Characterization of fluidized beds hydrodynamics by recurrence quantification analysis and wavelet transform

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Abstract

This paper reports the development of nonlinear time series analysis technique based on recurrence quantification analysis (RQA) method to characterize the hydrodynamic of gas-solid fluidized beds and a comparison with the obtained results by wavelet transform (WT) analysis method is made. An experimental work has been carried out at varying conditions, e.g. bed diameter (5, 9, 15 cm ID), particle size (150, 300 and 600 μm), bed height at aspect ratios (1, 1.5 and 2) and superficial gas velocities (ranging 0.1–1.7 m/s). Both methods show that by using larger particles and higher aspect ratios, the contribution of macro structures increases in the system. By increasing the gas velocity, finer structures in the bed first lose their contribution and after passing a transition velocity (of about 0.3, 0.5 and 0.7 m/s for sands with mean diameters of 150, 300 and 600 μm respectively) their contribution increases again. While the frequency domain analysis is not sensitive to the effect of scale; the RQA method shows an increase in meso structure contribution by increasing of the bed diameter.

Keywords:
Fluidized bed
Recurrence quantification analysis
Pressure fluctuations
Wavelet transform

Introduction

Fluidized bed reactors have numerous advantages over fixed bed reactors, making them an important class of reactors which are extensively used in various multiphase flow industrial applications such as chemical, agricultural, metallurgical, environmental and pharmaceutical. The wide application of fluidized beds is because of their efficient contact between fluid and particulate phases, thermal homogeneity, high mixing ability and high heat and mass transfer rates (Kunii and Levenspiel, 1977; Llauro and Llop, 2006; Schouten and Van den Bleek, 1998; Seemann et al., 2006; van Ommen et al., 2004). In spite of all these advantages, proper description of the hydrodynamics of gas-solid fluidized beds is difficult since they are a heterogeneous mixture of gas and solids exhibiting a liquid-like behavior (Rudisuli et al., 2012). Due to this complexity of interactions between particles and fluid, hydrodynamics of fluidized systems have been intensively studied over the years (Glicksman et al., 1994; Zhao and Yang, 2003; Chen and Fan, 1992; Sasic et al., 2007; Knowlton et al., 2005; Tahmasebpoor et al., 2013a). However, yet gas-solid fluidization has remained a notoriously difficult to quantify phenomenon and an improved understanding and accurate modeling are still important.

The equations governing fluidized beds system are relatively complex and sometimes unclear. Therefore, the hydrodynamics of gas-solid fluidized beds are usually studied using time series evaluation of the measured signals (Johnsson et al., 2000; Schouten and van den Bleek, 1998). Different time series signals can be used for studying the bed hydrodynamics such as pressure fluctuations (Johnsson et al., 2000; Schouten and van den Bleek, 1998; Sasic et al., 2007), bed vibration (Abbasi et al., 2009a,b; Azizpour et al., 2011), acoustic emissions (Vervloet et al., 2010; Salehi-Nik et al., 2009; Karimi et al., 2012) and local porosity (Yutani et al., 1986). However, pressure fluctuation measurements have some advantages that make them suitable for many practical applications. These advantages are ease of measurement and inclusion of the effect of various dynamic phenomena, such as bubble hydrodynamics, occurring in the bed (van Ommen et al., 2011; Johnsson et al., 2000).

Most researchers, who investigated fluidized beds based on pressure fluctuations, have considered that the hydrodynamic of a fluidized bed is non-linear, and could be even considered chaotic, although the latter was being debated (van der Schaaf et al., 2004). The technique that takes account of the nonlinearity of the dynamics is called chaos analysis, in comparison to statistical (e.g. standard deviation, averages) and spectral (e.g. Fourier transform, power spectrum) analysis. The chaos analysis is a way to describe the state of a nonlinear system by projecting the variables governing of the system into its multi-dimensional state space.
In this context, recurrence plot (RP) and recurrence quantification analysis (RQA) as new techniques based on the nonlinearity of the dynamics have been developed. The concept of the RP and RQA, which relies on the presence of recurring/deterministic structures underlying the data, is a basic property of dynamical systems, which can be exploited to describe the behavior of the system in the phase space (Marwan et al., 2007). Tahmasebpour et al. (2013a,b) demonstrated that RQA is a powerful tool to study the fluidization hydrodynamics.

