A reduced-order deterministic model describing an intermittency route to combustion instability

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Recently, there has been a growing interest in understanding and characterising intermittent burst oscillations that presage the onset of combustion instability. We construct a deterministic model to capture this intermittency route to instability in a bluff-body stabilised combustor by coupling the equations governing vortex shedding and the acoustic wave propagation in a confinement. A feedback mechanism is developed wherein the sound generated due to unsteady combustion affects the vortex shedding. This feedback leads to a variation in the time of impingement of the vortices with the bluff body causing the system to exhibit chaos, intermittency, and limit cycle oscillations. Experimental validation of the model is provided using various precursor measures that quantify the observed intermittent states.

Keywords: combustion instability; dynamical system; feedback; chaos; intermittency

1. Introduction

Combustion instability is a serious problem that affects the performance of rocket motors, jet engines, and gas turbines [1]. To understand combustion instability, the mechanisms that generate heat release, the hydrodynamics, the acoustic field, and their interplay need to be studied. The oscillations resulting from such an interaction can damage the combustor and also lead to other unwanted effects. The principle behind the formation of self-sustaining oscillations observed during combustion instability has not yet been understood completely, mainly because the interaction of these different components gives rise to a very complex dynamical system.

The coupling between the flow, acoustics, and heat release rate is the main factor that results in combustion instability. It is such a coupling that makes the system exhibit large-amplitude self-sustaining oscillations. If vortex shedding is present in the flow, the occurrence of self-sustaining oscillations is quite common and this has been perceived as a severe problem in propulsion [2,3]. A number of investigations have been performed to determine the effects of vortex shedding on combustion [4–7]. These experiments show the presence of vortical structures which carry unburnt fuel and indicate coupling between the heat release rate and the energy supplied by the hydrodynamic field. Hence, it becomes necessary to model combustion in the presence of vortex shedding. An excellent review of the topic can be found in [8].

Matveev and Culick [9] developed a model that describes the interaction between vortex shedding, heat release rate, and the acoustic field present in the chamber. This model depicts the acoustic modes as a kicked oscillator.

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It has been recently established that, in combustion systems involving vortex shedding, the transition to combustion instability occurs through intermittency [10]. Intermittency, here, is a dynamical state that displays large-amplitude periodic oscillations amidst low-amplitude aperiodic fluctuations. The model by Matveev and Culick [9] does not exhibit this intermittent behaviour. To capture the intermittent behaviour, a model was proposed by Nair and Sujith [11]. This model accounts for the presence of turbulence in the combustor; it portrays intermittency as a resultant of the interaction of this turbulent flow with the acoustic field and the heat release rate. Nair et al. [12] presented a case study where combustion noise was found to be deterministic chaos and showed that the transition to combustion instability is marked by loss of chaos. The model proposed by Nair and Sujith [11] qualitatively describes the phenomena that occur during this transition; however, the model uses stochastic elements.

As combustion noise is now shown to be deterministic in certain cases, it is necessary to develop a model that describes the transition to combustion instability without the use of any probabilistic quantities. The aim of this study is, therefore, to develop a simple deterministic model that is able to capture the distinctive features of the actual system. The model is intended to match qualitatively the results of experiments performed in [10, 12]; these experiments are performed using a turbulent combustor. Hence, the model developed in this study in fact captures the effects of hydrodynamics in a turbulent combustor without introducing too much complexity. Such a model would therefore be very helpful in providing insight into the physics of the transition to instability and in devising strategies to prevent and control such a transition.

In the present paper, the model by Matveev and Culick [9] is revisited. Changes to this model are proposed based on the interaction of acoustics and hydrodynamics. The main focus is to make minimal changes to the model but capture qualitatively the physics of the problem, the goal being to obtain a deterministic model that captures transition to combustion instability via intermittency. The dynamics of the system is investigated from the point of view of dynamical systems theory. The presence of various dynamical regimes reported in experiments [10] is confirmed using techniques available in dynamical systems theory. Further, the pattern of the impingement time of the vortices is investigated in the different dynamical regimes.

2. The model

The model described here is based on the model by Matveev and Culick [9] and captures the chaos and intermittency observed in thermoacoustic systems. A duct of length $L$ is modelled as closed–open, with a bluff body at a distance $L_c$ from inlet end (Figure 1). The backward facing step of height $d$ at the inlet end causes vorticity to build-up and shed vortices which are convected downstream. The vortices then impinge on the bluff body resulting in a localised, instantaneous heat release rate. The heat release rate affects the acoustic field while the acoustic field affects the hydrodynamics. Hence, there exists a three way coupling between the flow, acoustics, and heat release rate.

