Characterizing nonlinear dynamic features of self-sustained thermoacoustic oscillations in a premixed swirling combustor

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HIGHLIGHTS
• Nonlinear dynamics of a swirling thermoacoustic combustor is characterized.
• Effects of fuel-air equivalence ratio are studied on generating thermoacoustic instability.
• Periodic limit cycle and period-doubling bifurcation is experimentally found.
• ‘Kick-oscillator’ model is developed to capture deterministic and chaotic natures.
• Recurrence analysis is performed to characterize nonlinear dynamics of combustion instability.

ABSTRACT
To meet stringent NOx emission requirements, lean premixed swirling combustion technology is widely applied in power generation and propulsion systems. However, such combustion systems are more susceptible to nonlinear combustion instability due to the fluid flow-acoustics-combustion interaction. It is typically self-exited and characterized by large-amplitude pulsating oscillations. By applying experimental measurements and theoretical modelling, we explore the rich physics of how a methane-burnt swirling flame sustains periodic pulsating combustion oscillations, and its nonlinear dynamics features via recurrence plots (RP) and 0–1 chaotic test. The effects of (1) the equivalence ratio \( \Phi \), (2) the volume flow rate \( V_a \) of inlet air and (3) the swirling number \( S_n \) are examined. Hopf Supercritical and period-doubling bifurcation behaviours are experimentally observed with increased \( \Phi \) from lean to rich combustion. Same nonlinear features are theoretically modeled by using ‘kick-oscillator’ model representing combustion pulses and to capture periodic limit cycle behaviours followed by period-doubling bifurcation and transition to chaos. The deterministic or chaotic nature of the swirling combustor could be experimentally identified using classical approaches such as probability density functions (PDFs) and acoustics power spectrum in presence of combustion-sustained periodic fluctuations and 0–1 chaotic test method. When the combustor is experimentally operated under either lean (\( \Phi \leq 0.6 \)) or rich (\( \Phi \geq 1.1 \)) conditions, no self-sustained periodic acoustic fluctuations are generated. Furthermore the combustor is found to be more chaotic. The flame shapes (M- or V-shaped), colour, brightness and volumes are found to depend on \( S_n \) strongly. The present findings are physically insightful on understanding nonlinear features of swirling flow-acoustics-flame interaction.

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1. Introduction

Lean premixed combustion is a key technology to meet stringent regulations for nitrogen oxides (NOx) [1]. Thus such technology is widely applied in modern propulsion such as aero-engines and power generation (gas turbines) systems [2]. However, these gas turbines and aero-engines are more susceptible to self-sustained pulsating combustion (also known as thermoacoustic instabilities), when combustors are operated at such lean premixed conditions [3]. The generation mechanism is typically related to the interaction between turbulent/laminar flame and acoustics [4], as described by Rayleigh criterion [5]. Besides stationary gas turbines [6], self-sustained thermoacoustic instabilities are also found to occur in other thermodynamic systems [7–10] such as solid rocket motor [7], liquid rocket engines [8] and aero-engine afterburners [9]. The instability characterized with large-amplitude periodic pressure oscillations are highly undesirable [10], since it can cause structure vibration, flame flashback and reduce the lifetime and reliability of the engine combustors [11].

Extensive researches have been conducted on self-sustained thermoacoustic oscillations in premixed swirling combustors. Most gas-turbine engines utilize swirling flows to stabilize the flame for efficient and clean combustion [12,13]. Investigating and understanding the interaction of swirling flow- acoustics-flames are critical to design a stable combustor, which could be applied in power generation systems [14]. Karyeyen et al. [15] experimentally investigated the swirling flow-flame interaction in a hydrogen-rich gaseous fuel-involved combustor. CO and NOx emission were also evaluated. Wu et al. [16] conducted experimental study to examine the air-fuel equivalence ratio and inlet volume flow rate of fuel effect on a swirling combustor. Results show that equivalence ratio is an important operating parameter determining whether the swirling flame could sustain oscillations. Further, the ‘triggering’ phenomenon of dominant frequencies is discovered for a fixed equivalence ratio with varying the fuel flow rate. Khalil and Gupta [17] experimentally examined the acoustic signature and the heat release features of a swirling combustor. Flame fluctuations are observed at low frequencies. Huang et al. [18] performed large eddy simulations (LES) on a lean-premixed swirl-stabilized combustor. It is found that swirling intensity has dramatic effects on the pressure and heat release fluctuations in combustor. The recirculation zone of flow field is also found to play a critical role in the flow/flame interactions. Khalil and Gupta [19] experimentally evaluated the heat release fluctuations and visible emissions from a swirling combustor with different CO₂ dilution blended. Kim et al. [20] studied the propensity for self-sustained thermoacoustic instability in a dual swirl combustor. Effect of combustor length and thermal power were experimentally evaluated. Zhang et al. [21] conducted experimental investigation on a model industrial gas turbine combustor fuelled syngas of methane or carbon monoxide with different blended hydrogen. It is found that variations in fuel components have significant effect on the instability and flame characteristics.

