omega_t = \nu_t = 0.5\) for simplicity). The trader at \(\xi_0\) exchanges his investment attitudes and information to his neighbors. On the basis of the continuum percolation dynamics, neighbors can have an influence on each other and they may change their investment opinions and finally agree on the same attitude \(\psi_t(\xi_0)\). Then the investment attitude of cluster \(C(\xi_0)\) is defined by \(\text{sgn}(C(\xi_0)) = \psi_t(\xi_0)\). Thus \(\text{sgn}(C(\xi_0))\) takes value of either 1 or \(-1\), which means buying or selling attitude. \(|C(\xi_0)|\) represents the size of \(C(\xi_0)\), which denotes the number of spheres in the cluster \(C(\xi_0)\). And the corresponding aggregate of excess demand \(B_t\) for a stock is defined by a local interaction cluster as

\[
B_t = \text{sgn}(C(\xi_0))|C(\xi_0)|/l^3.
\]

The formula of a discrete time price evolution model is given as follows

\[
S_t = S_{t-1} \exp(\alpha B_t), \quad S_t = S_0 \exp(\alpha \sum_{k=1}^{t} B_k)
\]

where \(\alpha > 0\) denotes the depth of the stock market. \(S_t\) is the stock price at trading day \(t\), \(S_0\) is the initial stock price at trading day \(t = 0\). Stock logarithmic return \(R_t\) is proportional to aggregate of excess demand \(B_t\), the corresponding coefficient is \(\alpha\). Then the formula of stock logarithmic return from \(t - 1\) to \(t\) is as follows \([42–44]\)

\[
R_t = \ln S_t - \ln S_{t-1}.
\]

Fig. 2 shows an example of simulated price series and return series generated by the 3D multi-continuum percolation model.
2.3. Empirical mode decomposition algorithm

Empirical mode decomposition (EMD) is a sifting algorithm for nonlinear and nonstationary time series [45,46]. The central idea of EMD is that the time series is locally decomposed into a summation formula of a local detail with higher frequency and a local trend with lower frequency. The higher frequency part is known as the intrinsic mode functions (IMFs), which describes the different scales of the original series. EMD decomposes the time series \( \{X(t), t \in I\} \) (where \( I = [1, 2, \ldots, 2000] \) in this paper) into a finite number of IMFs, which must satisfy the following conditions: (i) in the whole dataset, the number of extrema and the number of zero crossings must be completely equal or differ by at most one; (ii) at any point, the mean value of the upper envelope defined by the local maximal value and the lower envelope calculated by the local minimal value must be equal to zero. The EMD method extracting IMFs is an elegant algorithm, which can be used in six steps: (1) Determine all the maximal values and minimal values of time series \( X(t) \); (2) Calculate the upper envelope \( U(t) \) and lower envelope \( L(t) \) of \( X(t) \) by cubic spline interpolation, and therefore both envelopes will cover all the time series \( X(t) \); (3) Compute the mean value \( m(t) = (U(t)+L(t))/2 \); (4) Generate a new time series \( g(t) = X(t) - m(t) \); (5) Check if \( g(t) \) satisfies the IMF conditions. If it is an IMF, take it as the \( i \)th IMF \( c_i(t) \) and replace \( X(t) \) with the residual value \( r(t) = X(t) - g(t) \). If not, let \( X(t) = g(t) \) and keep the sifting process; (6) Repeat steps (1)–(5) until the residual part satisfies the stopping criterion, and we finally have

\[
r_n(t) = X(t) - \sum_{i=1}^{n} c_i(t), \quad r_{i-1}(t) - c_i = r_i(t)
\]

where \( r_n \) is usually a constant or a monotonic function standing for the trend of the series \( X(t) \).

3. Cross recurrence quantification analysis of 3D financial price model

A great variety of nonlinear methods has been proposed to study the complex systems, for instance, some methods to estimate fractal dimensions, Lyapunov exponents or mutual information [47–50]. But they all need long data series. To overcome this difficulty, recurrence plots (RP) has been developed by Eckmann et al. [34], which is later expanded by N. Marman et al. [51,52] to cross recurrence plots (CRP). In order to investigate interrelations and quantify the multiscale synchrony of two systems, cross recurrence quantification analysis (CRQA) [33] is developed, which allows us to compare the financial model with the real markets.

