Sensor Dynamics in High Dimensional Phase Spaces via Nonlinear Transformations: Application to Helicopter Loads Monitoring

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Abstract—Accurately determining component loads on a helicopter is an important goal in the helicopter structural integrity field, with repercussions on safety, component damage, maintenance schedules and other operations. Measuring dynamic component loads directly is possible, but these measurement methods are costly and are difficult to maintain. While the ultimate goal is to estimate the loads from flight state and control system parameters available in most helicopters, a necessary step is understanding the behavior of the loads under different flight conditions.

This paper explores the behavior of the main rotor normal bending loads in level flight, steady turn and rolling pullout flight conditions, as well as the potential of computational intelligence methods in understanding the dynamics. Time delay methods, residual variance analysis (gamma test) using genetic algorithms, unsupervised non-linear mapping and recurrence plot and quantification analysis were used. The results from this initial work demonstrate that there are important differences in the load behavior of the main rotor blade under the different flight conditions which must be taken into account when working with machine learning methods for developing prediction models.

I. INTRODUCTION

The operational loads experienced by rotary-wing aircraft are complex due to the dynamic rotating components operating at high frequencies. As a result of the large number of load cycles produced by the rotating components and the wide load spectrum experienced by a rotary-wing aircraft’s broad range of maneuvers, the fatigue lives of many components can be affected by even small changes in loads, with repercussions on safety, component damage, maintenance schedules, etc. Operational requirements are significantly expanding the role of military helicopter fleets in many countries. This expansion has resulted in helicopters flying missions that are beyond the design usage spectrum, which was originally used to life fatigue critical components. Due to this change in usage, there is a need to monitor individual aircraft usage to compare with the original design usage spectrum in order to more accurately determine the life of critical components. One of the key elements to tracking individual aircraft usage and calculating component retirement times is accurate determination of the component loads.

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Fig. 1: Australian Army Black Hawk helicopter.

The rotor system components and attachments are some of the most fatigue-critical structural components on a helicopter. Direct measurement of the dynamic loads in these areas has traditionally been accomplished through slip rings or telemetry systems, but these techniques are difficult to implement and maintain. While advances in sensor technologies in the past decade have produced compact, lightweight, and economical devices, high equipment costs and large data storage requirements still make direct load monitoring impractical. An important goal is to determine if the dynamic loads on the helicopter could be predicted solely from the flight state and control system parameters (FSCS), as these parameters are already recorded by the flight data recorder found on most helicopters. Moreover, not all helicopters have the sensor infrastructure required for direct monitoring of all of the component loads of interest. Much research has been carried out using machine learning methods to model operational loads experienced by fixed-wing aircraft structure [1], [2]. In the case of rotary-wing aircraft, the loading spectrum experienced by the airframe structure is significantly more complex since the dynamic rotating components operate at frequencies several orders of magnitude higher than for fixed-wing aircraft. There have been a number of attempts at estimating these loads on the helicopter indirectly with varying degrees of success [3] [4] [5]. This is an important goal from a practical point of view with implications on aircraft safety, scheduling maintenance operations, costs reductions, etc.
However, in order to approach these tasks it is of outmost importance to understand the properties and the characteristics of these loads and particularly the influence of the different regimes of operation and flight conditions on the behavior of the component loads. Clearly, understanding the nature of their behavior, information content and dynamics would provide knowledge and insight when using machine learning techniques in order to build prediction models, as a pervasive problem is the introduction of noise, irrelevancies and spurious effects, among others, due to the use of other sensor data as predictors. While many of the helicopter dynamic loads are of interest, this study selected only one of these loads as the target parameter: the main rotor blade normal bending (MRNBX). Similarly while there were over 50 flight conditions performed during the flight loads survey, only three manoeuvres are considered here: forward level flight at full speed (LF), left steady turn (ST) at 45° and left rolling pullout (RP) at 1.5g. While it is clear that these should have quite different dynamics, a computational exploration should provide insight into different their influence on the MRNBX loads is.

This paper describes the preliminary study exploring the potential of using computational intelligence methods for helicopter loads estimation for understanding the behavior of component loads in helicopters under different flight conditions, particularly in the main rotor system of the Australian Army Black Hawk (Fig. 1). The objectives of this work are: i) to explore the behavior of one of the load related variables for a given component (the main rotor blade normal bending (MRNBX)), ii) to give a preliminary characterization of the influence of different flight condition of that behavior and iii) to introduce visual computational intelligence techniques in addition to analytical tools, in order to facilitate the understanding of these types of complex processes to users in the aerospace domain.

