Phase Synchronization Based Minimum Spanning Trees for the Analysis and Visualization of Currency Exchange Markets

Sivarit Sultornsanee\textsuperscript{a,*}, Arjun Duvvuru\textsuperscript{b}, Srinivasan Radhakrishnan\textsuperscript{c}, Harnita Chowdhary\textsuperscript{c}, Sagar Kamarthi\textsuperscript{d}

\textsuperscript{a}Department of Logistics Management, School of Business, University of Thai Chamber of Commerce, Bangkok, Thailand
\textsuperscript{b}Research Institute for Policy Evaluation and Design University of Thai Chamber of Commerce, Bangkok, Thailand
\textsuperscript{c}Symbiosis Institute of Management Studies, Pune India
\textsuperscript{d}Department of Mechanical and Industrial Engineering, Northeastern University, Boston, U.S.A

Abstract

Correlation based Minimum Spanning Tree (MST) networks are instrumental in capturing the basic workings of a system and have been accepted for use in the analysis of stock and currency exchange markets. Research in network analysis of financial markets shows that although correlations underlying MST networks capture essential information, they do not faithfully capture dynamic behavior embedded in the time series data of financial systems. We present a new Phase Synchronization (PS) based method for establishing correlations between nonlinear time series data, prior to constructing the MST. In this method, time series data generated by each entity in a system is transformed to a recurrence plot and the recurrence plots are further transformed to trajectories in phase space. For each pair of trajectories, phase synchronization (PS) is quantified based on the degree of phase locking observed. Distances for the MST are then computed as a function of the PS between each entity pair. We demonstrate the method using Thailand Baht exchange rates with 82 countries from 2004 to 2012. We further analyze and compare networks constructed using the PS method (PS-MST) and the existing cross-correlation method (CC-MST), to study the differences in market dynamics captured by these methods.

Keywords: Correlation Networks; Financial Networks; Foreign Currency Exchange Network; Phase Synchronization; Financial Analytics

1. Introduction

Several diverse fields such as genetics, epidemiology, transportation, sociology, finance, and economics have made use of the methods of network science to analyze, understand, and even control behavior of complex systems.

* Corresponding author. \textit{E-mail address:} sultornsanee@riped.utcc.ac.th
In finance, network science has been widely accepted as a practical and effective way to study dynamics of stock markets [2] [3], currency exchanges [4] [5], and overnight money markets [6]. Financial networks such as a stock market or a currency exchange network are constructed based on correlations between time series of the constituent financial entities, and are hence, also known as correlations networks. Typically, a correlation network is constructed using the following steps [3] [7],

1. Individual time series are considered as nodes in the network.
2. The cross correlation denoted by \( c_{ij} \) is computed for each pair of time series.
3. The correlation coefficients forming an \( N \times N \) correlation matrix with \(-1 \leq c_{ij} \leq 1\) are transformed into a \( N \times N \) distance matrix with elements \( d_{ij} = \sqrt{2(1-c_{ij})} \), such that \( 2 \geq d_{ij} \geq 0 \). The symmetric property of the distance formula ensures that \( d_{ij} = d_{ji} \). The triangular property reveals the relationship between the distance value and the correlation coefficient (smaller distance values indicate high correlation).
4. The distance matrix is essentially an adjacency matrix representing the correlation network. The distance matrix is then used to determine the minimum spanning tree (MST) – a simply connected graph that connects all the \( N \) nodes of the network with \( (N-1) \) edges such that the sum of all distances is minimum.

This method of constructing networks from correlations in time series data has been used in the analysis of stock markets [2] [8] and foreign currency exchange markets [5] [9] with the objective of determining the hierarchical structure of the respective systems, and how that structure changes during periods of turbulence and stability. Research in network analysis of financial markets shows that although correlations underlying MST networks capture essential information, they do not faithfully capture dynamic behavior embedded in the time series data of financial systems [10] [11]. In this paper we present a new Phase Synchronization (PS) based method as an alternate for establishing correlations between nonlinear time series data, prior to constructing the MST. This method is based on the principles of recurrence theory and has been shown in several studies, to effectively capture the dynamic behavior embedded in complex time series data [10].