3.1. Methodology

Compared to recurrence plot (RP), cross recurrence plot (CRP) is different, because two time series are simultaneously embedded in the same phase space. Given two time series \( \mathbf{X} = \{x_1, x_2, \ldots, x_N\} \) and \( \mathbf{Y} = \{y_1, y_2, \ldots, y_M\} \). The cross recurrence plot is for testing the closeness between points in the phase space. Then a \( N \times M \) array called the cross recurrence plot (CRP) is defined as

\[
\text{CR}_{ij} = \Theta(\varepsilon - \|x_i - y_j\|)
\]

where \( \Theta(\cdot) \) is the Heaviside step function and \( \| \cdot \| \) is the norm (e.g., the Euclidean norm). \( \varepsilon \) is a threshold which means the recurrence tolerance. It is obvious that the values of cross recurrence plot contain only one and zero. Long diagonal structures correspond to similar phase space behavior of two systems. The main diagonal line will occur black when the difference of both systems vanishes. In this paper, we suppose that the two time series have the same length \( N \) and therefore the CRP turns out to be a \( N \times N \) array.
Table 1
CRQA measures of returns of the real data and the simulation data.

<table>
<thead>
<tr>
<th></th>
<th>RR</th>
<th>DET</th>
<th>L</th>
<th>ENTR</th>
<th>LAM</th>
<th>TT</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSE vs. SZSE</td>
<td>0.0491</td>
<td>0.7924</td>
<td>3.4696</td>
<td>1.6474</td>
<td>0.4163</td>
<td>2.9375</td>
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<tr>
<td>$\lambda = 6$ vs. $\lambda = 7$</td>
<td>0.0396</td>
<td>0.7523</td>
<td>3.2133</td>
<td>1.5524</td>
<td>0.3235</td>
<td>2.6124</td>
</tr>
</tbody>
</table>

In order to quantify the synchrony of two systems, some measures are defined in the cross recurrence quantification analysis (CRQA). Let index $t \in [-T, \ldots, T]$ denote the number of the diagonal, where $t = 0$ refers to the main diagonal, $t > 0$ stands for the diagonals above and $t < 0$ marks the diagonals below. Let $P_t(l)$ represent the distributions of the length of the diagonal line, parallel to the main diagonal. The recurrence rate $RR$ is determined by

$$RR(t) = \frac{1}{N - t} \sum_{l=1}^{N-t} lP_t(l).$$

With a given delay $t$, $RR$ calculates the probability of occurrence of semblable states in two systems. Higher values of $RR$ correspond to higher density of recurrence points in a diagonal. In a CRP, stochastic behavior results in very short diagonals while deterministic behavior lead to longer ones. The measure determinism $DET$ is the ratio of recurrence points that comprise diagonal structures to all recurrence points.

$$DET(t) = \frac{\sum_{l=1}^{N-t} lP_t(l)}{\sum_{l=1}^{N-t} P_t(l)}.$$  

If both systems behave similarly in the phase space, longer diagonals will increase in a CRP. Another measure is the average diagonal line length $L$, which is determined as

$$L(t) = \frac{\sum_{l=1}^{N-t} lP_t(l)}{\sum_{l=1}^{N-t} P_t(l)}.$$  

$L$ reveals the length of time of synchrony in these two systems. Higher values of $L$ correspond to longer diagonals. Analogous to the recurrence quantification analysis (RQA), there are measures Shannon entropy $ENTR$, laminarity $LAM$ and trapping time $TT$ for vertical recurrence lines [53]. Values of $ENTR$ increase, the complexity of systems will increase. The rise in $LAM$ and $TT$ refers to the increase of stability.

3.2. CRQA for the 3D financial price model and the real market

In this section, CRQA is employed to quantify the synchrony of the 3D financial price model with $\lambda = 6$ and $\lambda = 7$ when other model parameters are fixed. Correspondingly, the cross recurrence behaviors of the SSE and the SZSE are displayed as comparison. In the phase space, the false nearest neighbors method [54] is applied to determine the embedding dimension $D$, and the average mutual information method [55] is used to find the time delay $\omega$. For the cross recurrence threshold $\varepsilon$, it is suitable to take $10\%$ of the maximal diameter of the phase space. Fig. 3(a)(c) show the distance plot and the cross recurrence plot for SSE vs. SZSE. Fig. 3(b)(d) show the distance plot and the cross recurrence plot for the price model $\lambda = 6$ vs. $\lambda = 7$. Different colors in the distance plot represent different distance, which is determined by $\|x_i - y_j\|$. In the cross recurrence plot, it is clear that both the real market data and the simulation data are composed of vertical and horizontal lines, which are comprised of recurrence points. We also calculate the CRQA measures showed in Table 1. We find all the measures of the simulation data are similar to those of the real markets, which reveals the rationality of the proposed model to some extent.

