Application of nonlinear time series analysis techniques to high-frequency currency exchange data

Fernanda Strozzi\textsuperscript{a}, José-Manuel Zaldívar\textsuperscript{b}, Joseph P. Zbilut\textsuperscript{c,}\textsuperscript{*}

\textsuperscript{a}Engineering Department, Quantitative Methods Group, Carlo Cattaneo University, 21053 Castellanza (VA), Italy
\textsuperscript{b}Institute for Environment and Sustainability, European Commission, Joint Research Center, 21020 Ispra (VA), Italy
\textsuperscript{c}Department of Molecular Biophysics and Physiology, Rush University, 1653 W. Congress, Chicago, IL 60612, USA

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Abstract

In this work we have applied nonlinear time series analysis to high-frequency currency exchange data. The time series studied are the exchange rates between the US Dollar and 18 other foreign currencies from within and without the Euro zone. Our goal was to determine if their dynamical behaviours were in some way correlated. The nonexistence of stationarity called for the application of recurrence quantification analysis as a tool for this analysis, and is based on the definition of several parameters that allow for the quantification of recurrence plots. The method was checked using the European Monetary System currency exchanges. The results show, as expected, the high correlation between the currencies that are part of the Euro, but also a strong correlation between the Japanese Yen, the Canadian Dollar and the British Pound. Singularities of the series are also demonstrated taking into account historical events, in 1996, in the Euro zone. © 2002 Elsevier Science B.V. All rights reserved.

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* Corresponding author. Tel.: +1-312-942-6008; fax: +1-312-942-8711.
E-mail address: joseph_p_zbilut@rsh.net (J.P. Zbilut).
1. Introduction

The seminal work of Nicolis and Prigogine [1] and Haken [2] has lead to the realisation that large classes of systems may exhibit abrupt transitions, hysteresis, spatio-temporal structures or deterministic chaos. This has questioned the reductionist paradigm, i.e., the reduction of observed phenomena to elementary entities at lower levels of hierarchy and organisation. Furthermore, it has been observed that nonlinear phenomena, that are not adequately described by linear approximations, are encountered in all areas of science.

Despite this rich variety of nonlinear dynamical systems, there is accumulating evidence that certain complex scenarios are frequently repeated between different fields of science. These findings indicate, that although complex systems may differ substantially in their detailed properties, significant analogies relative to their organisation and functioning exist. As a consequence, there has been an increasing interest in the study of “complexity” and in the search of a common background to all these systems [3,4]. The search of this common background has mainly concentrated in economics on two types of paradigms: Self-organized criticality (SOC) [5] and chaotic systems. SOC systems are deterministic nonequilibrium systems composed by many interacting parts which have the ability to develop structures and patterns in the absence of control or manipulation by an external agent [6]. This emergent behaviour, which the interacting parts cannot show alone, is not just the sum of their individual properties, and, although, it is dynamically complex, the statistical properties are described by simple power laws. SOC systems have been investigated in such diverse areas as geophysics (earthquakes), astrophysics (quasars), condensed matter physics, biological evolution and economics. The paradigm model for SOC is a sand pile [5].

The second class of complex systems are chaotic systems. Chaotic systems are deterministic systems governed by a “low” number of variables, which display apparently complex behaviour. Furthermore, even though chaotic systems are described by differential equations, which do not contain any random function, they are unpredictable in the long term due to their ability to amplify even a very small initial perturbation of initial conditions. Chaos theory has also been applied to a wide variety of fields, e.g., physics, chemistry, engineering, ecology and economics. The roots of economists’ interest in chaotic systems are to be found in the vast literature on business cycles. In fact, throughout the last century economists have postulated the existence of different dynamical behaviours in the form of economic cycles, including the business cycle [7], the Kuznets [8], and the Kondratieff cycle or economic long wave [9]. Since variations in amplitude and period have been observed, it is clear that they are not regular cycles, but is there a manifestation of chaotic behaviour? Is it possible to find the degrees of freedom that govern such behaviour?