II. HELICOPTER LOAD DATA

The data used for this work were obtained from a S-70A-9 Australian Army Black Hawk (UH-60/HH-60 variant) flight loads survey conducted in 2000 in a joint flight loads measurement program between the United States Air Force and the Australian Defence Force [6]. During these flight trials, 65 hours of useable flight test data were collected for a number of different steady state and transient flight conditions at several different altitudes and aircraft configurations. The strain data from the Black Hawk flight load survey were captured by 321 strain gauges, with 249 gauges on the airframe and 72 gauges on dynamic components. The airframe gauges were mounted on areas prone to cracking and structural distress, primarily in the upper cabin, tail cone, tail pylon, horizontal stabilator, external stores support system, and main rotor pylon. Accelerometers were installed to measure accelerations at several locations on the aircraft and other sensors captured flight state and control system parameters. The parameters were recorded at one of three sampling frequencies: 52 Hz, 416 Hz, and 832 Hz. Full details of the instrumentation and flight loads survey are provided in [6]. For the present study three flight records corresponding to MRNBX loads in three flight conditions, Level Flight, Steady Turn and Rolling Pullout were considered. Their lengths are comparable (453, 435 and 439 observations respectively).

III. COMPUTATIONAL AND MACHINE LEARNING METHODS

A. Time Delay Methods

In time series analysis coming from dynamic systems, phase space methods play an important role. They are given by vectors $\mathbf{X} \in \mathbb{R}^m$ where $m$ is a suitable dimension, given by an explicit system of $m$ differential equations in a purely deterministic system. The time evolution of a system can be studied by looking at the trajectories defined in the state (phase) space, where basins of attraction can be found when sets of initial conditions leads to the same asymptotic behavior of trajectories. Characteristic of chaotic systems is that attractors exhibit fractal structure. In real world systems, like the one considered here, phase spaces are not directly observable, but time series coming from monitored sensors are available. In these cases, phase space reconstruction is necessary using the method of delays. A one-dimensional time series is considered as a scalar quantity taken at multiples of a fixed sampling rate $\Delta t$, related to underlying state vectors $s_n = s(t(n \Delta t)) + \eta_n$ where $\eta_n$ represents a measurement noise. A delay reconstruction in $m$ dimensions is formed by the vectors

$$s_n = (s_{n-(m-1)\tau}, s_{n-(m-2)\tau}, \ldots, s_{n-r\tau}, s_n)$$

(1)

where $\tau$ is a multiple of the sampling rate representing a time difference in number of samples ($\tau \Delta t$ in time units). Embedding theorems [7] establish the conditions under which the behavior of $s_n$ in the delay space relate to the original trajectories of $x$. In general if $m$ is sufficiently large, that goal can be achieved. When finding such an embedding, the quantity $\hat{m} = m \tau$ represents the time span covered by the embedding vector $s_n$, but they must be estimated separately. In the case of $\tau$, typically the autocorrelation function and the mutual information [8] are used. If the signal is discretized into $p$ levels, the latter is given by

$$I_c = \sum_{i,j} p_{ij}(\tau) \ln(p_{ij}(\tau)) - 2 \sum_i p_i \ln(p_i)$$

(2)

where $p_i$ is the probability that that the signal is at the $i$-th level at time $t$ and $p_{ij}(\tau)$ is the probability that the signal is at the $i$-th level at time $t$ and at level $j$ at time $t + \tau$. Minima indicate lags with low redundancy. In the case of the autocorrelation function, the lower bound for $\tau$ is taken to be the value at which a 1/e decay is observed.

The false nearest neighbours technique [9] is typically used for estimating suitable embedding dimensions. It achieves this by revealing the points which become only neighbours as an artifact of a restricted embedding space, but drift apart as the embedding dimension is increased. Several statistics capture this concept and the one proposed by [10] is used in this paper:
\[ X_{fnn}(r) = \frac{\sum_{n=1}^{N-m-1} \Theta \left( \frac{|s_n^{(m+1)} - s_{k(n)}^{(m+1)}|}{\sqrt{k(n)}} - r \right) \Theta \left( \frac{\sigma}{\sqrt{m}} - \frac{|s_n^{(m)} - s_k^{(m)}|}{h(n)} \right)}{\sum_{n=1}^{N-m-1} \Theta \left( \frac{\sigma}{\sqrt{m}} - \frac{|s_n^{(m)} - s_k^{(m)}|}{h(n)} \right)} \]  

where \( m \) is the dimension, \( r \) is the threshold for the ratio of the distances between a point in \( m + 1 \) and in \( m \) dimensions, \( \sigma \) is the standard deviation of the series, \( s_{k(n)} \) is the closest neighbour to \( s_n \) in \( m \) dimensions, \( k(n) \) is the index of the time series element \( k \) different from \( n \) for which \( |s_n - s_k| = \min \) and \( \Theta \) is the Heaviside function.

