Adoption of continuous wavelet and recurrence plots to comparative studies of neutron monitor data sets from the period 1975 – 1992

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Abstract

The hourly pressure corrected data of individual neutron monitor station (NM) collected in WDC, from seventies of XX century to nowadays, are compared with records of other stations with similar cut-off rigidity in order to determine the quality of the studied data. Taking no account some short time details as GLE events, one can expect a similar NM intensity response at stations, with the similar cut-off rigidity, to the solar variability on moderate and long time scales. It could be reflected by similar values of the Pearson correlation coefficients and slopes of regression lines between the smoothed solar activity indices and NM data. The solar activity is represented by Ottawa/Penticon radio flux. If the correlation coefficients considerably wander away from the mean value for the determined epoch the data registered at such station in this time period should not be used to a further study of long-term variations. Powerful wavelet technique can also be applied to test the data in time - frequency space. We
provide such analysis for two sets of data with high and moderate cut-off rigidities. Periods with data gaps or with inconsistency of records ought to be corrected before further elaboration by means of the neuron network (NN) reconstructions. We propose the Elman NN as possibly the best available method for such correction. The results are presented in tables and on wavelet plots. Stations which have showed considerably different variations at some periods than other are indicated.

I. Introduction

Many time series of neutron component data are collected in World Data Centers from the fifties of 20\textsuperscript{th} century to nowadays. It is very important to believe that those data are typified as stabilized and generally of a good quality. In the prepared paper the correctness of the pressure corrected data of individual neutron monitor (NM) stations is examined on the monthly to yearly time scale separately for the periods of different heliospheric magnetic field polarity from 1958 to 2014. Taking no account some short time details as GLE events, one can expect a similar NM intensity response at every station with the similar cut-off rigidity, and possibly if such stations exist with similar asymptotic directions, to the solar variability on moderate and long time scales. It could be reflected by similar values of the Pearson correlation coefficients and slopes of regression lines between the smoothed solar activity indices and NM data registered at different stations. The solar activity is represented by Ottawa/Penticon radio flux. If the correlation coefficients considerably wander away from the mean value for the determined epoch the data registered at such station in that time period should not be used to a further study of long-term variations. Powerful wavelet technique can also be applied to test in time - frequency space the existing data. As examples, we show in this poster such the preliminary study of variability of 3 neutron monitor stations: Rome, Alma Ata B and Tbilisi in the time period 1975-1992.

II. Data used

We use here the hourly pressure corrected data from 3 neutron monitor stations with cut off rigidities equal about 6 GV as it is shown on Table 1. Only a few stations with this value of cut off exist. The knowledge of cosmic ray particle variations of rigidities equal and bigger than 6 GV is very important for modulation theories. The
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<table>
<thead>
<tr>
<th>NM station</th>
<th>Station ID</th>
<th>instrument</th>
<th>geog. latit. (deg.)</th>
<th>geog. long. (deg.)</th>
<th>Cut-Off Rigidity (GV)</th>
<th>Altitude (m)</th>
<th>time period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rome</td>
<td>ROME</td>
<td>9-NM-64</td>
<td>41.90</td>
<td>12.52</td>
<td>6.24</td>
<td>60</td>
<td>1975 – 1980</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12-NM-64</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1981 – 1984</td>
</tr>
<tr>
<td></td>
<td></td>
<td>17-NM-64</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1985 – 1992</td>
</tr>
<tr>
<td>Alma Ata B</td>
<td>ALMA B</td>
<td>18-NM-64</td>
<td>43.25</td>
<td>76.92</td>
<td>6.61</td>
<td>3340</td>
<td>1975 - 1992</td>
</tr>
<tr>
<td>Tbilisi</td>
<td>TBILIS</td>
<td>18-NM-64</td>
<td>41.72</td>
<td>44.80</td>
<td>6.66</td>
<td>510</td>
<td>1975 - 1991</td>
</tr>
</tbody>
</table>

As the basic level of original tabular hourly Rome data was changed after 1 Jan 1984, so we have improved them by multiplying the data from the period 1975 – 1984 by the factor 1.9298. Three time series are shown in Fig. 1

![Daily means of Tbilisi, Alma Ata B and Rome registration](image)

**Fig. 1**

In Fig. 1 the 24th point moving means of Ottawa/Penticton radio flux are plotted as measure of the solar activity. Every data of NM stations has been divided by 1000, for the clarity of the Rome records were multiplied by 1.43. For further analysis rather small effects of GLE’s in the NM records (GLEs nr 27 to 53) in above mentioned ranges of rigidities have been excluded and the gaps in data were filled up by means of interpolation.
### III. Methods of data analysis

We apply correlative analysis, wavelet technique and recurrence plots which lets to estimate the coherence between analyzed data. The magnitude-squared coherence between two time series $x(t)$ and $y(t)$ at the frequency $\omega$ is defined as

$$C_{xy}^2(\omega) = \left| P_{xy}(\omega) \right|^2 / P_{xx}(\omega) P_{yy}(\omega)$$  \hspace{1cm} (1)

where $t$ is time, $P_{xx}$ and $P_{yy}$ are power spectral densities of the series, $P_{xy}$ is the cross spectral density between these time series. The wavelet transform describes filtration of an analyzed signal $x(t)$ using real or complex-valued function $\psi(t, a)$, which is defined as

$$W_x(t, a) = (1/a^{0.5}) \int \psi^*(\tau-t)/a) \cdot x(\tau) \, d\tau$$  \hspace{1cm} (2)

where $*$ denotes complex conjugate, variable $a$ and $\tau$ are the dilation (frequency) and translation (position) scaling factors. In the case of one-dimensional original signal, the continuous wavelet transform generates a two-dimensional picture of the wavelet power (scalogram):

$$S_x(t, a) = \left| W_x(t, a) \right|^2$$  \hspace{1cm} (3)