Specifically, in the last decades there have been a considerable amount of discussion relating the theory of Brownian motion [10,11], fractional Brownian motion [12], non-linearity [13], chaos and fractals [14–16], scaling behaviour [17,18], and SOC [5,19] to the study of financial time series. The problem of characterising financial time series is still an open question [20]. Most of the tests developed in the area of economic theory, provide evidence of nonlinear dynamics, which is a necessary but not
sufficient condition for chaos. This nonlinearity may be deterministic or not determinis-
tic. In fact, there is no convincing evidence of deterministic low dimensionality in price
series [21] and the claims of low-dimensional chaos have never been well justified. For
example, Andreadis [22] analysing the S&P 500 index time series favours the stochastic
hypothesis, whereas Friederich et al. [23] using the high-frequency price changes of
the US Dollar–German Mark exchange rates support the analogy of turbulence and
financial data [17].

In this work we have applied nonlinear time series techniques to high-frequency
currency exchange data from the HFDF96 data set provided by Olsen & Associates.
The time series studied are the exchange rates between the US dollar and 18 other
foreign currencies from the Euro zone; i.e., Belgium Franc (BEF), Finnish Markka
(FIM), German Mark (DEM), Spanish Peseta (ESP), French Franc (FRF), Italian Lira
(ITL), Dutch Guilder (NLG), and finally ECU (XEU)—and from outside the Euro
zone—Australian Dollar (AUD), Canadian Dollar (CAD), Swiss Franc (CHF), Danish
Krone (DKK), British Pound (GBP), Malaysian Ringgit (MYR), Japanese Yen (JPY),
Swedish Krona (SEK), Singapore Dollar (SGD) and South African Rand (ZAR). Our
main interest was to classify these series and to determine if their dynamical behaviours
are in some way correlated. Furthermore, we were also interested in comparing the Euro
with the exchange currencies that are part of it to analyse its behaviour and properties
in comparison with them.

A prerequisite to analysis is the determination of stationarity of the series. To this
end space–time separation plots were performed and revealed that indeed the data
were transient. The absence of stationarity called for the application of recurrence
quantification analysis (RQA). This method is based on the definition of several pa-
rameters that allow for the quantification of the recurrence plots (RP) introduced by
Eckmann et al. [24]. The RP is based on the computation of a distance matrix between
reconstructed points in phase space, and was originally suggested to be a tool for the
determination of stationarity. This produces an array of distances in a square matrix.
In order to extend the original concept and make it more quantitative, Zbilut and Web-
ber [25] developed a methodology called RQA [26]. As a result, they defined several
variables to quantify RPs, which are: \( \% \text{recur} \) (percentage of neighbouring points in
recurrence plot); \( \% \text{deter} \) (percentage of recurrent points forming diagonal line struc-
tures);\( \text{entropy} \) (Shannon entropy of line segments distributions);\( \text{trend} \) (measure of the
paling recurrence points away from the central diagonal); \( 1/\text{line}_{\text{max}} \) (reciprocal of the
longest diagonal line segment which may relate directly to largest positive Lyapunov
exponent) [27].

The RQA analysis of each currency exchange rate time series has shown a certain
coherent structure. This structure allows a preliminary classification of the time series
in several clusters. Moreover, the RQA analysis was repeatedly performed on 336-point
epochs in order to analyse the dynamic information obtained. Neighbouring epochs were
shifted by 48 points and the nonlinear variables \( \% \text{recur} \), \( \% \text{deter} \) and \( \text{line}_{\text{max}} \) obtained
for the 18 time series analysed. As discussed in the text, it is possible to correlate
certain events that occurred during 1996 with those variables, for example, the entries of
the Finnish Markka and the Italian Lira in the European Monetary System. Furthermore,
the RQA method allowed for the study of the degree of correlation between several data
Table 1
Data considered: foreign exchange rates against US Dollar

<table>
<thead>
<tr>
<th>Data set</th>
<th>Name</th>
<th>Data set</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUD</td>
<td>Australian Dollar</td>
<td>GBP</td>
<td>British Pound</td>
</tr>
<tr>
<td>BEF</td>
<td>Belgium Franc</td>
<td>ITL</td>
<td>Italian Lira</td>
</tr>
<tr>
<td>CAD</td>
<td>Canadian Dollar</td>
<td>MYR</td>
<td>Malaysian Ringgit</td>
</tr>
<tr>
<td>CHF</td>
<td>Swiss Franc</td>
<td>JPY</td>
<td>Japanese Yen</td>
</tr>
<tr>
<td>DEM</td>
<td>German Mark</td>
<td>NLG</td>
<td>Dutch Guilder</td>
</tr>
<tr>
<td>DKK</td>
<td>Danish Krone</td>
<td>SEK</td>
<td>Swedish Krona</td>
</tr>
<tr>
<td>ESP</td>
<td>Spanish Peseta</td>
<td>SGD</td>
<td>Singapore Dollar</td>
</tr>
<tr>
<td>FIM</td>
<td>Finnish Markka</td>
<td>XEU</td>
<td>ECU</td>
</tr>
<tr>
<td>FRF</td>
<td>French Franc</td>
<td>ZAR</td>
<td>South African Rand</td>
</tr>
</tbody>
</table>