In the present study, nonlinear time series analysis techniques based on RP and RQA are explored to characterize the hydrodynamics of bubbling gas-solid fluidized beds using measured pressure fluctuation signals. Moreover, the effect of different operational parameters such as superficial gas velocity, size of particles, settled bed height and bed diameter are studied and a comparison is made with the obtained results by wavelet transform analysis method.

**Experiments**

The experimental setup is schematically shown in Fig. 1. Experiments were carried out in three different gas-solid fluidized beds made of Plexiglas pipe with bed diameters \( D \) of 5, 9 and 15 cm. Air at room temperature entered the columns through a perforated plate distributor of holes with 7-mm triangle pitch and its flow rate was measured by a mass flow controller. The type of used distributor in all beds was the same. A cyclone, placed at the column exit, would return the entrained solids back to the bed. Various initial aspect ratios of solids \( L/D = 1, 1.5 \) and 2) were used in experiments; \( L \) is settled bed height.

Sand particles (Geldart B) with a Sauter mean diameter of 150, 300 and 600 \( \mu \)m and a particle density of 2640 kg/m\(^3\) were used in the experiments. Since the focusing of this study was on the bubbling regime of fluidization, the superficial gas velocity for each particle was changed in the range of \( U_{mf} < U < U_c \). The amount of minimum fluidization velocity; \( U_{mf} \) (Puncochar et al., 1985) was considered about 0.018, 0.059 and 0.273 m/s and bubbling to turbulent transition velocity; \( U_c \) (Tahmasebpour et al., 2013b; Bi and Grace, 1995) was assumed about 0.92, 1.12 and 1.32 m/s for 150, 300 and 600 \( \mu \)m particles, respectively. According to literature, aspect ratio of 2 was not applied to 600 \( \mu \)m particles to avoid slug-ging condition (Rhodes, 2008). Each set of experiments was repeated two times and error bars calculated from the standard deviation of each set of measurements were plotted on the corresponding data points.

Pressure probe (model SEN-3248 (B075), Kobold Company) was screwed onto the gluing studs located at different distances above the distributor. This probe had a response time of less than 1 ms. Probes were located at the wall to avoid the effect of design parameters on obtained results. Measured absolute pressure signals were band-pass filtered at lower cut-off frequency of 0.1 Hz and upper cut-off Nyquist frequency of 200 Hz. The filtered signals were then amplified and sent to a data acquisition board system. Johnsson et al. (2000) and van der Stappen et al. (1993) recommended that the sampling frequency within the range of 50–100 times the average cycle frequency (typically between 100 and 600 Hz) is required for nonlinear evaluation of pressure fluctuations in bubbling fluidized beds. The sampling frequency of 400 Hz for pressure fluctuations was used in this work. Total number of data points in each sample was 120,000 corresponding to 300 s sampling time.

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**Fig. 1.** Schematic of the experimental setup.
Method of analysis

The methods of time-series analysis used in this work are briefly described in this section. Two recently introduced methods in phase space domain named RP and RQA methods are applied to examine time-series signals and also discrete wavelet transfer is used to compare the results.