In the next section the proposed PS-MST method is outlined and in Section 3 the method is demonstrated by analyzing the exchange rates of 82 currencies with the Thai Bhat from 2004 to 2012.

2. Method

Phase synchronization (PS) between a pair of entities is defined as the degree of phase locking observed between trajectories of the time series corresponding to these entities in phase space. In the context of a financial system, ‘the time series of an entity’ is essentially the fluctuation in value of stock or currency over time. Determining the PS between each entity pair in a system involves the following steps [10] [12],

1. Map time series of entities to trajectories in phase space. These trajectories are plots of the variable \( x \) in the time series data at time \( t \) versus the same variable at time \( t-\tau \), where \( \tau \) is the embedded time delay.
2. Compute recurrences for each trajectory using Equation 1. \( R_{ij} \) in the equation is a binary variable that takes a value of 1 if a state or data point \( \tilde{x} \) on the trajectory at time \( i \) falls within the neighborhood of \( \tilde{x} \) at time \( i+\tau \). In Equation 1, \( ||.|| \) denotes a suitable norm and defines the shape of neighborhood, \( \epsilon \) is the threshold that defines the size of neighborhood, and \( \theta \) is the Heaviside step function.

\[
R_{ij+\tau} = \theta(\epsilon - ||\tilde{x}_i - \tilde{x}_{i+\tau}||) \tag{1}
\]

3. Calculate the recurrence probability at time delay \( \tau \) for each trajectory using Equation 2. Note that the denominator on the right hand side of the equation is the sum recurrences calculated for \( N-\tau \) data points on the trajectory.

\[
P(\tau) = \frac{1}{N-\tau} \sum_{i=1}^{N-\tau} \theta(\epsilon - ||\tilde{x}_i - \tilde{x}_{i+\tau}||) \tag{2}
\]
4. Compute the PS between each trajectory pair as a cross correlation coefficient $CPR_{1,2}$ using Equation 3. In Equation 3, $\bar{P}_1(\tau)$ and $\bar{P}_2(\tau)$ are the mean-centered recurrence probabilities for trajectory 1 ($P_1(\tau)$) and trajectory 2 ($P_2(\tau)$); $\sigma_1$ and $\sigma_2$ are the standard deviations of $P_1(\tau)$ and $P_2(\tau)$ respectively.

$$CPR_{1,2} = \frac{\bar{P}_1(\tau)\bar{P}_2(\tau)}{\sigma_1\sigma_2}. \quad (3)$$

If two trajectories (corresponding to the time series of the entities) are phase synchronized, then the probability of recurrence is maximal at the same time and $CPR=1$. For N entities under consideration a $N \times N$ matrix of $CPR$ values is computed and then transformed into a $N \times N$ distance matrix with elements $d_{ij} = \sqrt{2(1-CPR_{ij})}$. The MST is then constructed from the distance matrix using Kruskal’s algorithm.

3. Data and Analysis

Thai currency exchange data was obtained from http://oanda.com/ for the year 1997 and for nine years from 2004 to 2012. 1997 was the year of the Thai currency crisis and the Asian financial crisis [13]; 1997 was included to compare the results between periods of crisis and stability. The time series data was divided into ten time windows, one window for each year. Minimum spanning trees were constructed for nine years (2004-2012) time period and for each of the ten time windows using PS-MST and CC-MST methods. Network measures from the nine time period were used as a baseline for comparison with measures from each of the ten time windows. In this paper we present results for networks from 1997, 2004, 2008 and 2012 time windows.

The MSTs generated by PS-MST and CC-MST were analyzed with the objective of gaining insights into the variations in network topology and measures, over time and between the two methods. Table 1 shows the list of network measures examined and their purpose.