sets by comparing the evolution of the $\%\text{recur}$ in the time series. The analysis shows, as anticipated, the high correlation among the exchange currencies in the Euro zone as well as their persistence. Based on these results, a derived indicator of correlation between financial time series is presented. This indicator shows, for example the high correlation between Japanese Yen and Canadian Dollar, and the fact that British Pound was, in 1996, more correlated to those series than to those from the Euro zone. In this, RQA methodology shows its potential for analysing high-frequency currency exchange rates, and how it can be used to find prediction windows and correlation windows between several currencies. In fact, the RQA methodology is able to detect changes in currency exchange rates states. Thus, it may allow for the improvement of forecasting capabilities by considering simultaneously correlated time series, as has been shown for the case of prediction in nonlinear dynamical systems [28].

2. Data

The data sets used were purchased from Olsen & Associates, called HFDF96, and is a subset of the O&A data bank that has been collected through real-time data-feeds using proprietary O&A data collection software. The data set HFDF96 consists of 43 intraday time series composed of:

1. half hourly bid and ask quotes for 25 major foreign exchange spot rates,
2. half hourly bid and ask quotes for four precious metals spot prices (XAU; XAG; XPT; XPD),
3. twelve half hourly series of transaction prices form six major Euromarket future contracts (USD, DEM, GPB, CHF, ITL, XEU),

The data span a period of 1 year from January 1996 to 31 December 1996. Each record includes the interpolation times (GMT). In this work, we have analysed half hourly bid ask quotes for 18 of the major foreign exchange spot rates, which are summarised in Table 1.
2.1. Data normalisation

We consider the logarithmic middle price $y_m$ as our primary time series, which can be calculated as follows:

$$y_m = \frac{\log(p_{bid}) + \log(p_{ask})}{2},$$  \hspace{1cm} (1)$$

where $p_{bid}$ and $p_{ask}$ are the bid and ask prices of the US Dollar with respect to some currency, respectively. In order to compare the different data sets analysed, we have normalised data sets between 0 and 1 and obtained a normalised logarithmic middle price $y$ as follows:

$$y = \frac{y_m - \min(y_m)}{\max(y_m) - \min(y_m)}.$$  \hspace{1cm} (2)$$

Fig. 1 represents a typical data set after transformation. Each data set contains 17568 points, which corresponds to 48 points each day times 366 days, and hence, for example, 10 February is in correspondence with the period 1968–2016.
2.2. Historical background

Although the first stage in the Economic and Monetary Union process in the EU began on 1 July 1990 with the liberalisation of capital movements, the entry into force of the Treaty on European Union on 1 November 1993 marked the genuine starting point of preparations for Economic and Monetary Union (EMU). In accordance with the Treaty, the second stage began on 1 January 1994 with, in particular, establishment of the European Monetary Institute (EMI), based at Frankfurt (Germany) as a first step towards the creation of the European System of Central Banks. The monetary turmoil experienced in 1995, largely caused by the slide in the value of the dollar, strengthened the Member States’ political determination to go ahead with EMU. That determination took shape at the Madrid European Council of 15 and 16 December 1995, which confirmed that the third stage of Economic and Monetary Union was to go ahead on 1 January 1999 in accordance with the convergence criteria, the timetable, the protocols and procedures laid down in the Treaty. The single currency was named the Euro.

Throughout 1996 and 1997 the economic upturn, against a background of closer nominal convergence, interest and inflation rates at exceptionally low levels, and stable exchange rates, enabled there to be a general improvement in the state of public finances, paving the way for the majority of Member States to switch to the Euro in 1999.