Recurrence plot

The recurrences plot (RP); derived from the nonlinear properties of the systems, is based on a graphical explanation of the dynamics of a system. Eckmann et al. (1987) introduced the RP as a graphical tool that can visualize the recurrences of dynamical systems. A RP provides a qualitative picture of the correlations between the states of a time series over all available time-scales. Usually, a phase space is a high dimensional space and can only be visualized by projection into two or three dimensional sub-spaces. Eckmann’s tool makes it possible to investigate the m-dimensional phase space trajectory through a two-dimensional representation of its recurrences. The RP is a 2-dimensional plot which is mathematically expressed as:

$$R_{ij} = \Theta(\varepsilon - ||x_i - x_j||) \quad i, j = 1, 2, 3, \ldots, N$$  \hspace{1cm} (1)

where $N$ is the number of considered states, $x_i, x_j \in \mathbb{R}^d$ represent the $i$-th and $j$-th points of the $d$-dimensional state space trajectory, $|| ||$ represent the norm, $\varepsilon$ is a threshold distance and $\Theta$ is the Heaviside function. The RP is obtained by plotting the recurrence matrix, Eq. (1); if $R_{ij} = 1$ it is considered a a recurrence point and appears as a black dot at the coordinate ($i, j$), if $R_{ij} = 0$ it is shown as a white dot. March et al. (2005) showed that the RP can be constructed without embedding. Thus, the embedding parameters of 1 based on the Takens’ theorem (Takens, 1981) were chosen for the analysis in this work.

Recurrence quantification analysis

Visual evaluation of the RP may be difficult. Therefore, the RQA was developed in order to quantify different appearing of RPs based on small-scale structures therein. The RQA involves estimation of some parameters (called recurrence parameters) that describe the structures in the RP. These structures are single dots, diagonal and vertical (or horizontal) lines (Marwan et al., 2007). The structures within a RP are related to different dynamics within the system. Recurrence rate, determinism and entropy are three RQA variables that were used in this study.

Recurrence rate (RR) expresses the density of recurrence points throughout the trajectory and is mathematically defined as:

$$RR = \frac{1}{N^2} \sum_{i,j=1}^{N} R_{ij}$$  \hspace{1cm} (2)

where $\sum R_{ij}$ is the total number of repeated points.

Determinism (DET) quantifies the fraction of repeated points forming diagonal line structures and is defined as:

$$DET = \frac{\sum_{l=l_{\text{min}}}^{N} l \cdot P(l)}{\sum_{l=1}^{N} l \cdot P(l)}$$  \hspace{1cm} (3)

Diagonal lines shorter than $l_{\text{min}}$ (usually $l_{\text{min}} = 2$) are not considered during calculation of DET. Length of a diagonal line ($l$) is defined as the number of its black dots and $\sum l \cdot P(l)$ is the number of black dots forming diagonal lines. Determinism is related to predictability of the system. For example, determinism is small for a stochastic system while it is large for a periodic system.

Entropy (ENT) refers to the Shannon information entropy of the probability distribution $(P(l) = P(l)/N_l)$ of the diagonal line lengths $l$, where $N_l = \sum_{l=l_{\text{min}}}^{N} P(l)$ is the total number of diagonal lines.

$$Entropy = -\sum_{l=l_{\text{min}}}^{N} P(l) \log_2 P(l)$$  \hspace{1cm} (4)

Entropy is expressed in bits/cycle and quantifies the complexity of the system. For example, entropy is zero for a periodic system in which all diagonal lines are of equal length, but relatively large within a chaotic system indicating its high complexity (Marwan et al., 2007).

![Fig. 2. Relative energies of macro, meso and micro structures in the pressure signals measured at 15 cm above the distributor for (a) 150 μm, (b) 300 μm and (c) 600 μm sand particles in 15 cm ID bed and $L/D = 1.5$.](image-url)
It should be mentioned that RQA method is sensitive to some of the input parameters like; length of epochs (L), radius threshold (ε) and minimal length of RP lines (lmin), which should be carefully determined before evaluating the RQA. These parameters were determined based on Tahmasebpour et al. (2013a) as values of 1024, 0.05 and 2 correspond to L, ε and lmin respectively.