Neglecting the effects of mean flow on wave propagation and the effects of viscosity and temperature gradients, the linearised momentum and energy equations are given as [13]

$$\frac{1}{\bar{\rho}} \frac{\partial p'}{\partial x} + \frac{\partial u'}{\partial t} = 0$$  \hspace{1cm} (1)
Figure 1. A schematic of the bluff-body stabilised combustor. $L_c$ shows the location of the point of combustion. $L$ is the length of the closed–open duct and $d$ is the height of the step. $\bar{u}$ is the mean velocity of the incoming flow. All horizontal distances are measured from the inlet end. $L_s$ refers to the location of the step; it is equal to zero according to the above schematic since the step is at the inlet end.

\[
\frac{\partial p'}{\partial t} + \gamma \bar{p} \frac{\partial u'}{\partial x} = (\gamma - 1) \dot{Q}',
\]

(2)

where the heat release is modelled as [9]

\[
\dot{Q}' = \beta \sum_j \Gamma_j \delta(x - L_c)\delta(t - t_j).
\]

(3)

Here $t_j$ represents the time when the $j$th vortex impinges on the bluff body and $\beta$ is a constant to balance the magnitude and units.

Following Balasubramanian and Sujith [14], a Galerkin expansion [15] is used to express the acoustic pressure and velocity in terms of the natural modes of a closed–open duct:

\[
p' = \bar{p} \sum_{j=1}^{N} \frac{\eta_j(t)}{\omega_j} \cos(k_j x) \quad \text{and} \quad u' = \bar{p}c \sum_{j=1}^{N} \eta_j(t) \sin(k_j x),
\]

(4)

where $k_j = \omega_j/c$.

Substituting the above expansion in the energy equation and introducing a damping term as suggested in [16], the following kicked oscillator equation involving heat release rate is obtained:

\[
\ddot{\eta}_n + \xi_n \dot{\eta}_n + \omega_n^2 \eta_n = b \omega_n \cos(k_n L_c) \sum_j \Gamma_j \delta(t - t_j),
\]

(5)

where $b = 2(\gamma - 1)\beta/L\bar{p}$.

The mode dependant damping coefficient is modelled as [11]

\[
\xi_n/\xi_1 = (2n - 1)^2,
\]

(6)

where $\xi_1$ can be obtained experimentally.
Further, the equation governing the production of vorticity at the step is given as \[ \dot{\Gamma} = \frac{1}{2} u(t)^2, \] (7)

and, when the circulation at the step reaches a critical value of \( \Gamma_{cr} = u(t)d/2S_t \), a vortex is shed \[9\]. Here, \( u(t) = \bar{u} + u'(L_s, t) \) is the velocity at the step; \( L_s \) is the location of the step from the inlet end, and \( S_t \) the Strouhal number. The downstream convection of the shed vortex is governed by the equation \[9\]

\[ \frac{dx_j}{dt} = \alpha \bar{u} + u'(x_j, t). \] (8)

It has been reported in the aeroacoustics literature that the impingement of a vortex leads to a feedback wave travelling upstream; this feedback wave affects the region of maximum receptivity of the shear layer, the step \[17,18\]. The perturbations introduced due to feedback eventually grow into a large-scale vortex \[17\] that impinges on the bluff body and thus closes the feedback loop. In combustion systems, as mentioned previously, the impingement of a vortex on the bluff body leads to heat release and the heat release rate affects the acoustic field. Even in such systems, it has been observed that the acoustic waves excite the separating shear layer and the resulting vortical fluctuations interact with the flame, leading to a variation in the heat release rate \[19,20\]. To capture this phenomenon, we propose that the change in the pressure at the point of impingement due to the burning of a vortex leads to feedback that affects the circulation building up at the step. Hence, if \( p_{Lc} \) is the pressure generated at the bluff-body location \( L_c \) due to the impingement of a vortex, the circulation at the step changes by an amount

\[ \Delta \Gamma = Bp_{L_c}, \] (9)

where \( B \) is a constant of proportionality. In order to match the experimental results qualitatively, the following form for the constant \( B \) is proposed:

\[ B = B_1 + B_2 S(\bar{u} - B_3), \] (10)

\[ S(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ 1 & \text{if } x > 0. \end{cases} \]

Here, \( S(x) \) is a step function. \( B_1 \) and \( B_2 \) are constants indicative of the feedback strength and \( B_3 \) represents the mean velocity at which there is a change in the feedback strength. It will be shown later in Section 3 ‘Results and discussion’ that, for such a choice of \( B \), the dynamics observed in the model as the system transitions to combustion instability is the same as that observed in the experiments.