In 1991 Finland, Norway and Sweden related their currencies to the Euro (ECU). In 1992 due to the instabilities in the financial markets a 15% fluctuation margin was conceded. Despite that, the British Pound as well as the Italian Lira abandoned the EMS in 1992. On 14 October 1996, the Finnish Markka joined the EMS exchange-rate mechanism. The Italian Lira returned on 25 November to the ERM. Also in 1996 the Austrian Schilling joined the EMS. Only Greece, Sweden, Denmark and the United Kingdom were not members. From 1 January 1995 Austria, Finland and Sweden entered the EU. From 1995 to 1997 the EU countries had to equate their economical parameters to those defined in the Maastricht Treaty. On 3 May 1998 the European Council confirmed the 11 countries that could adopt the Euro. However, the United Kingdom, Sweden and Denmark decided not to enter in the EMS.

At the start of 1996 there were preoccupations that even those EU members states that were most enthusiastic about monetary union would have great difficulties in meeting the conditions for taking part in the planned move to a single currency in January 1999. The tough qualification criteria including limits on government budget deficits and government debt levels and the timetable made the financial markets sceptical. The attitude began to change after the meeting of the finance ministers of the EU governments in Verona (Italy), in April. There, it became clear that all member nations were determined to make the goal of the EMU their economical and political priority. This was further confirmed when one after the other EU member states announced strong austerity measures during the summer of 1996 designed to reduce their budget deficits and meet the EMU criteria. Reflecting this remarkable political determination to achieve the single currency, the European financial markets gradually became less sceptical about the prospects for accomplishing it.
3. Analysis

3.1. Detecting nonstationarity

An important first step in the analysis of the data, is to determine if the series is stationary—all other calculations of invariants presume this fact—both linear and non-linear. Broadly speaking a time series is said to be stationary if there is no systematic change in mean (no trend), in variance, and, if strictly periodic variations have been removed. Most of the probability theory of time series is concerned with stationary time series, and for this reason time series analysis often requires one to turn a nonstationary series into a stationary one so as to use this theory. However, it is also worth stressing that the nonstationary components, such as the trend, may sometimes be of more interest than the stationary residual. Additionally, it should be remembered, that removal of nonstationary aspects of a series implies an a priori hint as to the form of the nonstationarity—something not always available. Furthermore, in the case of nonlinear series, there is always the concern that techniques used to “filter” the data, may, in fact, alter the representation of the dynamics. Indeed, most filters used are based on linear systems theory—to date, there is no universally accepted theory for filtering nonlinear systems, let alone in the presence of noise from acquisition or the dynamics.

In addition to the use of RPs as suggested originally by Eckmann et al. [24] to detect nonstationarity, we also report here a stationarity test, called space-time separation plot, introduced by Provenzale et al. [29]. The idea below is that in the presence of temporal correlations the probability that a given pair of state points in the reconstructed state space, \( \{s(t_i), s(t_i - \Delta t), s(t_i - 2\Delta t), \ldots\} \), has a distance smaller than \( r \), i.e., \( ||s_i - s_j|| < r \), does not depend only on the position of the state but also on the time that has elapsed between them. This dependence can be detected by plotting the number of neighbour points as a function of two variables, the time separation and the spatial distance. In principle, one can create for each time separation an accumulated histogram of spatial distances. In the case of power-law noises, only points with small spatial separation are dynamically near neighbours, i.e., the series is nonrecurrent in phase space. In this case the contour curves do not saturate. In the case of stationarity, we find a saturation in the plot.

Fig. 2 shows a typical result of the test for the analysed time series. In the graphic, the separation time is represented in the horizontal axis whereas the base 2 logarithm of the separation in space is represented in the vertical axis. For small \( \Delta t \), points are always near neighbours in space, as their time separation increases so does their separation in space, in principle [29]. Technically, it is necessary to create, for each time separation \( \Delta t \), an accumulation histogram of spatial distance \( \varepsilon \). We have used the program \( stp \) of Tisean [30] which returns level lines for 10%, 20%, \ldots, of pairs with a given temporal separation, \( \Delta t \).

It can be seen that the curves do not saturate at all, as was found for all of the series. Apart from the nonstationarity, this is another indication that the data we are analysing has significant power in the low frequency, such as \( 1/f \) noise or Brownian motion. In this case, all points in the data set are temporally correlated and there is no way of determining an attractor dimension from the sample. A similar situation arises if the
data set is too short. Then there are no pairs left after removing temporally correlated pairs. If we regard the problem from a different point of view, correlation times of the order of the length of the sample (nonsaturating curves) means that the data does not sample the observed phenomenon sufficiently [30].