Detecting the presence of chaos is important in any dynamic system and chaos is intuitively defined as sensitivity on initial conditions. It can be quantified as the rate at which two trajectories that start close to each other in the state space separate exponentially with time. Lyapunov exponents quantify the exponential divergence of initially close state space trajectories, thus providing information about the amount of chaos in the system [11], [12], [13]. The characteristic exponent describes the rate of separation of infinitesimally close trajectories in phase space, so that if \( x(t) \) and \( x(t) + \delta x(t) \) are two trajectories that start close in the phase space with a separation represented by \( \delta x(t) \), the sensitivity to initial conditions is given by \( |\delta x(t)| \approx e^{\lambda t} |\delta x_0| \), where \( x_0 \) is the initial state and \( \lambda \) the Lyapunov exponent, measuring the mean rate of separation of trajectories in the system [13]. Typically, positive Lyapunov exponents are associated to the presence of chaos in the system. However, positive exponents per se do not necessarily indicate chaos and it is better to see them as numeric measures that quantify the degree of “sensitivity to initial conditions” (i.e. local instability in a state space).

An important remark is that while Lyapunov exponents have been used extensively in the study of dynamic systems, it is known that the largest positive Lyapunov exponent does not indicate chaos in general, nor does the largest negative Lyapunov exponent indicate stability [14]. This situation becomes more complicated in real world systems where the Lyapunov spectrum is derived from observed data. Stochastic noise coming from the measurement process and/or the system itself may be a source of sensitivity with respect to initial conditions that is unrelated to chaotic behavior.

In the present case the estimations are made from time series, computed as [15], [16]

\[ S(\Delta n) = \frac{1}{N} \sum_{n_0=1}^{N} \ln \left( \frac{1}{|U(s_{n_0})|} \sum_{s_n \in U(s_{n_0})} |s_{n0+\Delta n} - s_{n+\Delta n}| \right) \]

where \( s_{n0} \) is a point of the series in the embedding space, \( U(s_{n0}) \) is a \( \epsilon \)-neighbourhood (for a given \( \epsilon \in \mathbb{R} \)), \( N \) is the number of point of the series, \( \Delta n \) is a a certain time away from \( n_0 \) and \( |.| \) denotes cardinality.

**B. Gamma Test (Residual Variance) Analysis**

A non-parametric approach for the exploration associated to phase space reconstructions based on time delay methods is the Gamma test [17], [18], [19]. It is a technique aimed at estimating the level of noise (its variance) present in a dataset. Noise is understood as any source of variation in a target/dependent variable that cannot be explained by a smooth model relating it with the input/predictor variables.

The fundamental information provided by this estimate is the feasibility of finding (fitting) a model that explains the dependent variable by a smooth deterministic function involving the observed input and output variables. Since model search is a costly, time consuming operation, knowing beforehand whether the information provided by the input variables is enough to build a smooth model is very helpful. It may give an indication that more explanatory variables should be incorporated to the data or that the underlying model may be very complex.

Let \( S \) be a system described in terms of a set of variables and with \( y \in \mathbb{R} \) being a variable of interest, potentially related to a set of \( m \) variables \( \mathbf{x} \in \mathbb{R}^m \) expressed as \( y = f(\mathbf{x}) + r \), where \( f \) is a smooth unknown function representing the system, \( r \) is a set of predictor variables and \( r \) is a random variable representing noise or unexplained variation. Under the assumptions of independency between \( \mathbf{x} \) and \( r \), continuity and bounded first and second derivatives for \( f \), it is possible to estimate the variance of the residual term using data obtained from \( S \).