It has to be noted here that the acoustic feedback is only responsible for creating perturbations at the step, but the eventual growth of perturbations is due to hydrodynamics \[17\]. Hence, the term \( \Delta \Gamma \) leads to changes in the time of vortex shedding but does not dictate the circulation of the shed vortex. The shed vortex still has a circulation of \( \Gamma_{cr} \). Equation (7) is then justly modified as

\[ \dot{\Gamma} = \frac{1}{2} u(t)^2 + \sum_j B p_{L_c}(t_j) \delta(t - t_j - \tau_s), \] (11)
Table 1. Values of parameters used to obtain data.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.575</td>
</tr>
<tr>
<td>$b$</td>
<td>$-6 \times 10^{-3}$ m/s</td>
</tr>
<tr>
<td>$\xi_i$</td>
<td>29 s$^{-1}$</td>
</tr>
<tr>
<td>$L_s$</td>
<td>0</td>
</tr>
<tr>
<td>$L_c$</td>
<td>0.05 m</td>
</tr>
<tr>
<td>$L_0$</td>
<td>0.7 m</td>
</tr>
<tr>
<td>$c$</td>
<td>700 m/s</td>
</tr>
<tr>
<td>$\bar{p}$</td>
<td>1 atm</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.4</td>
</tr>
<tr>
<td>$B_1$</td>
<td>$-13.9$ m$^2$/atm-s</td>
</tr>
<tr>
<td>$B_2$</td>
<td>$-5.36$ m$^2$/atm-s</td>
</tr>
<tr>
<td>$B_3$</td>
<td>9.4 m/s</td>
</tr>
</tbody>
</table>

where $t_j$ is the time at which impingement occurs and $\tau_s$ is the time it takes for the sound wave to travel from $L_c$ to $L_s$, i.e. the acoustic time delay.

The proposed model captures the dynamics of the combustion system without the use of any stochastic quantities. This is in contrast to the model proposed in [11], where probabilistic quantities have been used to describe the transition to combustion instability.

3. Results and discussion

The set of equations was integrated using the Runge–Kutta method of order 4, with $N = 10$ basis functions and a time step of $dt = 10^{-5}$ s. The pressure signal was computed at a distance $x = 0.09$ m from the step. The values of various parameters used to solve the equation are given in Table 1. These parameters, wherever possible, are chosen to match those utilised in the experiments by Nair et al. [12]. An initial value of $\eta_1 = 0.001$ was used in obtaining the plots. A transient signal of 0.4 s duration was removed from the pressure signal obtained numerically before computing the relevant quantities. After the impingement of a vortex on the bluff body, a delay of eight time steps was considered before changing the circulation at the step. This delay was obtained by dividing $L_c - L_s$ by the speed of sound.

Data obtained from the experiments performed by Nair et al. [10,12] is used for comparison with the results obtained from the model. The experimental setup consists of a turbulent combustor with the flame stabilised using a bluff body. The bluff body is located at 0.05 m and the pressure transducer at 0.09 m, respectively, from the backward-facing step. A more detailed description of the experimental setup can be found in [10].

We compare the pressure signal obtained from experiments with the signal computed numerically. As we can see from Figure 2, for low mean velocity ($\bar{u}$), the experimental system displays low-amplitude chaotic fluctuations. As $\bar{u}$ increases, the experimental system enters an intermittent state, where there are large-amplitude bursts of periodic oscillations along with low-amplitude aperiodic fluctuations; with further increase in $\bar{u}$, limit cycle oscillations are observed. The same behaviour is seen in the numerical model with increasing mean velocity. The system displays small-amplitude aperiodic fluctuations for $\bar{u} = 9.5$ m/s. On further increasing the mean velocity, intermittency is observed. Finally, the system displays limit cycle oscillations at 10.8 m/s. These dynamical regimes seen in the model are
Figure 2. Acoustic pressure signal variation at $x = 0.09$ m showing (a) chaos at $\bar{u} = 8.51$ m/s, (b) intermittency at $\bar{u} = 10.12$ m/s, (c) intermittency at $\bar{u} = 10.28$ m/s, and (d) limit cycle oscillations at $\bar{u} = 10.92$ m/s. The signals have been obtained from experiments.

shown in Figure 3. Figure 3(a) shows the presence of chaos in the model at low velocities. Figures 3(b) and 3(c) depict intermittency and it can be seen from these figures that the amplitude and duration of the intermittent bursts increase with increase in the mean velocity; the same is observed in the case of experiments. Figure 3(d) shows limit cycle oscillations.