Discrete wavelet transform

There are many analysis methods in the frequency domain such as discrete Fourier transform (DFT) and discrete short time Fourier transforms (STFT). DFT can be used to determine which spectral components exist in the signal, but it is not useful to identify the occurrence time of spectral components. The discrete short-time Fourier transform (STFT), which is a windowed version of the DFT, enables the time frequency analysis of a sampled signal. The problem in the STFT is to determine an appropriate width for this window. This is technically known as the support of the window. However, the window width may cause a problem with resolution: a narrow window gives a good time resolution but poor frequency resolution while a wide window gives a good frequency resolution but poor time resolution. The wavelet transform (WT) solves this problem (Abbasi et al., 2009a; Mallat, 1989). WT provides more flexibility in the time–frequency representation of a signal by allowing the use of variable sized windows. Thus, the WT can provide precise frequency information at both low and high frequencies. This makes the WT often referred to as a sort of mathematical microscope, since different parts of the signal in the analysis can be examined by an automatically adjusting focus. Due to its inherent time-level locality characteristics, the discrete wavelet transform (DWT) has received considerable attention in digital signal processing. The DWT is defined as:

\[
DWT(j, k) = \frac{1}{\sqrt{2^j}} \int x(t) \psi \left( \frac{t-k2^j}{2^j} \right) dt
\]

where \( \psi \) is mother wavelet function, \( j \) and \( k \) represent wavelet decomposed information level and time lag coefficient respectively and \( x(t) \) is pressure time series.

There are many types of wavelets, such as Harr, Daubechies, Mexican hat, and Spline wavelet. One can choose among them depending on the particular application. The quality of signal decomposition and reconstruction highly depends on selection of the mother wavelet and the choice of analyzing wavelets plays a significant role in signal processing. In the present study, Daubechies wavelets were used for decomposing pressure signals since these wavelets are good enough to have an engineering application. Details on how to choose the number of Daubechies wavelets and level of decomposition has been presented in our previous work (Tahmasebpour et al., 2013a). However, Daubechies 2 was considered as the optimum Daubechies wavelet among different Daubechies wavelets for decomposition of pressure fluctuations because it leads to the smallest difference between the original signal and the reconstructed signal. Also, the decomposition information level, \( j \), was set to 9 and then each pressure signal was decomposed into 9 sub-signals.

Mallat (1989) developed an efficient way of implementing above scheme by passing the signal through a series of low-pass and high-pass pair of filters called quadrature mirror filters. At each step of the decomposition process, the frequency resolution is doubled through filtering and the time resolution is halved through down-sampling. The sub-signals (decomposed) are then composed of the approximation sub-signal \( a_j(t) \) and the detail sub-signal \( D_j(t) \). Thus, the original signal \( x(t) \) can be recovered in terms of the sub-signals of different scales:

\[
x(t) \approx a_0(t) + D_1(t) + D_{-1}(t) + \cdots + D_9(t)
\]

The detail and approximation components, \( D_j(t) \) and \( a_j(t) \), lie within a frequency band of \([2^{-j}fs, 2^{-j+1}fs]\) and \([0, 2^{-j}fs]\), respectively, where \( fs \) is the time series sampling frequency. Thus, each wavelet decomposed information level \( j \) represents information of a different frequency band of original signals. The approximation component at a high level corresponds to information about the low frequency of the original signals and the detail component at a low level corresponds to information about the high frequency of the original signals (Zhao and Yang, 2003).

The energies of \( a_j(t) \) and \( D_j(t) \) sub-signals are defined as:

\[
E^a_j = \sum_{t=1}^{N} |a_j(t)|^2
\]

\[
E^d_j = \sum_{t=1}^{N} |D_j(t)|^2
\]

Therefore, energy distribution of \( x(t) \) can be calculated by Eqs. (7) and (8) at various levels. Based on the orthogonality and energy conservation of the WT, the total energy of the original signal \( x(t) \) can be calculated by summing energies of the sub-signals as follows (Zhao and Yang, 2003; Johnsson et al., 2000).

\[
E = \sum_{t=1}^{N} |x(t)|^2 = E^a_0 + \sum_{j=1}^{9} E^d_j
\]

Results and discussion

In this section, hydrodynamic of the system is studied by both frequency and RP techniques, and then similarities and differences in obtained results are compared. The influence of the experimental parameters on recurrence quantification variables (RR, DET and ENT) and also energy of the WT is discussed and quantified. It is shown that all the results of WT and RQA are in consistence with each other at different experimental conditions. However, in studying the scale effect, entropy of the system is a better measure for the characterization of the dynamical behavior of bubbling gas–solid fluidized beds in compared with WT.