3.2. Recurrence quantification analysis (RQA)

The actual methods developed in nonlinear time series analysis assume that the data series under analysis have reached their attractor, that they are not in a transient phase, that they are autonomous, and that their lengths are much longer than the characteristic time of the system in question. In the case of foreign exchange time series, this does not happen and it may be useful to have another procedure to analyse these data.

As previously stated, Eckmann et al. [24] introduced a new graphical tool, which they called an RP. The RP is based on the computation of the distance matrix between the reconstructed points in phase space, i.e., \( s_i = \{s(t), s(t-\tau), s(t-2\tau), \ldots, s(t+(d_E-1)\tau)\} \),

\[
d_{ij} = \|s_i - s_j\|. \tag{3}
\]
This produces an array of distances in a $N \times N$ square matrix, $D$, with $N$ the number of points under study. Once this distance matrix is calculated, in the original paper of Eckmann et al. [24], a plot was created by darkening pixels located at specific $(i, j)$ coordinates which correspond to a distance value between $i$ and $j$ lower than a predetermined cut-off, i.e., a ball of radius $r_i$ centered at $s_i$. Requiring $r_i = r_j$, the plot is symmetric and with a darkened main diagonal corresponding to the identity line. The darkened points individuate the recurrences of the dynamical systems and the RP provides insight into periodic structures and clustering properties that are not apparent in the original time series [24].

As also previously stated, to make this concept quantitative, Zbilut and Webber developed the RQA methodology [25,26]. They defined several variables to quantify RPs, which are: $\%\text{recur}$ (percentage of darkened pixels in recurrence plot); $\%\text{deter}$ (percentage of recurrent points forming diagonal line structures); $\text{entropy}$ (Shannon entropy of line segments distributions); $\text{trend}$ (measure of the paling recurrence points away from the central diagonal); $1/\text{line}_{\text{max}}$ (reciprocal of the longest diagonal line segment which relates directly to largest positive Lyapunov exponent) [27]. These five recurrence variables quantify the deterministic structure and complexity of the plot: $\%\text{recur}$ quantifies the amount of cyclic behaviour; $\%\text{deter}$ the amount of determinism through the counting of “sojourn points” [31]; $\text{entropy}$ the richness of deterministic structuring; $1/\text{line}_{\text{max}}$ scales with the maximum Lyapunov exponent; while $\text{trend}$ is essentially a measure of nonstationarity.

Since RQA methodology is, in principle, independent of limiting constraints such as data set size, data stationarity, and assumptions regarding statistical distributions of data, they started to apply RQA to physiological systems characterised by nonhomeostatic transients and state changes. Furthermore, they have extended the application to different systems ranging from molecular dynamics simulations [32] to human speech [33].

First, RQA was performed on all the time series using the time delay method of first minimum of the mutual information [34] and embedding dimension roughly according to the false nearest neighbors method of Kennel et al. [35], $d_E$ (FNN), as modified by Cao [36], $d_E$ ($E1&E2$), (Table 2) for the state space reconstruction. We

<table>
<thead>
<tr>
<th>Data set</th>
<th>$\tau$</th>
<th>$d_E$ (FNN)</th>
<th>$d_E$ ($E1&amp;E2$)</th>
<th>Data set</th>
<th>$\tau$</th>
<th>$d_E$ (FNN)</th>
<th>$d_E$ ($E1&amp;E2$)</th>
</tr>
</thead>
<tbody>
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<td>8</td>
<td>14</td>
<td>GPB</td>
<td>276</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>BEF</td>
<td>256</td>
<td>11</td>
<td>12</td>
<td>ITL</td>
<td>240</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>CAD</td>
<td>252</td>
<td>9</td>
<td>10</td>
<td>MYR</td>
<td>250</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>CHF</td>
<td>254</td>
<td>9</td>
<td>10</td>
<td>JPY</td>
<td>283</td>
<td>7</td>
<td>12</td>
</tr>
<tr>
<td>DEM</td>
<td>279</td>
<td>7</td>
<td>12</td>
<td>NLG</td>
<td>243</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>DKK</td>
<td>273</td>
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<td>13</td>
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<td>10</td>
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<tr>
<td>ESP</td>
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<td>12</td>
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<td>SGD</td>
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<td>11</td>
<td>10</td>
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<tr>
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<td>11</td>
<td>XEU</td>
<td>250</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>FRF</td>
<td>271</td>
<td>9</td>
<td>12</td>
<td>ZAR</td>
<td>270</td>
<td>15</td>
<td>14</td>
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Table 3
RQA values for the foreign exchange time series