The variation of the rms value of the pressure signal as a function of the control parameter is shown in Figure 4. We observe that the rms value increases with increase in the control parameter in the experiments as well as in the model. We also plot the FFT of the pressure signals obtained from the experiments and the model. We can see from Figure 5 that the instability frequency predicted by the model is close to 250 Hz, which matches the limit cycle frequency seen in the experiments. Note that absence of fluctuations in the
Figure 3. Acoustic pressure signal variation at $x = 0.09$ m showing (a) chaos at $\bar{u} = 9.5$ m/s, (b) intermittency at $\bar{u} = 10.4$ m/s, (c) intermittency at $\bar{u} = 10.65$ m/s, and (d) limit cycle oscillations at $\bar{u} = 10.8$ m/s. The signals were obtained using the model.

FFT of instability obtained from the model is due to the perfect limit cycle oscillations seen in the model. Further, the 0–1 test for chaos [21] was used to inspect the dynamical behaviour of the system as it transitions to combustion instability. A value of the measure ($k$) of the 0–1 test close to 1 indicates chaotic behaviour, while a value close to 0 indicates periodic behaviour of the system. Figure 6 shows the variation of $k$ with the mean velocity, and both the experiments and the model show a gradual loss of chaos as the limit cycle is approached.
Figure 4. Variation of the root-mean-square (rms) value of the pressure signal ($p_{\text{rms}}$) with the control parameter in (a) the experiments and (b) the model.

Figure 5. Frequency spectrum of limit cycle pressure oscillations computed using the Fast Fourier Transform (FFT) of the signal obtained from (a) the experiments and (b) the model. The frequency peak corresponds to 249 Hz in the case of the experiments, while the peak is at 250.54 Hz for the model.
Figure 6. Variation of the measure \( k \) of the 0–1 test for chaos with the control parameter in (a) the experiments and (b) the model. In either case, \( k \) goes to zero as the limit cycle is approached, indicating loss of chaos.

Figure 7. Recurrence plots for (a) a chaotic signal at \( \bar{u} = 8.03 \text{ m/s} \) with \( d = 8 \) and \( \tau = 1.2 \text{ ms} \), (b) an intermittent signal at \( \bar{u} = 10.12 \text{ m/s} \) with \( d = 11 \) and \( \tau = 1.1 \text{ ms} \), and (c) a periodic signal at \( \bar{u} = 10.92 \text{ m/s} \) with \( d = 12 \) and \( \tau = 1.1 \text{ ms} \). Here, \( d \) is the embedding dimension and \( \tau \) is the time delay. A threshold of \( \lambda/5 \) was used to compute the recurrence plots, where \( \lambda \) is the maximum distance between pairs of points in the reconstructed phase space. The Euclidean norm was used in computing the distances. The plots were obtained using experimental data.

Observing the patterns formed by recurring trajectories in the phase space gives a deeper insight into the dynamics of the system. The phase space of the system can be reconstructed (with an embedding dimension \( d \) and a time delay \( \tau \)) using the pressure signal. When the distance between a given vector and another vector in this reconstructed phase space is less than some threshold value, the trajectory is said to recur. The plot of the binary matrix thus obtained by computing the recurrence of vectors in the reconstructed phase space is termed a recurrence plot. A black point in this plot corresponds to the time instances when the trajectory recurs. Recurrence plots are described in detail in [22].

Recurrence plots have been obtained for the pressure signals that are chaotic, intermittent, and periodic. Figure 7 shows the recurrence plots for experimental data while
Figure 8. Recurrence plot for (a) a chaotic signal at $\bar{u} = 9.2$ m/s with $d = 11$ and $\tau = 1.1$ ms, (b) an intermittent signal at $\bar{u} = 10.4$ m/s with $d = 11$ and $\tau = 0.6$ ms, and (c) limit cycle oscillations at $\bar{u} = 10.8$ m/s with $d = 6$ and $\tau = 0.7$ ms. Here, $d$ is the embedding dimension and $\tau$ is the time delay. A threshold of $\lambda/5$ was used to compute the recurrence plots, where $\lambda$ is the maximum distance between pairs of points in the reconstructed phase space. The Euclidean norm was used in computing the distances. The plots were obtained using the model.