Let \( \mathbf{x}_{N[i,k]} \) denote the \( k \)-th nearest neighbor of \( \mathbf{x}_{i} \) in the input set \( \{ \mathbf{x}_1, \ldots, \mathbf{x}_M \} \). If \( p \) is the number of nearest neighbors considered, for every \( k \in [1, p] \) a sequence of estimates of \( \text{E} \left( \frac{1}{2} (y'-y)^2 \right) \) based on sample means is computed as

\[ \gamma_M(k) = \frac{1}{2M} \sum_{i=1}^{M} |y_{N[i,k]} - y_i| \]

In each case, an ‘error’ indication is given by the mean squared distances between the \( k \) nearest neighbors, given by

\[ \delta_M(k) = \frac{1}{M} \sum_{i=1}^{M} |\mathbf{x}_{N[i,k]} - \mathbf{x}_i| \]

where \( \text{E} \) denotes the mathematical expectation and \( |.| \) Euclidean distance.

The relationship between \( \gamma_M(k) \) and \( \delta_M(k) \) is assumed linear as \( \delta_M(k) \to 0 \) and then \( \Gamma = \text{Var}(r) \) is obtained by linear regression

\[ \gamma_M(k) = \Gamma + G \delta_M(k) \]

Of particular importance is the vRatio (\( V_r \)), defined as a normalized \( \Gamma \) value. It is measured in units of the variance of the output variable, and allows comparisons across different datasets:

\[ V_r = \frac{\Gamma}{\text{Var}(y)} \]
C. Nonlinear Space Transformations

Many dynamic systems can be embedded into spaces of relatively low dimension \((m \in [2,3])\), visualized as scatter plots where orbits, trajectories and attractors can be inspected.

When the embedding dimension of the spaces is larger than 3, standard graphical representations become unfeasible. The same situation arises when the number of series that describe the state of a system is large, as is the case in many real world problems. However, nonlinear transformations of these spaces into lower dimensional spaces could be pursued, particularly if the extent to which the transformed space preserves the internal structure of its native space could be quantified.

The construction of a smaller feature space can be performed via a nonlinear transformation that maps the original set of \(N\)-dimensional objects under study \(O\) into another space \(\hat{O}\) of smaller dimension \(\hat{d} < N\). This approach has been used for data representation and visual data mining (knowledge and data exploration) [20]. There are essentially three kinds of spaces generally sought [21]: \(i\) spaces preserving the structure of the objects as determined by the original set of attributes or other properties (unsupervised approach), \(ii\) spaces preserving the distribution of an existing class or partition defined over the set of objects (supervised approach), and \(iii\) hybrid spaces. Data structure is one of the most important elements to consider and it can be approached by looking at similarity relationships [22] between the objects, as given by the set of original attributes [20]. From this point of view, transformations \(\phi\) can be constructed that minimize error measures of information loss [23] based on similarities or distances.

If \(\delta(\vec{x},\vec{y})\) is a dissimilarity measure between any two objects \(\vec{x},\vec{y} \in O\), and \(\zeta(\hat{\vec{x}},\hat{\vec{y}})\) is another dissimilarity measure defined on objects \(\hat{\vec{x}},\hat{\vec{y}} \in \hat{O}\) \((\hat{\vec{x}} = \phi(\vec{x}), \hat{\vec{y}} = \phi(\vec{y}))\), a frequently used error measure associated to the mapping \(\phi\) is the Sammon error [23]:

\[
E_s = \frac{1}{\sum_{\vec{x} \neq \vec{y}} \delta(\vec{x},\vec{y})} \sum_{\vec{x} \neq \vec{y}} \frac{\left(\delta(\vec{x},\vec{y}) - \zeta(\hat{\vec{x}},\hat{\vec{y}})\right)^2}{\delta(\vec{x},\vec{y})} \tag{9}
\]

It is possible to minimize Eq. 9 with a wide variety of methods, ranging from classical optimization to computational intelligence-based techniques. Here the Fletcher-Reeves algorithm (FR) is used, which is a well known technique used in deterministic optimization [24]. This kind of optimization is prone to local extrema entrapment, therefore it is recommended to try different random initial parameter vectors.

The accuracy of the mapping depends on the final error obtained in the optimization process. Explicit mappings can however be obtained from these solutions using neural networks, genetic programming, and other techniques. In general \(\phi\) is a nonlinear function and in order to compare results from transformations obtained with different algorithms or different initializations, a canonical representation is preferred. It can be obtained by performing a principal component transformation \(P\) after \(\phi\), so that the overall transformation is given by the composition

\[
\hat{\phi} = (\phi \circ P) \tag{10}
\]

referred to as the canonical mapping. Since \(P\) does not change the dimension of the new space, an advantage of the canonical mapping is that it simplifies the comparison of different solutions. It also contributes to the interpretability of the new variables, as they have a monotonic distribution of the variance. The images of the mapped objects can be used for the construction of a 3D model using virtual reality for visual data mining and data exploration. It will be done here with the reconstructed phase spaces obtained by embedding operations.