It also should be mentioned as a different result that van der Schaaf et al. (2004) studied the similarity between chaos analysis and frequency analysis of pressure fluctuations in fluidized beds. They presented that correlation entropy (or Kolmogorov entropy) shows a relationship with power spectral density (PSD) function. They also concluded that the average cycle frequency of the pressure fluctuations can be directly interpreted in terms of physical phenomena in the fluidized bed and is therefore preferable for the characterization of fluidized bed dynamics.

Li and Kwaak (1994) demonstrated that the complex dynamics of a fluidized system may be reduced to three different structures in fluidized beds based on the multi-scale approach. These are large pressure fluctuations of low frequency correspond to the macro structure (a size comparable to the physical dimension of the bed diameter such as large bubble eruptions and movement of large bubbles), meso structure of higher frequencies (reflecting dynamic feature of clusters of dense phase and smaller bubbles or voidage changes close to the tip of the probe) and micro structure of very high frequencies which represents behavior of the interaction among single particles and between particles and fluid as well as noise effects. There are different suggestions in literature for selecting the frequency ranges corresponding to micro, meso and macro structures in fluidized beds (Zhao and Yang, 2003; Yang and Leu, 2008). By using the RQA, Tahmasebpour et al. (2013a) showed that the micro-scale can be represented by
frequencies in the range of 50–200 Hz ($D_1 + D_2$), the meso-scale can be captured by frequencies in the range of 3.125–50 Hz ($D_3 + D_4 + D_5 + D_6$) and the macro-scale can be described by low frequency range of 0–3.125 Hz ($D_7 + D_8 + D_9 + D_{10}$). They obtained these scales by investigating the similar RQA characteristics for different $j$ levels of pressure signals. The method was applied in this

Fig. 3. Recurrence plots for macro, meso, micro structures and raw signal measured at 15 cm above the distributor for (a) 150 µm, (b) 300 µm and (c) 600 µm sand particles in 15 cm ID bed, $L/D = 1.5$ and superficial gas velocity of 0.6 m/s.
work to pressure signals measured at different scales and gas velocities with different particles (results were not shown here). There was no any difference of the frequency boundaries at these different operating conditions. Therefore, above mentioned frequency ranges were applied to all the experiments.

Effect of superficial gas velocity and particle size

Fig. 2 shows relative energies of macro, meso and micro structures (with reference to the total energy) of the pressure fluctuations measured at 15 cm above the distributor for 150, 300 and 600 µm sand particles in the 15 cm ID bed and $L/D = 1.5$. The contribution of energies of different structures was calculated by Eqs. (7)–(9). For this reason, the main signals were decomposed up to the 9th level using the Daubechies 2 wavelet. The energy of decomposed sub-signals was calculated by Eqs. (7) and (8) and then by recombining the energies of sub-signals correspond to each structure, the energy of that structure was calculated by Eq. (9).