Figure 9. The variation with mean velocity of (a) the recurrence rate $r$, (b) the Shannon entropy $s$, and (c) the trapping time $\tau_p$. A fixed threshold of $\epsilon = 2000$ was used. The signals were embedded in a phase space of dimension $d = 19$, with a delay of $\tau = 1.2$ ms. The Euclidean norm was used in computing the recurrence quantities. These recurrence quantities are seen to fall at independent rates as the limit cycle is approached. These measures were computed using experimental data.
Figure 10. The variation with mean velocity of (a) the recurrence rate $r$, (b) the Shannon entropy $s$, and (c) the trapping time $\tau_p$. A fixed threshold of $\epsilon = 4000$ was used. The signals were embedded in a phase space of dimension $d = 18$, with a delay of $\tau = 1.1$ ms. The Euclidean norm was used in computing the recurrence quantities. These recurrence quantities are seen to fall at independent rates as the limit cycle is approached. These measures were computed using data obtained from the model.

Figure 8 shows the same for data obtained from the model. Figures 7(a) and 8(a) show the recurrence plot for a chaotic pressure signal. A grainy pattern in the plots suggests that the signal is chaotic. Figures 7(b) and 8(b) show the recurrence plot for an intermittent region, where regions containing diagonal lines can be seen amidst black rectangular patches. The pressure signal displaying limit cycle oscillations shows diagonal lines in the recurrence plots as can be seen from Figures 7(c) and 8(c). This indicates similarity in the dynamics of the experimental system and the model.

The recurrence plots can be further used in quantifying some aspects of the dynamics of the system. This is called Recurrence Quantification Analysis (RQA). It has been shown that quantities such as the recurrence rate, entropy, and the average passage time (or trapping time) obtained from RQA can act as precursors that identify the onset of combustion instability [10]. The recurrence rate measures the density of the black points in the recurrence plots and is given as $r = \sum_{i,j=1}^{N} R_{ij}/N^2$ where $R_{i,j}$ is the $(N \times N)$ recurrence matrix. The Shannon entropy ($s$) measures the amount of order in the system and is obtained from the diagonal line distribution in the recurrence plot. If $P(l)$ is the histogram of diagonal lines of length $l$ and $N_l = \sum_{l=1}^{N} P(l)$ is the total number of diagonal lines present, then $p(l) = P(l)/N_l$ gives the probability that a diagonal line has length $l$. The Shannon entropy is then given as $s = -\sum_{l=1}^{N} p(l) \ln(p(l))$. Further, the trapping time ($\tau_p$) gives an estimate of the time spent in the aperiodic state and is given as $\tau_p = \sum_{v=1}^{N} v P(v)/\sum_{v=1}^{N} P(v)$, where $P(v)$ is the histogram of vertical lines of length $v$. Figure 9 shows the variation of $r$, $s$, and $\tau_p$ with the mean velocity for experimental data, while Figure 10 shows the variation of these recurrence quantities with the mean velocity for data obtained from the model. It can be seen that these quantities fall prior to the system reaching combustion instability, thus acting as precursors. This has been observed previously in experiments [10].
In order to investigate the dynamical nature of the system further, the pattern of impingement of the vortices is studied. The change in circulation at the step on the impingement of a vortex at the bluff body leads to variation in the impingement times of the vortices. This variation is the reason for the observed dynamical behaviour of the system. Therefore, observing the time between two impingements, and also how this time difference varies as time progresses, gives a way to quantify the observed behaviour of the system. This idea is illustrated in Figure 11, where the time difference between two impingements has
been plotted against time for data obtained from the model. Whenever the time difference becomes orderly, i.e. whenever the time difference between two vortex impingements is within 20% of the time difference between the previous two impingements, the point is marked with a filled circle; otherwise, an empty circle is used as the marker. As can be seen from Figure 11(a), for low velocities where the signal is chaotic, the time difference between two bursts does not show any pattern. As the velocity increases, large-amplitude bursts are observed amidst aperiodic fluctuations. It is observed in Figure 11(b) that, for an intermittent signal, the time difference between impingement of vortices become orderly in the region of high-amplitude bursts, but appears to be disorganised in the aperiodic region. Figure 11(c) shows that the limit cycle corresponds to the impingement of vortices at the acoustic frequency. This is because the time period of acoustic oscillations is 0.004 s and it can be seen from this figure that the time difference between impingement of two consecutive vortices is the same. This indicates that the vortices impinge in an ordered fashion in the limit cycle regime. Also, the merging of the two branches of impingement times present in Figure 11(c) shows that the vortex impingement gradually locks on to the acoustic field. Hence, the presence of feedback between acoustics and hydrodynamics leads to the eventual coupling between the two. Since the vortices impinge at the acoustic frequency, the heat release rate also oscillates at the same frequency. Thus, the locking-on of acoustics, hydrodynamics, and combustion is seen to occur at combustion instability.