<table>
<thead>
<tr>
<th>Data set</th>
<th>%recur</th>
<th>%deter</th>
<th>Entropy</th>
<th>Maxline</th>
<th>Trend</th>
<th>Data set</th>
<th>%recur</th>
<th>%deter</th>
<th>Entropy</th>
<th>Maxline</th>
<th>Trend</th>
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</thead>
<tbody>
<tr>
<td>AUD</td>
<td>50.2</td>
<td>93.6</td>
<td>8.5</td>
<td>1943</td>
<td>−74.9</td>
<td>GBP</td>
<td>45.4</td>
<td>85.7</td>
<td>8.0</td>
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<td>−20.9</td>
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<tr>
<td>BEF</td>
<td>42.3</td>
<td>70.5</td>
<td>5.8</td>
<td>2552</td>
<td>−26.5</td>
<td>ITL</td>
<td>47.1</td>
<td>74.8</td>
<td>7.4</td>
<td>2858</td>
<td>−42.8</td>
</tr>
<tr>
<td>CAD</td>
<td>50.7</td>
<td>86.9</td>
<td>7.7</td>
<td>2990</td>
<td>−47.9</td>
<td>MYR</td>
<td>60.5</td>
<td>82.7</td>
<td>7.5</td>
<td>2508</td>
<td>−57.3</td>
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<tr>
<td>CHF</td>
<td>51.5</td>
<td>81.3</td>
<td>7.9</td>
<td>2972</td>
<td>−24.2</td>
<td>JPY</td>
<td>51.3</td>
<td>87.8</td>
<td>7.6</td>
<td>2145</td>
<td>−68.0</td>
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<td>88.2</td>
<td>8.0</td>
<td>2189</td>
<td>−56.4</td>
<td>NLG</td>
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<td>85.2</td>
<td>8.0</td>
<td>3557</td>
<td>−19.4</td>
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<td>90.1</td>
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<td>1982</td>
<td>−65.4</td>
<td>SEK</td>
<td>48.1</td>
<td>86.4</td>
<td>8.0</td>
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<td>−40.9</td>
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<td>73.5</td>
<td>7.9</td>
<td>2508</td>
<td>−4.4</td>
<td>SGD</td>
<td>48.2</td>
<td>79.0</td>
<td>7.7</td>
<td>3017</td>
<td>−37.0</td>
</tr>
<tr>
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<td>56.3</td>
<td>93.0</td>
<td>8.7</td>
<td>2480</td>
<td>−53.8</td>
<td>XEU</td>
<td>51.7</td>
<td>87.9</td>
<td>8.3</td>
<td>3008</td>
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<tr>
<td>FRF</td>
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<td>82.9</td>
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<td>2227</td>
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<td>ZAR</td>
<td>52.3</td>
<td>90.7</td>
<td>6.8</td>
<td>1748</td>
<td>−84.0</td>
</tr>
</tbody>
</table>

Fig. 3. RP analysis of the Australian–US Dollars foreign exchange time series. (Example of constant RP.) RQA parameters: time delay and embedding dimension (see Table 4), distance cut-off: mean distance between points, line definition: 48 points (1 day).

Note that given the nonstationary nature of the data, calculation of a “correct” delay and embedding is specious, since theoretically, the dynamics can move through different dimensions. What is important is that the embedding be high enough to include all the relevant dynamics (including the effects of noise). It is clear from Table 2, that an alternative procedure, choosing an embedding of 15 empirically, would also suffice [37].

The RQA computed values are summarised in Table 3. By visual inspection, it is possible to distinguish between several general patterns in the recurrence plots. This allows an empirical classification of the series as follows: (1) constant RP over time (AUD and JPY) (Fig. 3); (2) higher recurrence during the first 6 months (MYR and ZAR) (Fig. 4); (3) higher recurrence during the last 6 months (CAD, DEM, DKK, FIM, SEK, SGD) (Fig. 5); (4) aeroplane structure (BEF, CHF, ESP, FRF, GPB, ITL,
Fig. 4. RP analysis of the Malaysian Ringgit–US Dollar foreign exchange time series. (Example of higher recurrence during the first 6 months.) RQA parameters: time delay and embedding dimension (see Table 4), distance cut-off: mean distance between points, line definition: 48 points (1 day).