Considering the nonlinear behaviors of thermoacoustic system [22], conventional linear analysis such as power spectral analysis [23] may be inadequate to intensively understand and interpret the physics underlying the thermoacoustic instability. Nonlinear time series study is extensively used as a reliable tool for analysing nonlinear behaviours [24]. Lieuwen [25] tracked a combustor’s dynamic stability margin by applied autocorrelation method. Gotoda et al. [26] performed a detailed nonlinear analysis by on thermoacoustic instabilities in a gas-turbine model combustor. A nonlinear time series analysis in combination with a surrogate data method are performed. It is found that the nonlinear behaviour of the thermoacoustic instability transitioned from random fluctuation to periodic oscillation. Tony et al. [27] analysed pressure signal obtained from both bluff-body and swirling combustors. Using correlation dimension and correlation entropy, it is demonstrated

Fig. 1. Schematics of (a) the experiment rig, (b) the swirler and experimental rig segment with acoustic pressure sensors implemented, (c) the two-microphone technique.
that pressure fluctuations generated by combustion has the features of a high-dimensional chaotic data contaminated with white and coloured noise. Recently a method named ‘0-1’ chaotic test distinguishing a dynamical system is in a regular or chaotic state was proposed in Ref. [28]. Then Nair et al. [29] demonstrate this method is robust for analysing pressure fluctuation in combustion instability. Besides used to analyse thermoacoustic instabilities system, the 0–1 test method has been successfully applied to many other fields and systems, such as transportation system [30], Duffing system [31], internal combustion engine [32]. There are several advantages of this method: (1) it is a universal method analyse to any dynamical system; (2) it does not need the phase space reconstruction, and (3) the output is only one value, ‘0’ or ‘1’ which depends on whether the input time series signal is regular or chaotic. Albeit continuous efforts, the nonlinear dynamics features of complex combustion system and the insightful physical mechanism of self-sustained oscillations are not fully understood, which motivates the present experimental investigation.

According to nonlinear dynamic theory, when the control parameter exceeds the nonlinear the bifurcation point, there are two different types of Hopf bifurcation: (1) a supercritical (soft) bifurcation characterised by a gradual increasing amplitude (2) a subcritical bifurcation (hard) characterised by a sudden jump amplitude [33]. In practical combustor, with operating parameter, i.e. equivalence ratio [34], stratification ratio [35,36], inlet velocity [37,38] and temperature [39], varying and passing through critical values, bifurcations occur with oscillations amplitudes changing dramatically. The occurrence of such Hopf bifurcation is a great challenge to the safe operation of the combustion system. Therefore, it is practically important to determine the stability and nonlinear characteristics of a lean premixed combustor. Extensive studies have been conducted to investigate the above-introduced bifurcation phenomenon. Kashinath et al. [40] numerically examined the thermoacoustic oscillations and bifurcations of a ducted premixed flame for flame position, duct length and mean velocity. Bonciolini et al. [41] experimentally investigate thermoacoustic instability in a lab-scale swirling type combustor under transient operation. It is found that the system undergo two successive and mirrored supercritical Hopf bifurcations. A method of changing quickly the bifurcation parameter to avoid undesired large amplitude limit cycles is also reported and demonstrated. Han et al. [42] experimentally evaluated the inlet temperature effect on combustion instabilities in a pressurised kerosene fuelled Lean Premixed Pre-vaporized (LPP) combustor. A supercritical bifurcation without hysteresis behaviour triggered by the air inlet temperature is observed.

In this work, experimental studies are conducted to analyze the nonlinear characteristics of self-excited thermoacoustic oscillations in a methane-air premixed swirling combustor. Emphasis is placed on evaluating the effects of (1) equivalence ratio (2) air flow rate and (3) swirling intensity on nonlinear dynamic behaviors of the combustor. The paper is organized as: The experimental setup, fuel-air supplying system and measure system are depicted in Section 2. The measured pressure signal is analyzed by applying phase reconstruction, recurrence analysis and 0–1 chaotic test as described in Section 3. Finally, key findings are summarized in Section 5.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Combustor size and transducer position parameters.</th>
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<tr>
<td>Parameter</td>
<td>Value (unit: mm)</td>
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<tr>
<td>L₁</td>
<td>100</td>
</tr>
<tr>
<td>L₂</td>
<td>50</td>
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<tr>
<td>L₃</td>
<td>328</td>
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<tr>
<td>L₄</td>
<td>466</td>
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Fig. 2. (a-c) Photos of the 3 swirlers, (d-f) top view photo of swirling flame, (g-i) side view photo of swirling flame for different swirling number at \( \Phi = 0.9 \), \( Vₐ = 215 \text{ L/min.} \) (a), (d) and (g), \( Sₙ = 0.42 \), (b), (e) and (h), \( Sₙ = 0.73 \), (c), (f) and (i), \( Sₙ = 1.27 \).
2. Description of experimental test

Experimental studies are performed on a swirling combustor which is placed horizontally. The schematic of the experimental set-up is shown in Fig. 1(a). There are three main components consisting of (1) a swirler, (2) fuel-air supplying system and (3) diagnostic packages. The experimental rig is made of stainless steel, except a window section made of silica glass used for laser/camera diagnostic measurement. To capture the flame dynamics, two digital cameras are implemented to obtain side view and top view of the swirling flame. Acoustic perturbations are captured by using pressure sensors. The upstream pressure fluctuations $p_1(t)$ and $p_2(t)$ are monitored in real-time by using two microphones (PCB Model 378C10, 1.0 mV/Pa). The downstream pressure signals $p_3(t)$ and $p_4(t)$ are measured by 2 sensors (PCB Model 106B, 43.5 mV/kPa). To attenuation acoustic reflection effect, a semi-infinite technique [20] is used. Water cooling technique is applied to protect the pressure sensors. The acoustic data are logged by Log Instruments NI-USB-6366 with a 20,000 Hz sampling rate. Fig. 1(b) shows the profile and main dimensions of the rig and the axial positions of these pressure sensors. The detailed parameters are summarized in table 1. Pressure signal $p_3(t)$ are used for the analysis in the remainder of the present work. The microphones were calibrated against a known signal before every experiment.