<table>
<thead>
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<th>Network measures analyzed</th>
<th>Purpose of analysis</th>
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<tr>
<td>Average Distance as a function of Endpoint Degree</td>
<td>Determine how edge and node properties influence overall topology of the currency exchange network</td>
</tr>
<tr>
<td>Average Betweenness Centrality as a function of Degree</td>
<td>Determine how node properties and position in the network influence topology of the currency network</td>
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<tr>
<td>Node Degree</td>
<td>Examine if node properties change with external events</td>
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<tr>
<td>Network Diameter and Average Shortest Path</td>
<td>Examine if edge properties change with external events</td>
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First, we compare endpoint degree $k_i k_j$ as a function of average distance $<d_{ij}>$. Endpoint degree of an edge is the product of the degrees of nodes the edge is incident on. Fig. 1a and 1b are plots of average distance versus endpoint degree for networks constructed from PS-MST and CC-MST respectively. A first look at the plots suggests that the relationship between the endpoint degree and average distance is random. A closer observation of the data points corresponding to 1997 reveals that the average distance for almost every value of $k_i k_j$ is below 0.2, and the average distances for other time windows predominantly occupy levels between 0.2 and 0.45. This observation is an indication of change in the structure of the currency exchange network during times of turbulence. In the case of CC-MST, the average distances for the 1997 time window also predominantly occupy levels below 0.2, but there is also a considerable occupation in this level by data points from other time windows.

Second, we compare the variation of betweenness centrality of a node with respect to its degree. Betweenness centrality of a node is the fraction of shortest paths between node pairs that pass through that node. In simple terms it is the measure of influence a node has on controlling flow through the network. In the case of both PS-MST (Fig. 2a) and CS-MST (Fig. 2b), the betweenness centrality is observed to increase with node degree. But the upward trend appears to hold stable up to the third degree nodes.
There is a considerable variation in centrality observed between time windows for nodes above degree four. For instance, nodes of degree 5 in Figure 2b have an average centrality between 0.25 and 1.0 whereas nodes of degree 3 have an average centrality between 0.22 and 0.4. These observations indicate the presence of two types of high degree nodes or hubs – prominent players in the network and inactive hubs lying outside the main spine of the minimum spanning tree [3]. In the case of PS-MST, 1997 and 2008 time windows have an average centrality lower than the other time windows indicating an influence of external events – the Thai currency devaluation and the subprime mortgage crisis, on the network’s structure.

A comparison of high degree nodes or hubs across all time windows showed that the position of hubs in the minimum spanning tree changes from time to time. Fig. 3a and Fig. 4a show how the positions of the high degree nodes changed for the time windows under consideration. For both PS-MST and CC-MST there are very few nodes that always take the top three degrees. In the case of PS-MST, North Korean Won, UAE Dhiram, Chinese Yuan, Singapore Dollar, and Taiwan Dollar are the major hubs in the minimum spanning tree as they fall within the top three degrees in at least three of the ten time windows (see Fig. 3b). In the case of CC-MST, North Korean Won, Chinese Yuan, Qatari Riyal, Danish Krone, Singapore Dollar, and Macau Pataca are the major hubs (see Fig. 4b).
A Comparison of diameters of currency networks and average shortest paths for the ten time windows including 1997 is shown in Fig. 5. Network diameter is the minimum of all the shortest paths computed in the MST and the average shortest path is the mean of shortest paths between every pair of nodes in a network. Both PS-MST and CC-MST show a low point during periods of market turbulence – 1997 and 2008. For the time windows after 2008, the networks responded with a general increase in diameter and average shortest path with the exception of the 2011 time window corresponding to CC-MST. This observation shows that the exchange networks tend to reduce distances between nodes (or shrink as a whole) during crisis events; and increase distances (or expand) during periods of stability.
4. Conclusion

Currency exchange networks generated by PS-MST and CC-MST were analyzed using different network measures such as endpoint degree, betweenness centrality, diameter, and average shortest path. The comparative analysis showed that both PS-MST and CC-MST methods are equally good in capturing the topology; in other words, both methods have the ability to produce network maps representative of the Thai currency exchange. Analysis of endpoint degree and betweenness centrality showed that PS-MST produced characteristic low values on both metrics during the Thai currency crisis of 1997 and U.S. subprime mortgage crisis of 2008. This shows that PS-MST, unlike CC-MST, has the ability to capture changes in structure of the Thai currency exchange network during crisis events. Analysis of network diameter and average shortest path showed that networks produced by both PS-MST and CC-MST respond to market conditions in a similar way – they shrink in size during market turbulence and expand during the more stable periods. Examination of just the node degrees revealed no more information than the major hubs in the exchange networks.

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References