The cross wavelet transform of two time series $x$ and $y$ is defined as

$$S_{xy}(t, a) = W_x(t, a) W_y(t, a)$$  \hspace{1cm} (4)

the wavelet coherence (WTC) of two signals or time series is defined as

$$C_{xy}^2(t,a) = \left| A(a^1 S_{xy}(t,a)) \right|^2 / (A(a^1 \left| S_x(t,a) \right|) \cdot A(a^1 \left| S_y(t,a) \right|)),$$  \hspace{1cm} (5)

where $A$ is a smoothing operator along time axis and scale axis. It is a complex function and is resembled the formula for correlation coefficient, $0 \leq C_{xy}^2(t,a) \leq 1$. When coherence values for two time series are close to one it can be concluded that both data sets are very similar in a wide range of frequencies. In the paper we use the

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packages prepared by Torrence & Compo, Grinsted et al., and Rwave program, which employs the Morlet wavelet in order to evaluate scalograms and WTC of two data sets

\[ \psi(t,a) = \pi^{-1/4} e^{i\omega_0 \eta} e^{-a^2 \eta^2}, \]  

where \( \omega_0 \) is the dimensionless frequency, \( \eta \) is the dimensionless time.

Recurrent analysis was originally proposed by Eckmann et al. as a graphical realization of the basic property of recurrence (repeatability of the trajectory of a dynamical system in the phase space). A recurrence plot (RP) is represented by a two-dimensional square matrix with elements taking the values of zero and unity

\[ R_{ij} = \theta(\varepsilon_i - |x_i - x_j|) \]  

where \( i,j = 1,\ldots,N \) is the number of \( x_i \) states under consideration; \( \varepsilon_i \) is the boundary distance parameter which denotes the density of recurrence points, \( \theta \) is the Heaviside function. If the state of a system at the time \( i \) is recurrent to another state at the time \( j \), the corresponding element \( (i,j) \) of the recurrence matrix is unity, otherwise it is zero. On a recurrence plot, units are imaged by black dots, while zeros are imaged by white dots. Both the ordinate and abscissa axes of the recurrence plot represent the time.

Extension of RP is Cross-Recurrence Plot (CRP). In this case two time series \( (i = 1,\ldots,N_x) \) \( (j = 1,\ldots,N_y) \) are considered in the same phase space. Commonly \( N_x \neq N_y \), then CRP is non-square matrix. Depending on kind of relationships between time series the main diagonal can be broken, curved or even vanished. In special case when \( x = y \), the main diagonal is the line of synchronization (LOS).

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\[ CR_{ij} = \theta (\epsilon_i - |x_i - x_j|) \quad i,j = 1,...,N \quad (8) \]

Construction of the cross recurrence plot implies the analysis of two time series rather than one.

The presence of diagonal structures (in the simplest case, lines) on the recurrence or cross recurrence plot is evidence of a synchronous evolution of the processes under consideration.

Table 3. Pearson correlation coefficients */

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Daily means</td>
<td>no</td>
<td>Bartels rotation means</td>
<td>n</td>
</tr>
<tr>
<td>Ottawa - Rome</td>
<td>-0.67</td>
<td>6520</td>
<td>-0.73</td>
<td>-0.79</td>
</tr>
<tr>
<td>Ottawa - Tbilisi</td>
<td>-0.67</td>
<td>6316</td>
<td>-0.73</td>
<td>-0.34</td>
</tr>
<tr>
<td>Ottawa – Alma Ata</td>
<td>-0.51</td>
<td>6520</td>
<td>-0.63</td>
<td>-0.16</td>
</tr>
<tr>
<td>Rome - Tbilisi</td>
<td>0.80</td>
<td>6312</td>
<td>0.76</td>
<td>0.30</td>
</tr>
<tr>
<td>Rome – Alma Ata</td>
<td>0.70</td>
<td>6540</td>
<td>0.83</td>
<td>0.15</td>
</tr>
<tr>
<td>Alma Ata - Tbilisi</td>
<td>0.80</td>
<td>0.85</td>
<td>6312</td>
<td>0.58</td>
</tr>
</tbody>
</table>

*/ All coefficients have a significance level p < 0.05, n – number of daily values used

For the first selected period: 1975-1980 the correlation between NM stations is rather weak, also it is seen from Fig. 1. It is seen that the station Rome well correlated with solar activity. The longer and short term variability of the neutron monitor intensities during the period studied are seen from the scalograms of the 12th hour means of data presented in Fig 2 and 3.5 where the variability of them is much better illustrated than through the simple correlation.
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Fig. 3. Wavelet coherence between hour NM data during part of the mentioned period. Thick contours separate statistically significant regions (red, dark red) on the level of 95% against red noise and designate cones of influence of edge. Horizontal axis: 12 hour means, vertical axis: period of signal.

Dark red and red colors show the periods of good consistency between Alma Ata and Tbilisi data.

Fig. 4. Recurrence plots of NM data for: a) Rome, b) Alma Ata; c) cross recurrence plot of two time series for these observatories. Vertical and horizontal axes: time in 12 hour means.

Conclusions

Wavelet technique and recurrence plots can be useful for comparisons showed above. From presented 3 stations Rome and Alma Ata are recommended for further work.

References