Moreover, the EMD is applied to the SSE, the SZSE and the simulation data. In this section, we focus on IMF1, IMF2, IMF3, IMF4, IMF5 and IMF6. We further investigate the cross recurrence plots of their corresponding IMFs (using the distance plot instead of CRP). Fig. 4(a)–(f) show the CRPs of the IMFs of SSE vs. SZSE. Fig. 5(a)–(f) show the CRPs of the IMFs of $\lambda = 6$ vs. $\lambda = 7$. We find the pattern of the IMF1 of the real market is similar to that of the original return, this is mostly because IMF1 holds main characteristic of the original return. From IMF1 to IMF5, the cross recurrence plots show more vertical and horizontal lines. Especially for IMF6, with more short lines parallel to main diagonal lines, the recurrence pattern of IMF6 is obviously different from other five IMFs, because the IMF6 loses more information of the original return. IMF2 to IMF5 also lose information of return which can be observed in their CRPs. The IMFs of the simulated return also display similar properties to that of the real market, which reflects the proposed model can reflect some recurrence properties of real stock markets.

CRQA is also applied to the IMFs of SSE vs. SZSE and the proposed price model $\lambda = 6$ vs. $\lambda = 7$. The empirical results are displayed in Table 2. For RR, IMF5 of the real market has higher values, which indicates the higher density of recurrence
Fig. 3. (a) The distance plot of SSE vs. SZSE. (b) The distance plot of the proposed price model $\lambda = 6$ vs. $\lambda = 7$. (c) The cross recurrence plot of SSE vs. SZSE. (d) The cross recurrence plot of the proposed price model $\lambda = 6$ vs. $\lambda = 7$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 2
CRQA measures of IMFs of the real data and the simulation data.

<table>
<thead>
<tr>
<th></th>
<th>RR</th>
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<td>SSE vs. SZSE</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>IMF1</td>
<td>0.0015</td>
<td>0.2387</td>
<td>3.8261</td>
<td>0.9521</td>
<td>0.0180</td>
<td>2.0385</td>
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<tr>
<td>IMF2</td>
<td>0.0602</td>
<td>0.6558</td>
<td>5.6700</td>
<td>2.4207</td>
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<tr>
<td>IMF3</td>
<td>0.1937</td>
<td>0.9732</td>
<td>8.8343</td>
<td>2.8887</td>
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<tr>
<td>IMF4</td>
<td>0.2275</td>
<td>0.9941</td>
<td>13.1367</td>
<td>3.2017</td>
<td>0.9601</td>
<td>11.6528</td>
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<tr>
<td>IMF5</td>
<td>0.2641</td>
<td>0.9988</td>
<td>19.8692</td>
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<tr>
<td>IMF6</td>
<td>0.2369</td>
<td>0.9998</td>
<td>29.7156</td>
<td>4.1447</td>
<td>0.9999</td>
<td>27.2776</td>
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<tr>
<td>$\lambda = 6$ vs. $\lambda = 7$</td>
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<tr>
<td>IMF1</td>
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<td>2.3124</td>
<td>0.4534</td>
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<td>2.0328</td>
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<tr>
<td>IMF2</td>
<td>0.0683</td>
<td>0.6443</td>
<td>6.3219</td>
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<td>IMF4</td>
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</table>

points. Similarly, IMF1 has lowest density of recurrence points. For DET and L, IMF1, IMF2 and IMF3 have lower values than other IMFs, which shows that they are less predictable and stable. This is because they contain more information of the original return. For ENTR, IMF1 has the lowest values of all the IMFs, and therefore IMF1 has lowest complexity. For LAM and TT, IMF3, IMF4 and IMF5 with higher values are much more stable and predictable. For all the CRQA measures, IMF1 has obviously lowest values, which reveals IMF1 has weakest determinism behaviors of all the IMFs. We find that CRQA measures of the proposed price model $\lambda = 6$ vs. $\lambda = 7$ show similar behaviors to the real market.
4. Multiscale cross-sample entropy analysis