In order to produce swirling flow, an axial swirler is applied at the inlet of the combustor. Typically, the swirling intensities are characterized by number swirling number $S_N$, which is a non-dimensional parameter [43]. In physics, $S_N$ denotes the ratio of the axial flux of swirl momentum to the product of inlet radius and axial flux of axial momentum [44]. In present work, the thickness of vanes is 1 mm which can be considered very thin and axial and azimuthal velocities are assumed to be uniform, then the swirl number can be approximated as in Ref. [43] as:

$$S_N = \frac{\int_{D_s/2}^{D_s/2} u_s r dr}{(D/2) \int_{D_s/2}^{D_s/2} u_s^2 r dr} \approx \frac{2 \left[ 1 - (D_s/D)^2 \right]}{3 \left[ 1 - (D_s/D)^2 \right]} \tan \beta$$

(1)

where $D_s$ and $D$ are the diameter of the center shaft and the inlet; $u_s$ and $u_a$ are axial and tangential velocity; $\beta$ is the swirler vane angle. It can be concluded from above equation that swirling number $S_N$ is only determined by 3 parameters $D_s$, $D$, and $\beta$. In experiment, $D_s$ and $D$ remain constant. As Fig. 2(a-c) shown 3 different swirlers with $\beta = 30^\circ$, $45^\circ$ and $60^\circ$ are applied, which correspond $S_N \approx 0.42$, 0.73 and 1.27.

Consider the fuel-air supplying system, the fuel methane are characterized by number swirling number $S_N$, which is a non-dimensional parameter [43]. In physics, $S_N$ denotes the ratio of the axial flux of swirl momentum to the product of inlet radius and axial flux of axial momentum [44]. In present work, the thickness of vanes is 1 mm which can be considered very thin and axial and azimuthal velocities are assumed to be uniform, then the swirl number can be approximated as in Ref. [43] as:

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Consider the fuel-air supplying system, the fuel methane and the oxidizer air are storage and supplied by compressed gas cylinder [45]. With the assumption that the nitrogen $N_2$ is not be involved in combustion, the combustion reaction of the premixed methane and air can be written as:

$$CH_4 + 2(O_2 + 3.76N_2) \rightarrow CO_2 + 2H_2O + 7.52N_2$$

(2)

when the methane and air are fuelled at stoichiometric ratio, i.e. $\Phi = 1.0$ [46], air is supplied enough to completely burn all methane to form CO$_2$ and H$_2$O. In this case, the flow rate ratio in volume between the fuel and air is given as:

$$\left(\frac{V_f}{V_a}\right)_{\Phi=1} = \frac{1}{2 \times (1 + 3.76)} = \frac{1}{9.52} = 0.105$$

(3)

in practice, equivalence ratio $\Phi$ as defined in Eq. (4) is used to characterize the fuel-air ratio:

$$\Phi = \left(\frac{V_f}{V_a}\right)_{real} / \left(\frac{V_f}{V_a}\right)_{\Phi=1} = 9.52 \times \frac{V_f}{V_a}$$

(4)

when $\Phi < 1.0$, lean combustion occurs, while $\Phi > 1$ denotes rich combustion. The air flow rate $V_a$ and the fuel flow rate $V_f$ are controlled by D07-60B and D07-9E flow meters. The accuracy of the flow meters is less than $\pm 2\%$.

The present experimental investigations focus on characterizing the nonlinear dynamic features of self-sustained thermoacoustic oscillations. The effects of (1) equivalence ratio $\Phi$, (2) air flow rate and (3) the swirling number $S_N$ are examined one at a time. The equivalence ratio $\Phi$ is varied from 0.6 to 1.1 with the interval of 0.1. Five different air flow rates between 130 and 250 L/min are tested. The swirling number $(S_N = 0.42, 0.73$ and $1.27)$ is changed by setting $\beta$ to $30^\circ$, $45^\circ$ and $60^\circ$, as shown in Fig. 2(a-c). Note that to evaluate the accuracy of the experimental measurements, experimental measurements are repeatedly conducted. Thus measurement error analysis is performed on the measured acoustic pressure data. All the operating conditions and the error analysis are summarized in Appendix.

3. Data-processing methodologies

3.1. Root-mean-square of the acoustic measurements

The RMS (root-mean-square) value of a sample pressure signal data $p_{rms}$ can be determined as:

$$p_{rms} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} p(t)^2}$$

(5)

where $N = 50,000$ is the total sample number.