Fig. 5. RP analysis of the Finnish Markka–US Dollar foreign exchange time series. (Example of higher recurrence during the last 6 months.) RQA parameters: time delay and embedding dimension (see Table 4), distance cut-off: mean distance between points, line definition: 48 points (1 day).

NLG, XEU) (Fig. 6). Of course, for some time series this classification is clearer than for others and there are some cases that fall in between. We note that this qualitative classification method has been previously verified [38].

Afterwards, RQA was repeatedly performed on 336-point epochs, which corresponds to a week of data, generating 305 values for each of the five RQA variables, i.e., $\%\text{recur}$, $\%\text{deter}$, $\text{entropy}$, $\text{trend}$, $1/\text{line}_{\text{max}}$. Neighbouring epochs were shifted by
Fig. 6. RP analysis of the British Pound–US Dollar foreign exchange time series. (Example of aeroplane structure.) RQA parameters: time delay and embedding dimension (see Table 4), distance cut-off: mean distance between points, line definition: 48 points (1 day).

48 points, which corresponds to one day. The nonlinear variables \(\%\text{recur}\), \(\%\text{deter}\) and \(\text{line}_{\text{max}}\) are plotted as function of time (see Figs. 7–9, for an example). \(\%\text{recur}\) quantifies the percentage of the plot occupied by recurrent points, which corresponds to the proportion of recurrent pairs over all possible pairs below the chosen radius. \(\%\text{deter}\) denotes the percentage of recurrent points that appear in sequence, forming diagonal line structures in the distance matrix. It corresponds to the amount of patches of recurrent behaviour in the studied time series, and indicates portions of the state space where the system resides for a longer time than expected by chance alone [39]. This is an important indication of deterministic signature in the time series. Following its variation it is possible to distinguish several levels and variations, and hence, it is possible to study how our forecasting capabilities would change with the level of \(\%\text{deter}\) we observe in the time series. For example, there is a generalised increase in the \(\%\text{deter}\) in summer around July (\(\text{time} = 10000\)) for a considerable number of exchange rate time series, mainly in the Euro zone (for an example see Fig. 9). As was emphasised in the Historical Background Section, exactly in summer 1996 the EU member states decided to reduce their budgets deficits in order to meet the EMU criteria. \(\text{line}_{\text{max}}\) is the length in terms of consecutive points of the longest recurrent line in the plot scales with the maximum Lyapunov exponent. Trulla et al. [27] found that \(\text{line}_{\text{max}}\) was able to accurately predict the value of the maximum Lyapunov exponent and locate bifurcation points in a logistic map going from regular to chaotic regime. For example, it is possible to see in Figs. 7–9, that the entrance of the Finnish Markka on October 1996 (around 14 000) and of the Italian Lira on November 1996 (around 16 000) in the Exchange-Rate Mechanism, changed \(\text{line}_{\text{max}}\) not only in both currencies but also in the ECU time series.
4. Comparison of currency exchange rates

In order to compare the different currency exchange rates time series, the \%recur values were calculated using the same embedding parameters for the 18 time series. A mean value for the time delay, $\tau = 260$, and embedding dimension, $d_E = 11$, were used. The radius cut-off was defined as 10\% of the mean distance between points in all the time series, $r_c = 0.043$. Then the calculated \%recur values were plotted against each other. This guarantees that we are comparing the same time periods in the currency exchange rates time series. The data sets were fitted by linear regression and the values of the regression coefficients, $r$, are summarised in Table 4 (we recall that $r$ provides a measure of how well the linear regression model can reproduce the actual output, i.e., the $y$-axis).

As can be seen, currencies in the Euro zone were highly correlated during 1996. This is due to the fact that those currencies were part of the European Monetary
Fig. 8. Nonlinear metrics of the Italian Lira–US Dollar foreign exchange time series: %deter (upper panel), %recur (middle panel) and line_{max} (lower panel). Values are computed from a 336 point window (epoch), data are shifted 48 points between epochs. RQA parameters: time delay and embedding dimension (see Table 2), distance cut-off: mean distance between points/10, line definition: 16 points (8 h). The vertical line represents the entry of the Italian Lira in the European Monetary System.