Figure 12 shows the solution for \( \bar{u} = 10.5 \text{ m/s} \) with \( B = 0 \) (without feedback in the build-up of circulation) and \( B \neq 0 \). It can be clearly seen that the intermittent behaviour is lost when there is no feedback. This observation emphasises the need to incorporate feedback in the build-up of circulation. We see from the preceding paragraphs that the choice of a step function for \( B \) leads to dynamics that is qualitatively similar to the experiments. However, it has to be noted at this point that the choice of \( B \) here is only empirical. The variation
of $B$ with mean velocity might be different in the actual system and a better estimate can be obtained through particle image velocimetry (PIV) experiments.

On a final note, we elucidate the main differences between the proposed model and the model by Nair and Sujith [11] since the latter also describes the transition to combustion instability. First, we wish to highlight that the experimental observations point to deterministic dynamics in the considered system [12]. The model given in [11] uses stochastic elements to describe the transition to instability while the model presented here is completely deterministic. The proposed model therefore shows that we do not need any probabilistic quantities in describing the transition from combustion noise to combustion instability via intermittency.

We further note that the proposed model incorporates acoustic pressure feedback in the circulation. The presence of such feedback in aeroacoustic as well as combustion systems has been investigated previously [17,19]. It is an important physical mechanism that influences the formation of vortical patterns in the system. But the model described in [11] does not account for such feedback. In fact, in the model by Nair and Sujith [11],
there is no major influence of the acoustic sub-system on the hydrodynamic sub-system. The importance of feedback can be illustrated by comparing the time series depicting intermittent pressure oscillations, shown in Figure 13. Experimentally, we see that the burst of periodic oscillations is large in duration and sandwiched in between low-amplitude chaotic oscillations (Figure 13(a)). A similar observation is seen in the case of intermittent oscillations obtained from the proposed model (see Figure 13(b)). However, for oscillations obtained from the model given in [11] we can see from Figure 13(c) that, although the oscillations are intermittent, the pattern and duration of the bursts are different from what is seen in the experiments. The reason for this is the absence of feedback in this model; feedback helps in prolonging the duration of the intermittent burst. Hence, we see that the proposed model is necessary as it incorporates the necessary physical mechanisms and also gives a deterministic description of the transition as required by the experiments.

4. Conclusions
A simple deterministic phenomenological model was constructed to capture the transition to combustion instability. This model shows the presence of chaos at low velocities. Then, as the velocity is increased, large-amplitude intermittent bursts are seen amidst aperiodic fluctuations, and finally a large-amplitude limit cycle is observed. Hence, the model is seen to capture the rich dynamics present in combustion systems that have vortex shedding.

It has to be noted that the model proposed in this paper is completely deterministic. Previous work in this direction [11] has incorporated elements of stochastics. However, it has been established through experiments that the considered system is deterministic in nature. Further, this reduced-order model captures the effects of hydrodynamics in combustion systems that have vortex shedding. However, the great amount of complexity associated with turbulence present in such systems has been avoided. Thus, this model serves as a tool to understand the physics of such complex systems in a relatively easy manner.

We show that feedback between heat release, acoustics, and vortex shedding causes the system to exhibit chaos, intermittency, and limit cycle oscillations. The sound generated by combustion affecting the vortex shedding is responsible for such feedback. The presence of this kind of feedback has been reported in the aeroacoustics literature. The system shows chaos for low velocities and there is a loss of chaos as instability is approached. This loss of chaos occurs through the formation of intermittent bursts of periodic oscillations. Further, recurrence quantities such as the recurrence rate, the Shannon entropy, and the trapping time fall as the system approaches instability. These quantities are therefore seen to behave in the same manner as observed in the experiments.

Vortex shedding and impingement happen in an aperiodic fashion for low mean velocities. As the mean velocity increases, the impingement of vortices is seen to become in tune with the periodic intermittent oscillations. Finally, at the onset of instability, we observe that the vortices impinge at acoustic frequencies. Thus, vortex shedding and impingement lock on to the acoustic field in the chamber, leading to self-sustaining oscillations in the combustor.

Disclosure statement
No potential conflict of interest was reported by the authors.

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