3.2. Recurrence plot analysis

Recurrence analysis in phase space is a typical method used to understand nonlinear dynamical systems such as swirling combustors [47]. In such combustors, the physical processes include hydrodynamics, acoustics, and chemical combustion, which are characterized by acoustic pressures, flow velocity and unsteady heat release and other variables [48]. The phase space of such combustors consists of these governing variables, which are evolving and encompassing useful information of combustion-driven noise and trainset behaviours. The acoustic data obtained from the swirl combustor is a superposition of (1) combustion noise, (2) hydrodynamically generated noise (for example vortex-driven noise) and (3) acoustics generated due to flow or flame-acoustic coupling [49]. Thus acoustic pressure measurements could be applied to capture the dynamic features of the original phase space via a proper reconstruction of the phase space obtained from pressure signals in time domain. It is known that pulsating behaviours are characterized by phase space trajectories [50,51]. The trajectory revisits itself after a period of pulsation. If random noise is present, the trajectory is still visiting its neighbourhood after each period. The trajectory has a diverging behaviour, such system is chaotic. The diverging behaviours depend on how rapidly the trajectory loses its neighbourhood, which is defined by a reference point and a predefined neighbourhood size. Thus recurrence analysis is an insightful and promising method to capture the combustor dynamic physics. Typically, recurrence plot (RP) represents the recurrence of trajectories, which evolves within a multi-dimensional phase space on a two-dimensional plot [26].

A lag-reconstructed vector $P(t)$ is created by using the measured acoustic pressure $p(t)$ in time domain as:

$$P(t) = (p(t), p(t + \tau), \ldots, p(t + \tau(m - 1)))$$

$$\tau = 1, 2, 3, \ldots, N - \tau(m - 1)$$

(6)

where $\tau$ is the time delay parameter which can be estimated with mutual information method with a mutual information method [52]. The embedding dimension $m$ can be determined by the false nearest neighbour algorithm [53]. There are two kind s of the RP plot, unthreshold and thresholded recurrence plot. Here thresholded recurrence plot is used. The thresholded RP is defined as:

$$R_e(i, k) = \Theta(\epsilon - \|P(i) - P(k)\|)$$

(7)

where $P(i)$ and $P(k)$ are lag-reconstructed vector of pressure signal, $\epsilon$ is
3.3. 0-1 chaotic test

In dynamic combustion systems, it is quite critical to distinguish the dynamic behaviours between regular and chaotic. Here we apply 0-1 chaotic detection technique on our measured pressure signal \( p(t) \). A new couple of translation data is obtained as:

\[
\begin{align*}
    u(n) &= \sum_{i=1}^{n} p(t) \cos(\pi r n) \\
    v(n) &= \sum_{i=1}^{n} p(t) \sin(\pi r n)
\end{align*}
\]

(8a) (8b)

where \( r \) in \((0, 1)\) is a random constant. The Mean Square Displacement (MSD) can then be determined as:

\[
M_r(n) = \lim_{n \to \infty} \frac{1}{N} \sum_{i=1}^{N} [(u(t+n) - u(t))^2 + (v(t+n) - v(t))^2]
\]

(9)

with \( n \ll N \). It is seen that \( n \leq n_{cut} \) where \( n_{cut} = N/10 \) yields good results. Finally, the asymptotic growth rate \( K_r \) can be calculated from \( M_r \). There are two typical methods to calculating \( K_r \). One is linear regression and the other is correlation coefficient. In practical applications, the correlation method greatly outperforms the regression method [28]. Thus we choose the correlation coefficient method. Then \( K_r \) can be estimated with defining define vectors \( \xi = (1, 2, 3, \ldots, n_{cut}) \) and \( \Delta = (M_1(1), M_1(2), M_1(3), \ldots, M_1(n_{cut})) \):

\[
K_r = \text{corr}(\xi, \Delta) = \frac{\text{cov}(\xi, \Delta)}{\sqrt{\text{var}(\xi) \text{var}(\Delta)}}
\]

(10)

where \( \text{corr}() \) is the correlation coefficient, \( \text{cov}() \) represents the covariance and \( \text{var}() \) denotes variance. \( K_r \) is a useful measure to describe whether the time series signal is chaotic or regular quantitatively. \( K_r = 1 \) represents that the time series has chaotic characteristics, while \( K_r = 0 \) donates that the time series is regular.

4. Results and discussion

4.1. Effect of equivalence ratio \( \Phi \)

As dynamical system theory is considered, the change of a critical parameter can cause the behavior or structure of the system sudden qualitative or topological change. This behavior is typically known as "Hopf bifurcation". The parameter values at which such bifurcation occurs is called the bifurcation point. The system bifurcation map of the swirling combustor is shown in Fig. 3(b). Here, as the equivalence ratio \( \Phi \) is chosen to be the bifurcation parameter. It can be seen that when \( \Phi \leq 0.5 \) the flame cannot exist steadily, thus there is no pressure oscillations. When \( \Phi \) is increased to up to 0.7, low amplitude acoustics are present in the combustor. The \( p_{rms} \) is maximized at \( \Phi = 0.9 \), and is decreased with further increase of \( \Phi \) above 1.0. As \( \Phi \) is decreased from 1.1 to 0.4, \( p_{rms} \) will be slowly increased and then decreased quickly and finally to 0 as \( \Phi \) is decreased below 0.5. The variation behaviors confirm that the present combustor is associated with Hopf supercritical bifurcation. To capture the supercritical bifurcation behavior, a "kick-oscillator" model [54] is developed as

\[
P_{n+1} = \gamma R p_n(1 - \gamma p_n)
\]

(11)

The model is a mathematical expression of a nonlinear ‘kicked oscillator’. It is in ‘free acoustic oscillation’ until ‘kicked’ by instantaneous heat release. In physics, a vortex structure is shed from the swirler and mixing of the reactants with combustion products from the previous cycle occurs, when the velocity increases during each cycle of oscillations. After a time delay in mille-seconds (due to mixing and chemical reaction), a pulse of unsteady heat release is generated. The combustion in the ‘dominant’ vortices is not accomplished during the ‘original’ pulse and it is suppressed with decreased acoustic pressure. However, as the acoustic pressure is rising, the remaining reactants associated with the original vortex structure respond strongly to the increased pressure and thus provides a second peak in the unsteady heat release. Eq. (11) is an introductory example for studying bifurcations and chaotic behaviors [54].