4.1. Multiscale cross-sample entropy

Cross-sample entropy is introduced by Richman and Moorman [56] to measure the degree of synchrony of two time series, which shows more consistency comparing to cross-approximate entropy [57,58]. Multiscale cross-sample entropy (MSCE) is based on the cross-sample entropy to investigate the synchrony behaviors in different time scales. Smaller values of MSCE correspond to greater synchrony, while larger values correspond to weaker synchrony.

Let \( \mathbf{x} \) and \( \mathbf{y} \) denote two different time series with the same length \( N \), \( \mathbf{x} = (x(1), x(2), \ldots, x(N)) \) and \( \mathbf{y} = (y(1), y(2), \ldots, y(N)) \). Firstly we construct the corresponding coarse-grained series \( \mathbf{u}^{(r)} \) and \( \mathbf{v}^{(r)} \). Each element of \( \mathbf{u}^{(r)} \) is defined as
Fig. 5. The distance plots of IMF1, . . . , IMF6 of the model $\lambda = 6$ vs. $\lambda = 7$, respectively.

\[ u_s(\tau) = \frac{1}{\tau} \sum_{s=1}^{N/\tau} x(i), \quad 1 \leq s \leq N/\tau. \]  

(11)

Similarly, each element of $v_s^{(\tau)}$ is defined as

\[ v_s^{(\tau)} = \frac{1}{\tau} \sum_{j=s(\tau-1)+1}^{s \tau} y(i), \quad 1 \leq s \leq N/\tau. \]  

(12)

It is obvious that the coarse-grained series is exactly the original time series when $\tau = 1$. 
4.2. MSCE analysis of real data and the financial model with different $\beta$.

Then, at the same scale factor $\tau$, we calculate the cross-sample entropy of each pair of coarse-grained series. Let $M$ denote the length of each coarse-grained series, where $M$ equals to $N/\tau$. For a pair of given coarse-grained series $u^{(\tau)} = (u_1^{(\tau)}, u_2^{(\tau)}, \ldots, u_M^{(\tau)})$ and $v^{(\tau)} = (v_1^{(\tau)}, v_2^{(\tau)}, \ldots, v_M^{(\tau)})$, we fix the vector length $m$ and define

$$f_m^{(\tau)}(i) = \{u_{i+k}^{(\tau)} : 0 \leq k \leq m - 1\}, \quad 1 \leq i \leq M - m + 1.$$  

$$g_m^{(\tau)}(i) = \{v_{i+k}^{(\tau)} : 0 \leq k \leq m - 1\}, \quad 1 \leq i \leq M - m + 1.$$  

We give the definition of the distance of $f_m^{(\tau)}(i)$ and $g_m^{(\tau)}(i)$ as

$$d(f_m^{(\tau)}(i), g_m^{(\tau)}(i)) = \max\{|u_{i+k}^{(\tau)} - v_{i+k}^{(\tau)}| : 0 \leq k \leq m - 1\}, \quad 1 \leq i \leq M - m + 1.$$  

Let $n_i^{(m)}$ represent the number that vectors $f_m^{(\tau)}(i)$ are within the tolerance distance $r$ of $g_m^{(\tau)}(i)$. Correspondingly, let $n_i^{(m+1)}$ be the number of length of $m + 1$. Then we have the formula of the cross-sample entropy as

$$\text{Cross-SampEn}(u^{(\tau)}, v^{(\tau)}, m, r) = -\ln\left[\frac{\sum_{i=1}^{M-m} n_i^{(m+1)}}{\sum_{i=1}^{M-m+1} n_i^{(m)}}\right].$$  

In conclusion, MSCE($x, y, \tau, m, r$) = Cross-SampEn($u^{(\tau)}, v^{(\tau)}, m, r$).