IV. Recurrence Plots and Recurrence Quantification Analysis

Recurrence plots [25], [26], [27] is a graphic tool to visualize the recurrence of states in a phase space. They investigate the m-dimensional phase space trajectory through a two-dimensional representation of its recurrences. Such recurrence of a state at time \(i\) and a different time \(j\) is actually a representation of a squared matrix defined as

\[R_{ij} = \Theta(\epsilon - ||\vec{x}_i - \vec{x}_j||), \quad \vec{x}_i, \vec{x}_j \in \mathbb{R}^m, i,j = 1, \ldots, N\]

where \(N\) is the number of considered states, \(x_i, \epsilon\) is a distance threshold, \(||.||\) is a distance norm (e.g. Euclidean) and \(\Theta\) is the Heaviside function. Usually state space vectors are not available and in that case a phase space can be reconstructed from a time series by using the time delay embedding of Eq. 1.

The features present in a recurrence plot can be quantified in several ways and there is a variety of measures proposed: Recurrent rate (RR) is the frequency with which a given state will recur:

\[RR = \frac{1}{N^2} \sum_{i,j=1}^{N} R_{ij} \tag{11}\]

Percentage determinism (DET): percentage of recurrence points for which diagonal lines in the recurrence plot are of minimal length \(\ell_{\text{min}}\)

\[DET = \frac{\sum_{\ell = \ell_{\text{min}}}^{\ell} P(\ell)}{\sum_{i,j=1}^{N} R_{ij}} \tag{12}\]

where \(P(\ell)\) is the frequency distribution of the lengths of diagonal lines \(\ell\) of minimal length \(\ell_{\text{min}}\). It is related to the predictability of the dynamical system (white noise has a recurrence plot with almost only single dots and very few diagonal lines, whereas a deterministic process has a recurrence plot with very few single dots but many long diagonal lines).

Trapping Time (TT) is an indication of how long the system remains in a specific state:

\[TT = \frac{\sum_{v = \ell_{\text{min}}}^{v} P(v)}{\sum_{v = \ell_{\text{min}}}^{v} P(v)} \tag{13}\]

where \(P(v)\) is the frequency distribution of the lengths of vertical lines \(v\) of minimal length \(\ell_{\text{min}}\).

Considered as inversely related to the maximum Lyapunov exponent, maxLine (LMAX) serves as an indication of chaos, but the relationship has been questioned:

\[L_{\text{max}} = \max_{\ell} \{v_i, \quad i = 1, \ldots, N_\ell\} \tag{14}\]

where \(N_\ell\) is the number of diagonal lines (excluding the main diagonal).
V. EXPERIMENTAL SETTINGS

The experimental methodology consists of the application of computational intelligence and machine learning techniques in various phases: (I) estimation of the key elements of the delay space reconstruction of the phase space associated to the sensor signals from each flight condition (determining the elements $\tau$ and $m$ of Eq. 1), (II) residual variance analysis in order to assess the noise variance levels and other properties related to potential modeling and complements the findings of stage (I). It includes a more detailed analysis of the previous findings in order to produce irregular embeddings which will further reduce the dimensionality of the representation and prediction problems. Phase (III) involves a visual representation and analysis of the reconstructed phase spaces using nonlinear mappings and recurrence plots.

A. Data preprocessing

Each of the MRNBX sensor signals corresponding to the different flight conditions were normalized to z-scores (zero mean and unit variance), thus making their values comparable across flight conditions. In order to explore the structure of the time dependencies, tuples describing the state of the system in terms of the time lagged values along the time series with respect to each time $t$ can be formed as

$$\{S(t-\hat{\tau}), \ldots, S(t-2), S(t-1)\}, \ S(t)$$

where $S$ is the MRNBX sensor series for a given flight condition and $\hat{\tau}$ is a maximum embedding lag. The curly brackets separate the predictor from the target components of the tuple.