Fig. 3(a) shows the modelled bifurcation diagram. It is obtained by varying the bifurcation parameter \( R \) as \( \gamma_1 = 3 \) and \( \gamma_2 = 1 \). It can be seen that as \( R \) is increased to \( R_b \) (bifurcation point), a Hopf supercritical bifurcation occurs, which changes the system from stable state to periodic motion. Then the modelled system undergoes a period-doubling bifurcation, when \( R \) is further increased to \( R_a \). This theoretical finding is not consistent with our present swirling combustion measurements. Our present experimental tests reveals that the combustor intrinsically choose the bifurcation behaviors. In other words, rich combustion corresponding to \( \Phi \geq 1.1 \) does not lead to such period-doubling bifurcation as theoretically predicted at \( R \geq R_a \). Further increasing \( R \) will lead to a transition to chaos, which is not shown in Fig. 3(a). The dimensionality of the dynamical system is increased dramatically and the high-dimensional combustion system collapse onto an attractor of low dimension.

Furthermore, the effect of the equivalence ratio on combustion-flow-acoustic interactions are evaluated by applying the classical frequency spectrum and PDF (Probability density function) methods. This is shown in Fig. 4, as \( \Phi \) is varied from 0.6 to 1.1 with the interval of 0.1. Here, the volume flow rate of air is set at \( V_a = 215\, \text{L/min} \) and the swirling number \( S_n \) is 1.27. It can be seen from Fig. 4(a) that when \( \Phi = \)
0.6, the PDF distribution has a signal peak and a Gaussian bell shape, with some noise superimposed, centred on \( p = 0 \). This histogram shape is characteristic of random walk time series. And there is no obvious peak in the frequency spectrum. When \( 0.7 \leq \Phi \leq 0.9 \), as the equivalence ratio increases, PDF distribution evolved into a bimodal peaks. The amplitude of acoustic fluctuations is maximized at \( \Phi = 0.9 \).

Fig. 4(b) shows that as \( \Phi \) is increased, \( p_{\text{rms}} \) and the dominant frequency both increased first and then decayed. The maximum \( p_{\text{rms}} \) and the dominant frequency \( \omega/2\pi \) occurs at \( \Phi = 0.9 \), which are 1766 Pa and 183 Hz. The measured acoustic pressure signals and corresponding phase diagram and recurrence plots (RPs) at \( \Phi = 0.6, 0.9 \) and 1.1 are shown in Fig. 5. Fig. 5(d) shows that orbits converge on the core of the attractor at \( \Phi = 0.6 \), representing system undergo a stochastic oscillation. In Fig. 5(g), irregular structures in RP plot means turbulence noise are dominant and there is a lack of determined structures. At \( \Phi = 0.9 \), we observed a clear annulus trajectory with some noise in the phase diagram (Fig. 5(e)), which indicate that there is a periodic oscillation in combustion system. The self-excited state is a limit cycle oscillation, resulting from a Hopf bifurcation. In RP plot Fig. 5(h), regularly arrayed geometrical structures are observed. At \( \Phi = 1.1 \), a strange attractor is seen in Fig. 5(f). Such oscillation resulting from period-doubling bifurcation is qualitatively different from limit cycle oscillation. The loop turns into a dense toroidal structure indicating quasi-periodic oscillation which also reflected by an incommensurate frequency peak \( (\omega_2) \) in the power spectrum (Fig. 4(b)). Irregular structures content in Fig. 5(i) also hints towards the presence of chaotic tendencies.

### 4.2. Effect of air flow rate

The effect of incoming air flow rate is examined by obtaining the PDF and frequency spectrum of the acoustic pressure measurements as \( V_a \) is set to 5 different values and \( \Phi = 0.7 \) and \( S_N = 1.27 \). This is shown in Fig. 6. It can be seen from Fig. 6(a) that the PDF has a distribution of a single peak at \( V_a = 130 \) and 155 L/min and \( p_{\text{rms}} \) values are also quite low. Correspondingly, no obvious dominant peaks are observed in the frequency spectrum as shown in Fig. 6(b). As the air flow rate is further increased, bimodal distributions are observed to occur in PDF. The dominant frequency and \( p_{\text{rms}} \) are found to increase with increased \( V_a \). Maximum values are observed at \( V_a = 250 \) L/min, which are 180 Hz and 750.3 Pa.

Similar phase reconstruction are conducted on the operation condition of \( V_a = 130, 180 \) and 250 L/min, as shown in Fig. 7. At \( V_a = 130 \) L/min, the trajectories fill the core of the phase space, and the corresponding pressure fluctuation signal does not appear to exhibit a deterministic nature. As \( V_a \) is increased to 180 L/min, attractor size increases. A torus-like structure with a large width of the trajectories is observed. Also the main peak in the power spectra starts to appear, indication the periodicity of the dominant periodic fluctuations. When \( V_a \) is 250 L/min, the outer-limit cycle remains, and its periodicity becomes prominent.