4.2. MSCE analysis of real data and 3D financial price model with different $\beta$

In this section, we apply the multiscale cross-sample entropy (MSCE) analysis to explore the synchrony behaviors of the financial model and the real stock market. Since the model parameter $\beta$ is of great importance, it represents the proportion relation of investors with different abilities of spreading information. Hence, we focus on the values of MSCE of the proposed model when $\beta$ changes from 0.6 to 0.8 with a step of 0.05, where we fix other model parameters as $\rho_1 = 0.6, \rho_2 = 0.8, \lambda = 5.5$ and $l = 30$. Fig. 6(a) shows the MSCE plot of SSE returns with SZSE returns compared to the financial model with different $\beta$. As the value of $\beta$ increases, the MSCE values increase at each time scale, which means that the larger the value of $\beta$ gets, the weaker the synchrony is. Fig. 6(b) shows the MSCE plot of the SZSE returns vs. the financial model with different $\beta$, which has similar synchrony behavior as the SSE returns. This also shows that the statistical behavior of the SSE data and the SZSE data are similar, which agrees with the fact. Also, with the time scale $\tau$ increasing, the values of MSCE of returns gradually decrease and finally reach constant, which reveals that they may contain correlations and complex structures. This property of real data vs. simulation data is similar as SSE vs. SZSE, which shows that the proposed model is reasonable to some extent.

Next, the EMD is used to SSE vs. $\beta = 0.6$ and SZSE vs. $\beta = 0.6$, and we investigate the synchrony behaviors of their IMFs. The MSCE values of IMF1, IMF2, IMF3, IMF4 and IMF5 for SSE vs. $\beta = 0.6$ are showed in Fig. 7(a). The MSCE values of both the IMF1 and the original return series gradually decrease and remain constant, as time scale gets larger. Except for IMF1, the MSCE values of other IMFs firstly increase at smaller time scales and then almost decrease when time scales become larger. This is because IMF1 seizes main structures of the original return series, and therefore has much more similar behaviors to the original return itself, while IMF4 and IMF5 contain less information and therefore behave a bit differently. The MSCE values of IMF1, IMF2, IMF3, IMF4 and IMF5 for the SZSE vs. $\beta = 0.6$, which is exhibited in Fig. 7(b), have similar empirical results to them of the SSE vs. $\beta = 0.6$. This also reveals that the simulation data has similar statistical behaviors as the SSE data and the SZSE data.

Furthermore, we study the MSCE results of volatility series $|R_t|^q$ for the real market data and the simulation data. Fig. 8(a) shows the MSCE values of $|R_t|^q$ of SSE vs $\beta = 0.6$ in different time scales. It is obvious that all MSCE values decrease as the
time scale increases. When \( q < 1 \), at each given time scale, the MSCE values decrease as \( q \) becomes smaller, manifesting an increase of synchrony behaviors with smaller \( q \), which can also be clearly observed in three-dimensional MSCE plot in Fig. 8(c). Whereas, for \( q \geq 1 \), the MSCE values decrease as \( q \) gets larger at each time scale, which is also presented in Fig. 8(e). These properties are similar to the MSCE values of \(|R_t|^q\) of SZSE vs. \( \beta = 0.6 \), which can be found in Fig. 8(b)/(d)/(f).

4.3. MSCE analysis of real data and 3D financial model with different \( \lambda \)

In this section, we investigate the MSCE values of the real data and the financial price model with \( \lambda \) changing from 5 to 7 in a step of 0.5, where other model parameters are fixed as \( \rho_1 = 0.6 \), \( \rho_2 = 0.8 \), \( \beta = 0.6 \) and \( l = 30 \). Since \( \lambda \) represents the density of investors in the model, different values of \( \lambda \) will play an important role in MSCE values. Fig. 9(a) exhibits the MSCE values of returns of SSE vs. SZSE and SSE vs. simulation data with different \( \lambda \). It is found that as \( \lambda \) becomes smaller, the MSCE values of SSE vs. simulation data also turn to be smaller. It indicates that the synchrony behavior of the proposed model becomes greater if \( \lambda \) decreases. Moreover, with the increase of time scales, the MSCE values of both the real market data and the simulation data are showed a downward trend, which also manifests the synchrony behavior of the proposed model is similar to that of the real market. Fig. 9(b) displays the MSCE values of returns of SSE vs. SZSE and SZSE vs. simulation data with different \( \lambda \), which shows the similar results.