In this study $\hat{\tau}$ was set according to the $\tau$ and $m$ values found in stage (I) in order to cover the suggested maximum lag found by the Gamma-test increased embedding exploration. They were explored using genetic algorithms (GA) (Eq. 8). The problem representation chosen was based on binary chromosomes coding the characteristic functions of each predictor variable, so that the position of the 1 bits indicate whether the lag corresponding to the given bit index is retained. Table I shows the GA parameters used in the experiments. The number of nearest neighbors used for the computation of $V_r$ was determined for the delay space series of each flight condition prior to the application of the GA.

The fitness function ($\mathcal{F}$) used by the genetic algorithm is the one used in [28]

$$\mathcal{F} = W_T F_T + W_G F_G + W_L F_L$$

where $F_T$, $F_G$ and $F_L$ are the contributions to the overall fitness coming from the Vratio, the Gradient and the number of non-zero lags in the chromosome respectively, weighted by the corresponding factors $W_T = 1$, $W_G = 0.1$ and $W_L = 0.1$. The contributions to the fitness are given by

$$F_T = \begin{cases} 1 - (1 - 10V_r)^{-1} & \text{if } V_r < 0 \\ 2 - 2(1 + V_r)^{-1} & \text{otherwise} \end{cases}$$

$$F_G = 1 - (|G|/\text{range}(G))^{-1}$$

$$F_L = I(C)/\text{card}(C)$$

where $C$ denotes an individual in the population, card$(C)$ its cardinality and $I(C)$ a function returning the number of non-zero elements in $C$.

VI. RESULTS

The values of $\tau$ from Eq. 2 and inspection of the autocorrelation function (Table II) suggest that sampling the series with $\tau = 2$ would considerably reduce redundancies.

TABLE II: First three lag values of the autocorrelation function for each flight condition.

<table>
<thead>
<tr>
<th>Lag</th>
<th>Level Flight</th>
<th>Steady Turn</th>
<th>Rolling Pullout</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.488</td>
<td>0.329</td>
<td>0.227</td>
</tr>
<tr>
<td>2</td>
<td>0.320</td>
<td>0.367</td>
<td>0.403</td>
</tr>
<tr>
<td>3</td>
<td>-0.068</td>
<td>-0.208</td>
<td>0.123</td>
</tr>
</tbody>
</table>

Using this delay, the behavior of $X_{fnn}$ (Eq. 2) for ratio thresholds $r \in \{1, 2, 3\}$ is shown in Fig. 2. For Level Flight (LF) and Steady Turn (ST) it indicates that values around $m = 12$ are appropriate. For Rolling Pullout (RP), $X_{fnn}$ stabilizes at the lowest values at about $m = 15$. Accordingly, a total time lag of $\hat{\tau} = (3\tau m) = 30$ would be sufficient but 40 was chosen for a more conservative estimate.

![Fig. 2: Behavior of the $X_{fnn}(r)$ function (Eq. 3) for all flight conditions and thresholds given by $r = 1, 2, 3$.](image)

Lagged vectors $\{S(t-40), S(t-39), \ldots, S(t-1), S(t)\}$ were formed using the $\hat{\tau}$ value in order to investigate potential
irregular embeddings with the Gamma test based genetic algorithm. The irregular embedding results corresponding to the best GA-individual in the final population are shown in Table III.

**TABLE III: Genetic Algorithm embedding results for the MRNBX sensor series under three flight conditions**

<table>
<thead>
<tr>
<th>Flight Condition</th>
<th>VRatio ($V_r$)</th>
<th>Gradient (G)</th>
<th>Number of Contributing Lags (max = 40)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level flight</td>
<td>2.8632E-7</td>
<td>0.0205</td>
<td>26</td>
</tr>
<tr>
<td>Steady Turn</td>
<td>4.1976E-7</td>
<td>0.0268</td>
<td>20</td>
</tr>
<tr>
<td>Rolling pullout</td>
<td>4.5020E-6</td>
<td>0.0289</td>
<td>25</td>
</tr>
</tbody>
</table>