More detailed insights on the chaotic or deterministic nature of the combustor are obtained by conducting 0–1 tests. This is shown in Fig. 8. The 0–1 test method is a dualistic test method and it provides a simple and an effective way to test the chaotic characteristics of the time-domain pressure measurements. It can be seen that \( K_e \) is almost constant and closed to 1.0 when the air flow rate is set to \( V_a = 130 \) L/min, which indicates that the combustor is completely chaotic. However, when the air flow rate is set to \( V_a = 250 \) L/min, \( K_e \) we calculated is approximately equal to 0. It reveals the time series are regular which can also be seen in Fig. 7(f). It differ from the above two situations when the air flow rate is set to \( V_a = 180 \) L/min. \( K_e \) cannot be stabilize near 0 and fluctuates between 0 and 0.5. It is concluded that the swirling combustor is chaotic.

### 4.3. Effect of the swirling number \( S_N \)

The implementation of a swirler as shown in Fig. 2 is aimed to create more intensified vortex-involved turbulence and enhance the combustion efficiency. The swirling flow is 3 dimensional and interacts with the dynamic flame to affect its colour, volume and brightness. The flame response to the swirling flow is shown in Fig. 2(d-f) with the top- and side-view as captured by the camera. It can be seen that as the SN is increased from 0.42 to 1.27, the flame shape is changed from M to V. At \( SN = 0.42 \), the premixed flame lifts away the combustor inlet and is anchored at \( x = 70 \) mm. When \( SN = 1.27 \), the V-shaped flame is obtained, which anchored on the central rod tip. It can be seen that the central rod is in red colour indicating a higher temperature. Furthermore, as \( S_N \) is increased from \( S_N = 0.42 \) to 1.27, the flame colour is changed from dark blue to light blue. This indicates that temperature is increased with \( S_N \).

Correspondingly, the acoustic pressure measurements in time-domain and phase diagram are shown in Fig. 9. It can be seen that the pressure fluctuation amplitude is increased with increased \( S_N \). The periodic nature and nonlinearity could be clearly observed from the phase plot attractors’ shapes are loops at \( S_N = 0.73 \) and 1.27. The main difference of these two attractors is width of ‘ring’ are smaller at \( S_N = 0.73 \). At lower swirling intensity, the attractor’s shape is still a closed loop, but is not contained in a 2 dimensional plane anymore. This indicates the chaotic trajectory. Fig. 9(c) shows the acoustic pressure fluctuations upstream of the combustor \( p_1(t) \) and \( p_2(t) \). It can be seen that as the \( S_N \) is increased, the amplitude of pressure fluctuations upstream of the combustor decreases. Then the effect of the swirling number \( S_N \) on acoustic pressure fluctuations is evaluated by conducting PDF and frequency spectrum analysis. This is shown Fig. 10, as \( S_N \) is set to 3 different values and \( \Phi = 0.9 \) and \( V_a = 215 \) L/min respectively. It can be seen from Fig. 10(a) that the PDF of the measured pressure signal has a bimodal distributions for any given \( S_N \). Closer observation reveals that the PDF distribution of \( S_N = 0.73 \) and 1.27 are almost symmetric. However, the PDF corresponding to \( S_N = 0.42 \) has a taller positive pressure side peak. Fig. 10(b) shows that \( p_{\text{rms}} \) and the dominant frequency are increased as the swirling number is increased. \( p_{\text{rms}} \) and the dominant frequency are maximized at \( S_N = 1.27 \), and they are 1766 Pa and 183 Hz respectively.

Finally, physical insights are obtained on the dominant frequency of

![Fig. 4. Stationary operations of the combustor for different equivalence ratio at \( V_a = 215 \) L/min \( S_N = 1.27 \) (a) PDF (Probability Density Function) of pressure (b) Frequency spectrum.](image)
the periodic acoustic pressure fluctuations present in the combustor by predicting its mode shape [55]. Here we use the classical two-microphone method [56] to conduct the travelling plane wave decomposition. As illustrated in Fig. 1(c), the measured acoustic pressure signal $p_3(t)$ and $p_4(t)$ can be used to determine the decomposed incident $p_I$ and reflected travelling wave $p_R$ in frequency domain as:

$$p(t) = p_I(t) + p_R(t)$$

(12)

where $\rho = j\omega$ is the sound speed, $u$ is the mean velocity in the combustor, $\hat{\cdot}$ denotes Fourier transform of the time-domain signals. Then the pressure at $x$ in frequency domain $p_x(\omega)$ can be written as:

$$p_x(\omega) = p_I(\omega)e^{-j(\omega t + \tau)} + p_R(\omega)e^{j(\omega t + \tau)}$$

(13)

Fig. 11 shows the comparison of theoretically predicted and experimentally measured longitudinal mode shapes [57,58] at $\Phi = 1.1$, $V_a = 215$ L/min, $S_n = 0.42$. The corresponding pressure measurements $p_I(\omega)$ and $p_R(\omega)$ in frequency domain at $\omega/2\pi = 227$ Hz are used to conduct travelling plane waves to obtain the longitudinal mode shape along the axial direction of the combustor. According to Eq. [18], the $n$th resonant frequency $f_n = (2n - 1)\omega/4L_c$. The theoretical 2nd longitudinal mode shape at $3\omega/2\pi = 681$ Hz is also presented. It can be seen that the theoretical and experimental results agree well in general, although some discrepancy near the combustor outlet can be observed.
This is most likely due to the end correction effect. This reveals that periodic pressure fluctuations resulting from the swirling flow-acoustics-flame corresponds to the acoustic resonance nature of the combustor in terms of the 1st longitudinal mode [59,60].