It can be seen in Fig. 2 that the relative energy of the macro structures initially increases by increasing the gas velocity and then gradually decreases. Energy contribution of meso structures shows an opposite trend, i.e., decreases by increasing the gas velocity at low gas velocities, then increases at high gas velocities. Furthermore, relative energy of micro structures seems approximately constant and negligible compared to other structures. It seems that finer structures in the bed first lose their contribution by increasing the gas velocity when compared to the macro-structures. After passing a transition point at a minimum energy, their contribution increases with increasing gas velocity. This change point can be considered as the velocity at which the generating rate of macro and finer structures changes. In other words, energy contribution of macro structures to the total energy initially increases by increasing the superficial gas velocity, indicating the growth of bubbles, and then the energy is slowly transferred from large bubbles to finer structures like smaller bubbles, clusters, and solid particles interaction and their motion. Fig. 2 illustrates that this change point occurs at gas velocities of about 0.3, 0.5 and 0.7 m/s for sands with mean diameters of 150, 300 and 600 µm, respectively, in the 15 cm ID bed and $L/D = 1.5$. This trend suggests that larger particles produce larger and more stable bubbles at bubbling regime of fluidization and consequently change point velocity between macro structures and finer structures shifts to higher velocities by increasing the particle size. Moreover, energy of finer structures is higher for smaller particles which may confirms that formation and stability of finer structures has more importance for smaller particles. This result is in agreement with literature (Johnsson et al., 2000).

Because of this fact that RPs are newly used for study of the hydrodynamic in this work, it is necessary to check whether or not the RPs are capable of recognizing various structures in a fluidized bed. RPs have been made of two different local patterns; local white areas which are representative of macro-scale structures (or
bubble contribution) and local bold areas which are representative of finer structures (meso-scale and micro-scale structures) (Babaei et al., 2012). Fig. 3a–c shows RPs of pressure fluctuations signals measured at 15 cm above the distributor for 150, 300 and 600 μm sand particles in the 15 cm ID bed with L/D = 1.5 and at superficial gas velocity of 0.6 m/s. As it can be seen, increase in size of particles from 150 to 600 μm leads to larger local white areas. This is due to higher amplitudes of pressure fluctuations that larger bubbles generate in presence of larger particles. To explain better, RPs corresponding to macro, meso, micro structures as well as the raw signals are also shown in Fig. 3a–c. To draw these figures, firstly the original signal x(t) is recovered by Eq. (6) in terms of macro, meso and micro sub-signals in the ranges of 0–3.125 Hz, 3.125–50 Hz and 50–200 Hz respectively. Then, with comparing the similarity of RP of raw signal with each one of those sub-structures, the main structure is determined. These figures demonstrate that by increasing the size of particles, the macro structures become more important since the RP of the raw signal becomes more similar to that of the macro structures when the size of particles increases.

Qualitatively, RP figures of measured pressure fluctuations signals at different superficial gas velocities show similar results obtained by frequency domain (not shown here). The size of local white areas increases at first, when superficial gas velocity increases, and subsequently the sizes remain approximately constant until superficial gas velocity reaches transition velocity, and finally they shrink to smaller sizes as superficial gas velocity increases more. This behavior is confirmed quantitatively by computing the values of RR against superficial gas velocity for pressure fluctuations signals measured at 15 cm above the distributor for different particles in 15 cm ID bed, as shown in Fig. 4. As mentioned previously, the bubbles have more growth in presence of bigger particles, and this might be a reason that smaller recurrences are observed for bigger particles in gas velocities correspond to bubbling fluidization regime. In addition, for all signals, the behavior of RR versus U shows three regions. A decrease at first that indicates growth of macro structures, then very small slope that shows large bubbles are slowly transferred to finer structures and finally an increase that represents the regime transfer from bubbling to turbulent. These changes in slope at higher velocities is considered as transition velocity from bubbling to turbulent regime; Uc (Tahmasebpoor et al., 2013b) which is in good agreement with the literature (Bi and Grace, 1995). It is obvious that RR can also illustrate the change points of about 0.3, 0.5 and 0.7 m/s for sands with mean diameters of 150, 300 and 600 μm, respectively and these values are in agreement with the results obtained from the energy of the frequency domain shown in Fig. 2.

Effect of settled bed height

Typical results of determinism and entropy are presented in Fig. 5a and b for pressure signals measured at 10 cm above the distributor for 150 μm sand particles and 15 cm ID bed at various aspect ratios.