Interestingly, whereas the number of contributing lags exhibits no important variations with the flight condition, the $V_r$ and $G$ show a progression from low to higher values as the flight condition spans the sequence LF→ST→RP. This indicates a simultaneous decrease in determinism and increase in underlying model complexity. Even though the total time lag remains at $\tau=40$, about one half of the lags are meaningful for model building. Moreover, an important reduction of the dimensionality of the reconstructed phase spaces was achieved (26, 20, 25 vs 40). As such, they are still too high-dimensional and not amenable to direct inspection, but a nonlinear mapping (Eq. 10) could provide a reasonable low-dimensional representation provided that the error mapping is reasonably low. Using Euclidean distance as a dissimilarity measure on both the original and the target spaces, five random initializations were tried for each flight condition when solving Eq. 9. The resulting $\mathbb{R}^3$ spaces are shown in Fig. 3. Coloured spheres indicate delay vectors corresponding to each point of the original time series whereas line segments link consecutive time points, thus indicating the trajectories in the phase space. The red, green, blue colors correspond to values of the time series according to $\{<\bar{s}-\sigma), (\bar{s}-\sigma), (\bar{s}+\sigma)>, (\bar{s}+\sigma)\}$, where $\bar{s}$ is the mean of the time series and $\sigma$ its standard deviation (0 and 1 respectively, as the series is normalized to z-scores).

The mapping error $S_e$ in Eq. 9 increases along LF→ST→RP sequence, indicating the greater complexity of the structure of the original $\mathbb{R}^{25}$ space for RP. It is clear that recurrent orbits are more widespread, with greater interdistance between neighboring delay vectors, consistent with the Gamma test parameters.

A representation of the high-dimensional approximations to the phase spaces of MRNBX via its nonlinear transformation to a virtual reality 3-D space are shown in Fig. 3. It is impossible to reproduce 3-D structures on hard media. Therefore only 2-D snapshots from fixed perspectives can be presented and two of them are presented for the space corresponding to each flight condition. In all cases, it can be seen that the initial and final states of the system are very different (note that distance is inversely related to similarity) and that the system’s trajectory in the mapped phase space passes through the similar states at different times.

A similar picture is provided by the recurrence plots ([29]) and the quantification analysis RQA (Fig. 4). They exhibit an increased degree of complexity as the flight condition spans LF→ST→RP, like the spaces obtained with nonlinear mapping. The subjective impression derived from the visual inspection is complemented with the quantitative measures associated with the recurrent plot matrix. They are shown in Table IV. The RQA parameters DET, RR and maxLine sharply decrease along LF→ST→RP. The Lyapunov exponents obtained were 2.83, 4.03, 3.74 for LF, ST and RP respectively. The value for Level Flight is much smaller than those for the other two flight conditions, which is consistent with the relatively simpler complexity of the LF manoeuvre as...
Fig. 3: Nonlinear transformation of high dimensional delay-spaces associated to the MRNBX sensor for different flight conditions. In all cases the left figure corresponds to a snapshot of the resulting $\mathbb{R}^3$ space and the right one to a slight rotation over the X axis as a second perspective. The initial and final points of the original MRNBX time series for each flight condition are indicated with arrows. The MRNBX sensor values in each of the original series are indicated with colors (Red = low, Green = medium, Blue = high). Line segments join consecutive points in time and their sets define the orbits in the delay-spaces.
TABLE IV: Recurrent Quantification Analysis results for all flight conditions

<table>
<thead>
<tr>
<th>RQA Parameter</th>
<th>Level Flight</th>
<th>Steady Turn</th>
<th>Rolling Pullout</th>
</tr>
</thead>
<tbody>
<tr>
<td>RR</td>
<td>2.202</td>
<td>1.273</td>
<td>1.03</td>
</tr>
<tr>
<td>DET</td>
<td>0.734</td>
<td>34.921</td>
<td>13.725</td>
</tr>
<tr>
<td>MaxLine</td>
<td>88</td>
<td>22</td>
<td>7</td>
</tr>
</tbody>
</table>

compared to ST and RP and one would expect the Lyapunov exponent value to increase with the manoeuvre complexity. However, these last estimates should be considered only for orientation purposes, given the relatively short size of the time series analyzed.

VII. Conclusions

A preliminary exploration of loads in critical helicopter components like the main rotor blade normal bending in level flight, steady turn and rolling pullout flight conditions exhibits a differential behavior from the point of view of the dynamics of the physical process. Several techniques from the fields of computational intelligence and non linear systems analysis coincide in identifying an increasing degree of complexity in the sequence level flight, steady turn and rolling pullout maneuvers. These differences must be taken into account when applying machine learning techniques for building predicting models of component loads using flight control and system parameters model as predictors. Despite the complexities of the process, visual data mining tools proved to be very effective at understanding the behavior of the phase spaces associated to the processes. More flight records and more flight conditions must be analyzed in order to have a better characterization of the flight dynamics.

References