Besides, after performing the EMD algorithm, we explore the synchrony behaviors of IMFs of SSE vs. simulation data with \( \lambda = 5 \) and SZSE vs. simulation data with \( \lambda = 5 \), which are presented in Fig. 10(a)/(b), respectively. It is clear that except for IMF1, the MSCE values of other IMFs have a rising trend at smaller time scales. However, all MSCE values of IMFs show a downward trend with the increase of time scale factor. Whether SSE vs. simulation data or SZSE vs. simulation data, the MSCE values of IMF1 have much more similar behaviors to the original return series than other IMFs, which also demonstrates that the IMF1 contains main structures of the original return series, and other IMFs have more noises. In addition, the MSCE values of IMF1 and IMF2 are lower than those of return series, especially for larger time scales, which indicates IMF1 and IMF2 have greater synchrony behaviors than the original return series. Whereas, IMF3, IMF4 and IMF5 behave in the opposite trend, which means they have weaker synchrony behaviors.

Finally, we further perform the MSCE analysis of volatility series \(|R_t|^q\) for different \( q \). Fig. 11(a)/(b) show the MSCE values of volatility series of SSE vs. simulation data with \( \lambda = 5 \) and SZSE vs. simulation data with \( \lambda = 5 \), respectively. It is clear to find that all MSCE values of volatility series gradually decrease with time scales increasing, indicating volatility series have greater synchrony behaviors under smaller time scales. For \( q < 1 \), the synchrony behavior of volatility series becomes weaker under larger \( q \), which is presented in Fig. 11(c)/(d). But, for \( q \geq 1 \), as \( q \) increases, the volatility series show an increase of synchrony, which is exhibited in Fig. 11(e)/(f). Synchrony behaviors of the proposed model are similar to the real market data, demonstrating the rationality of this model, to some extent.

5. Conclusion

In this paper, the 3D multi-continuum percolation system is developed to model the financial price dynamics, in order to have a deeper understanding of the mechanism of the stock market. In this proposed model, the cluster of multi-continuum percolation is used to stand for a cluster of the investors who share the same investment attitudes. Two radii parameters are introduced to describe various abilities of investors to spread investment information. And we add a ratio parameter \( \beta \) to represent the proportion relation between these two kinds of spheres. Then we comparatively investigate the multiscale synchrony of paired return series and corresponding IMFs of the financial model and real stock markets. Cross recurrence quantification analysis is applied to measure multiscale synchrony, and it is found that the simulation data has similar synchrony behaviors to that of real market SSE vs. SZSE for both return series and IMFs. Furthermore, the multiscale cross-sample entropy analysis is utilized to investigate the synchrony of paired financial series derived from the proposed model with varying parameter \( \beta \) and radius parameter \( \lambda \). We find that for return series, with the increase of time scales, the MSCE
Fig. 8. (a) MSCE plot of volatility series of the SSE and the financial model with $\beta = 0.6$. (b) MSCE plot of volatility series of the SZSE and the financial model with $\beta = 0.6$. (c)/(e) Three-dimensional MSCE plots of volatility series (for $q < 1$ and $q \geq 1$) of the SSE and the financial model with $\beta = 0.6$, respectively. (d)/(f) Three-dimensional MSCE plots of volatility series (for $q < 1$ and $q \geq 1$) of the SZSE and the financial model with $\beta = 0.6$, respectively.

Fig. 9. (a) MSCE plot of returns of SSE vs. SZSE and SSE vs. simulation data with different $\lambda$. (b) MSCE plot of returns of SSE vs. SZSE and SZSE vs. simulation data with different $\lambda$. 
values of both the real market data and the simulation data are showed a downward trend. For the IMFs, the MSCE values of nearly all IMFs have a rising trend at smaller time scales and have a downward trend at larger time scales, which is similar to real data. For volatility series $|R_t|^q$, the MSCE values decrease as $q$ becomes smaller when $q < 1$, whereas, the MSCE
values decrease as $q$ increases when $q \geq 1$. All these empirical results show that the simulated data has similar synchrony behaviors to those of real market data, which shows that the proposed 3D financial model is reasonable. In this paper, we make it possible to model the stock market by continuum percolation system in higher dimensional space, which is a new attempt. And further study of this proposed model will be performed in the near future.

Acknowledgment

The authors were supported in part by National Natural Science Foundation of China Grant No. 71271026.

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