Self-excited thermoacoustic oscillations may occur in practical swirl combustors. They are applied in gas turbines [19,61]. Such oscillations are characterized by large-amplitude pressure and velocity (harmonic or non-harmonic modes) perturbations in the engines. Thus these oscillations are unwanted, since they can cause costly and catastrophic damage to the engine systems [17]. To ensure the engine system being operating stably, a real-time diagnostic/monitoring package is applicable. This is typically conducted by applying a pressure transducer as we did here. Furthermore, the real-time acoustic pressure measurements show that the transition from stable to periodic and to chaotic motions can occur in the present swirling combustor, as the bifurcating parameter is varied, such as fuel-air ratio, swirling number. This is confirmed by the corresponding frequency spectrum analysis and is consistent with the finding reported on a bluff-body-applied gas turbine combustor. The other common feature between the current swirling combustor and gas turbine combustors is that the Hopf bifurcation and limit cycles oscillations are experimentally confirmed in both types of power generation systems.

Conventionally, it is assumed that there is no such self-excited thermoacoustic oscillation, when the combustion mode is in rich domain, i.e. fuel-air ratio $\Phi$ is greater than 1.0. However, the present work shows that combustion-sustained limit cycles can be produced as $\Phi \geq 1.1$. Furthermore, to investigate the nonlinear dynamics of the swirling combustor, a model expressed as a nonlinear ‘kicked oscillator’ is developed. As the fuel-air ratio is increased, a Hopf supercritical bifurcation occurs, which changes the combustor from stable state to periodic motion and then undergoes a period-doubling bifurcation, as $\Phi$ is further increased. However, this finding is not consistent with the present experimental observation. To the best knowledge of the present authors, such theoretical and experimental bifurcation studies have not been compared and reported before on such swirling combustors [62–64].

5. Conclusions

The present work investigates the fundamental physical mechanism and nonlinear behaviors of self-sustained thermoacoustic oscillation in a premixed methane-burnt swirling combustor. For this, experiment measurements of acoustic pressure fluctuations, theoretical bifurcation modelling and nonlinear dynamics analysis are conducted. The effects of (1) equivalence ratio (2) air flow rate and (3) swirling intensity are examined at a time. As the nonlinear behaviors described by a ‘kick-oscillator’ model of $P_{n+1} = \gamma R P_n (1 - \gamma P_n)$, the system undergo supercritical Hopf and period-doubling bifurcation successively as equivalence ratio is increased from 0.6 to 1.1. The model is representing combustion heat release pulses that occur equally spaced in time. When such model is applied to study one acoustic mode, it is found that the dimensionality of the combustion system is increased to three or greater and even chaotic oscillations are theoretically possible. Periodic limit cycle characteristics are observed first. It is then followed by period-doubling bifurcations. This is experimentally confirmed by our experimental measurements, as the bifurcation parameter $R$ is chosen to be equivalence ratio $\Phi$. In theory, a transition to chaos will occur, as $R$ is further increased. The high-dimensional combustion system will collapse onto an attractor of low dimension. Experimentally validating this is quite challenging. Since a very large $\Phi$ leads to the swirling flame...
being blow out of the combustor due to the high volumetric flow rates. When the combustor is operated at lean (\(\phi \leq 0.6\)) or rich (\(\phi \geq 1.1\)) conditions, there is no self-sustained thermoacoustic oscillation with relatively negligible \(p_{\text{rms}}\) and SPL (sound pressure level). The corresponding RP (recurrence plot) and phase diagram reveals the system’s chaotic feature. The largest-amplitude pulsating oscillation is observed at \(\phi = 0.9\). The \(p_{\text{rms}}\) and dominant frequency are approximately 1766 Pa and 183 Hz respectively. PDF has unimodal distribution otherwise bimodal distribution, when combustion instability occurs. When a low flow rate air is supplied, i.e. \(V_a = 130\) and 155 L/min, there are no limit cycle oscillations. The combustor is more chaotic as revealed by applying the 0–1 chaotic test and phase diagram analysis. As \(V_a\) is increased to 250 L/min, \(p_{\text{rms}}\) and the dominant frequency are increased to 750.3 Pa and 180 Hz respectively. The corresponding PDF undergoes a transition from a single peak to symmetric bimodal distribution.

Finally, the effect of the swirling number \(S_N\) is examined by changing the vane angle of the swirler. Increasing \(S_N\) is shown to lead to the flame shape being changed dramatically from ‘M’ to ‘V’ shape. At lower swirling number, the premixed flame has a less brightness and a smaller diameter and volume. As \(S_N\) is increased, the flame moves form downstream to the inlet and anchored on the central rod tip. Central rod can be observed to be red at \(S_N = 1.27\). In practice, a stable swirling...
combustor is desirable for power generation and energy sectors. Thus, it is important for industrial gas turbines to carefully vary critical parameters to avoid nonlinear limit cycle oscillations being generated.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgement

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Author contributions

YZ.S. and ZM.R. conducted experimental measurements. D. Z. and YZ.S. processed and analysed the experimental data and discussed the results with B. W., DK. S. and XF. S. D. Z. conceived and initialized the project. All authors contributed to the paper writing.