Fig. 5. Determinism (a) and entropy (bits/cycle) (b) of the pressure signals measured at 10 cm above the distributor for 150 μm sand particles and 15 cm ID bed at various aspect ratios.

Effect of settled bed height

Typical results of determinism and entropy are presented in Fig. 5a and b for pressure signals measured at 10 cm above the distributor for 150 μm sand particles in the 15 cm ID bed at various L/Ds. The behavior of DET and ENT versus U can be divided into three regions. At first region, low superficial gas velocities, the DET increases and ENT decreases, indicating the growth of macro

![Fig. 4. Recurrence rate of pressure fluctuations signals measured at 15 cm above the distributor for different particles in 15 cm ID bed, L/D = 1.5.](image)

![Fig. 6. Relative energies of macro and meso structures in the pressure signals measured in three beds (5, 9 and 15 cm) for 300 μm sand particles and L/D = 2.](image)
structures. This is because of this fact that with increasing the portion of macro-structures in a fluidized bed, pressure fluctuations exhibit a periodic behavior, resulting in increasing DET and decreasing ENT (Tahmasebpoor et al., 2013a). In the second region, the changes slope becomes very small, because at this range the finer structures become more significant than before. Finally, at

![Fig. 7. Recurrence plots for macro and meso structures and raw signal measured for 300 μm sand particles at L/D = 2 and U^2/D of 5.4 m/s² in (a) 5 cm ID bed, (b) 9 cm ID bed, and (c) 15 cm ID bed.](image-url)
the higher superficial gas velocities, determinism and entropy begin to increase and decrease again respectively due to the regime change.

Also, Fig. 5a demonstrates that the determinism increases with increasing bed height and consequently entropy decreases which shows presence of more macro structures in the case of higher settled heights (Tahmasebpour et al., 2013b). Thus, the complexity of the system decreases when the bed height increases (less and larger bubbles), which is expressed in both determinism and entropy. This result is in agreement with the bubble size measured via correlation presented in the literature (Cai et al., 1994; Karimipour and Pugsley, 2011). The bubble size is dependent to the height (typically about $L^{0.8}$). It means that the bubbles have more growth at higher aspect ratios. Due to the presence of more macro structures in higher $L/D$s, the pressure signal approaches periodic behavior; so the $DET$ increases and $ENT$ decreases.

**Effect of bed diameter**

One of the well-known methods to build two reactors that behave dynamically similar is Glicksman’s method in which a set of dimensionless scaling numbers are tried to be kept the same in both units. To ensure hydrodynamic similarity between three used beds in this work, Glicksman’s so-called ‘simplified set’ containing dimensionless scaling groups of $(U^2/gD, \rho_d p, U/U_{mf, L/D, bed\ geometry, \phi, \text{PSD}})$ is applied (Glicksman et al., 1993; Yang, 2003). By using the same particle size in all three beds, keeping one of the numbers $U^2/gD$ or $U/U_{mf}$ constant will be achieved with percentage of error. However, it has been mentioned that priority should be given to the constancy of the Froude number (van den Bleek et al., 2002). Therefore we choose to keep Froude number and bed aspect ratio the same at all three beds filled with specific particle.

Fig. 6 shows relative energies of macro and meso structures measured in three bed sizes for 300 μm sand particles and $L/D = 2$. The horizontal axis of this figure is considered as $U^2/D$ based on Glicksman’s method. Probe was placed in these beds in such a way that the ratio of the probe position to the settled height of solids was the same. As can be seen in this figure, the relative energy of the structures in these three beds is quite similar at $U^2/D$ values higher than about 3. Although it is expected that the bed size influences the contribution of structures due to change in the wall effect for different bed sizes, Fig. 6 does not reflect this fact, which shows that this analysis method is not very sensitive to the effect of scale. Of course, the wall effect on the hydrodynamics decreases by increasing the bed diameter such that it vanishes beyond a certain diameter. However, the wall effect cannot be neglected in small diameter beds since contact with the wall of reactor affects particle–gas interactions (Rudisuli et al., 2012).