System so their relative oscillations were controlled by their respective Central Banks. Surprisingly, a similar strong correlation has been found for the system Canadian Dollar, Japanese Yen and British Pound. This group even though it was not controlled, exhibited the same type of behaviour. On the other side, the Swiss Franc is the currency, which has lower regression coefficients with all the other currencies. The results from Table 4 allow us to define a measure of the degree of correlation between exchange currency rates time series.

Furthermore, it is possible, by comparing the %recur, to show the existence of correlation in time between several time series. For example, Fig. 10 shows that the Belgium Franc was correlated during the entire year to the ECU; for the case of the Italian Lira, Fig. 11, a different correlation is seen with the ECU before and after (crosses) its entrance in the EMS; whereas for the Swiss Franc it is possible to establish two periods (January–March, left line) and (May–December, right line) with a transition between
Fig. 9. Nonlinear metrics of the ECU–US Dollar foreign exchange time series: %deter (upper panel), %recur (middle panel) and line_{max} (lower panel). Values are computed from a 336 point window (epoch), data are shifted 48 points between epochs. RQA parameters: time delay and embedding dimension (see Table 2), distance cut-off: mean distance between points/10, line definition: 16 points (8 h). The vertical lines represent the entry of the Finnish Markka and the Italian Lira in the European Monetary System, respectively.

them (crosses), (Fig. 12). Hence, we can establish correlation periods between several exchange rates time series. This helps to improve the degree of precision in forecasting by combining both series, when correlated.

5. Conclusions

The lack of saturation in the space time separation plots shows that the time series are either nonstationary or that they have not been sampled long enough, and hence, the application of the surrogate data tests which assume a stationary Gaussian linear process as a null hypothesis is not adequate.

Representing the data using the RP allows for the individuation of four different structures: constant RP over time, high recurrence during the first 6 months or in the last 6 months, aeroplane structure. Evidently, some time series are in the borderline of these classification schema and to classify them in one group or in another is rather
Table 4

Regression coefficients for the high frequency currency exchange rates time series

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Fig. 10. Plot of the %recurrence for the ECU and the Belgium Franc.
subjective. Individual analysis of high-frequency currency exchange rates from 1996 using RQA has allowed us to establish a correlation between changes in the system and external factors. The fact that there are considerable variations in the measure of $%deter$ during the year may be indicative of changes in our forecasting potential. This may be related to singular events, which effect an important change in the dynamics [39]. Our current research is oriented along these lines to quantify further this effect.

In order to compare the results obtained for all the time series we have calculated the RP on a weekly basis shifted by a day, and we have generated 329 values for each of the five RQA variables ($%det$, $%recur$, entropy, trend, $1/maxline$) as a function of time, using the same state space reconstruction parameters. We have plotted $%recur$ of each time series as a function of $%recur$ of the other currencies and used the regression coefficient to define a new measure of nonlinear correlation. We observe that in some cases, for example $%recur$ (BEF) with respect to $%recur$ (XEU) there is a strong correlation (linear behaviour) whereas in other cases, for example $%recur$ (CHF) with respect to $%recur$ (XEU), we have something like a phase transition from one line to another.

The results show, as expected, the high correlation between the currencies that were part of the Euro zone in 1996. This can be considered as a confirmation of the technique, but also an unexpected strong correlation between the Japanese Yen, the Canadian Dollar, and the British Pound. Furthermore our method has been found that,
in the Euro zone, the most correlated currencies in the ECU, in 1996, were the Spanish Peseta and the Belgium Franc. However, the higher correlation coefficient corresponds to the Danish Krone, which even though it was not in the Euro zone had the typical behaviour of one of the currencies inside. At this point, we have to remember that in 1996 Italian Lira and Finnish Markka entered in the Euro zone only at the end of the year, i.e., respectively the 25 of November and 14 of October, and data on Portuguese Escudo exchange rates were not available.

Finally, by inspection of the different plots, it is possible to observe changes in the degree of correlation during the year. This technique can find an application whenever we are interested in finding correlation in a temporal series, for example, when we want to minimise the portfolio’s risk by investing in noncorrelated markets.

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References