Appendix A

To evaluate the accuracy of the experimental measurements, the experimental error analysis is conducted on the $p_{\text{rms}}$ calculated by measurement acoustic pressure. The results are summarized in tables below.

Here $\sigma$ is defined as $\sigma = \frac{1}{N} \sum_{i=1}^{N} |p_i - \frac{1}{N} \sum_{i=1}^{N} p_i|$ and $\chi_{\text{max}} = \max \left\{ \sum_{i=1}^{N} \left| p_i - \frac{1}{N} \sum_{i=1}^{N} p_i \right| \right\}$ $N = 50,000$ is the total number of measurements.

See Tables A1‒A3 and Table B1.

Table A1
A summary of experimental measurement max error and variance, as equivalence ratio is varied.

<table>
<thead>
<tr>
<th>Equivalence ratio test $p_{\text{rms}}$ (Pa)</th>
<th>Test1</th>
<th>Test2</th>
<th>Test3</th>
<th>Variance $\sigma$</th>
<th>Max Error $\chi_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Phi = 0.6$</td>
<td>32.14</td>
<td>31.77</td>
<td>30.54</td>
<td>0.40</td>
<td>2.3%</td>
</tr>
<tr>
<td>$\Phi = 0.7$</td>
<td>624.08</td>
<td>599.75</td>
<td>622.07</td>
<td>182.43</td>
<td>2.5%</td>
</tr>
<tr>
<td>$\Phi = 0.8$</td>
<td>1532.29</td>
<td>1587.50</td>
<td>1585.29</td>
<td>977.05</td>
<td>2.3%</td>
</tr>
<tr>
<td>$\Phi = 0.9$</td>
<td>1762.96</td>
<td>1768.92</td>
<td>1765.42</td>
<td>8.96</td>
<td>0.2%</td>
</tr>
<tr>
<td>$\Phi = 1.0$</td>
<td>1147.78</td>
<td>1141.58</td>
<td>1146.33</td>
<td>10.53</td>
<td>0.3%</td>
</tr>
<tr>
<td>$\Phi = 1.1$</td>
<td>340.48</td>
<td>333.66</td>
<td>335.99</td>
<td>12.00</td>
<td>1.1%</td>
</tr>
</tbody>
</table>

Table A2
A summary of experimental measurement max error and variance, as air flow rate $V_a$ is varied.

<table>
<thead>
<tr>
<th>Air flow rate test $p_{\text{rms}}$ (Pa)</th>
<th>Test1</th>
<th>Test2</th>
<th>Test3</th>
<th>Variance $\sigma$</th>
<th>Max Error $\chi_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_a = 130$ L/min</td>
<td>72.96</td>
<td>72.50</td>
<td>74.76</td>
<td>1.42</td>
<td>1.8%</td>
</tr>
<tr>
<td>$V_a = 155$ L/min</td>
<td>75.66</td>
<td>76.59</td>
<td>78.10</td>
<td>1.52</td>
<td>1.7%</td>
</tr>
<tr>
<td>$V_a = 180$ L/min</td>
<td>234.20</td>
<td>245.31</td>
<td>238.29</td>
<td>31.59</td>
<td>2.5%</td>
</tr>
<tr>
<td>$V_a = 215$ L/min</td>
<td>624.08</td>
<td>599.75</td>
<td>622.07</td>
<td>182.43</td>
<td>2.5%</td>
</tr>
<tr>
<td>$V_a = 250$ L/min</td>
<td>743.49</td>
<td>746.97</td>
<td>752.48</td>
<td>20.54</td>
<td>0.6%</td>
</tr>
</tbody>
</table>

Table A3
A summary of experimental measurement max error and variance, as swirling number $SN$ is varied.

<table>
<thead>
<tr>
<th>Swirling number test $p_{\text{rms}}$ (Pa)</th>
<th>Test1</th>
<th>Test2</th>
<th>Test3</th>
<th>Variance $\sigma$</th>
<th>Max Error $\chi_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SN = 0.42$</td>
<td>614.38</td>
<td>600.02</td>
<td>614.20</td>
<td>67.87</td>
<td>1.6%</td>
</tr>
<tr>
<td>$SN = 0.73$</td>
<td>1193.89</td>
<td>1176.49</td>
<td>1193.24</td>
<td>97.28</td>
<td>1.0%</td>
</tr>
<tr>
<td>$SN = 1.27$</td>
<td>1762.96</td>
<td>1768.92</td>
<td>1765.42</td>
<td>8.96</td>
<td>0.2%</td>
</tr>
</tbody>
</table>

Table B1
Experimental conditions.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equivalence ratio $\Phi$</td>
<td>0.6, 0.7, 0.8, 0.9, 1.0, 1.1</td>
</tr>
<tr>
<td>Air flow rate $V_a$</td>
<td>130, 155, 180, 215, 250 (L/min)</td>
</tr>
<tr>
<td>Swirling number $SN$</td>
<td>0.42, 0.73, 1.27</td>
</tr>
</tbody>
</table>
Appendix B. Supplementary material

Supplementary data to this article can be found online at https://doi.org/10.1016/j.apenergy.2020.114698.

References