RPs for macro and meso structures as well as the raw signal measured for 300 μm sand particles in three beds at $U^2/D$ of about 5.4 m/s (superficial gas velocities of 0.5, 0.7 and 0.9 m/s for 5, 9, 15 cm ID beds respectively) are shown in Fig. 7a–c. As can be seen in these figures, structures and their contribution are different in beds with different sizes. These figures illustrate that by increasing the bed diameter, gradually; the RP of the raw signal becomes more similar to that of the meso structure. And, it can be concluded that meso structures become dominant by increasing the size of the bed while contribution of macro structures are more important in smaller beds. In other words, macro structure is the dominant structure in small beds while by increasing size of the bed, contribution of meso structures increases such that in large beds it becomes the dominant structure of the bed. It also should be mentioned that above result does not mean bubbles are bigger in smaller beds and as it can be seen the size of local white areas in RP of 15 bed is bigger than the other two and also the amount of
increasing in bed size and this is in consistency with the amount of meso structures. This is the main concern of using entropy at determining hydrodynamic behavior of fluidized beds compared to frequency method, since frequency method could not identify this difference in hydrodynamic of small beds (see Fig. 6). Also, the entropy of the system shows that by increasing the size of particles, more macro structures can be observed in the bed because of lower entropies. This is the same trend observed in the RPs shown in Fig. 3. The larger particles seem to create larger bubbles which lead to formation of more macro structures in the bed. In other words, complexity of the system decreases by employing larger particles in the bed. Furthermore, Fig. 8c demonstrates that there is no considerable difference between the entropy of the system in when using different particles in the 5 cm bed. This can be related to the strong wall effect in this bed. The wall effect is very strong in this bed and this may cover the effect of size of particles. The wall effect decreases by increasing the size of the bed, hence, the effect of other parameters can be identified.

Conclusions

It was shown how RQA method provides valuable tool to improve the understanding of the hydrodynamic of multiphase reactors. RR, DET and ENT variables calculated from the pressure time series of the system at different experimental conditions as well as the energy calculated by WT showed the following:

- In the bubbling regime of fluidization, larger particles produce larger and more stable bubbles and the contribution of macro structures is higher accordingly. This trend was confirmed by WT method and RP. Also RQA was in good agreement with this behavior and larger particles lead to higher determinism and lower entropy.
- Both WT and RQA results showed that contrary to macro structures, the contribution of finer (micro and meso) structures of the bed initially decreases with increasing the gas velocity and then their contribution increases with increasing the gas velocity after a transition velocity. Corresponding transition velocity occurs at gas velocities of 0.3, 0.5 and 0.7 m/s for sands with mean diameters of 150, 300 and 600 μm, respectively.
- The contribution of meso structures increases by increasing in size of bed. In other words, by decreasing the wall effect which happens by increasing size of bed, entropy of the systems comes up. Frequency domain analysis was not able to recognize the wall effect; however, RP and RQA methods showed increase of meso structure contribution by increasing of bed diameter.
- Shannon entropy which is on the base of system’s information leads to formation of more macro structures in the bed. In other words, complexity of the system decreases by employing larger particles in the bed. This shows that the complexity of the system increases by

RR values confirm this observation (not shown here). Therefore based on our conclusion in section ‘Effect of superficial gas velocity and particle size’, the size of bubbles seems to be bigger in larger bed sizes provided that \( \frac{U}{U_{mf}} \) is almost the same and regime of fluidization is bubbling. On the other hand, contribution of meso structures increases by increasing the size of bed and it becomes the considerable contribution of structures. This causes more complexity in larger beds. This qualitative result should be discussed more quantitatively by the RQA method.

Entropy of the system for 150, 300 and 600 μm sand particles in 5, 9 and 15 cm ID beds have been presented in Fig. 8a–c. As it can be seen in this figure, for all three mentioned particles, the entropy of the system in bed of larger diameter is higher than smaller beds. This shows that the complexity of the system increases by

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