Analysis of cycle-to-cycle pressure oscillations in a diesel engine

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Abstract

In this paper we study the fluctuations of mean indicated pressure (MIP) in a diesel engine. Using recurrence plots, recurrence quantification analysis and continuous wavelet transform, we investigate the cycle-to-cycle pressure variations for six rotational speeds of the crankshaft. We find that depending on the speed of rotation, the pressure variations may have a strongly periodic component and/or be intermittent. The strong periodicities appear in low-frequency bands and may persist over many cycles, whereas the intermittency is present at higher frequencies. The results may be useful to develop effective control strategies for efficient engine performance.

Keywords: Diesel engine; Pressure fluctuations; Recurrence plots; Recurrence quantification analysis; Wavelet analysis

1. Introduction

The physicochemical processes in an internal combustion engine are influenced by many factors leading to complex dynamics. These factors include composition of the fuel–air mixture, amount of recycled gases supplied to the cylinder, and engine aerodynamics. They affect the process variables such as in-cylinder pressure, temperature and heat release, which in turn, result in changes in output power. Understanding the cycle-to-cycle fluctuations in the process variables has been an active topic of research for many years. Earlier work focused mainly on spark ignition (SI) engines \[6–9,11,14,16–18,22–25,35,39,44,45\], but more recent studies have directed their attention also to diesel engines \[2,4,13,19,26,32,36,39\]. In order to develop effective control strategies for efficient engine performance, it is important to gain a good understanding of the complex dynamics of the process variables. There have been many attempts to analyze the cycle-to-cycle variations by applying deterministic methods from nonlinear dynamical systems and chaos theory \[6–9,11,25,44,45\]. Other efforts have included the effect of noise and estimated the noise level contributing to stochastic variations \[18,22–24,35\].
We have performed an experimental study on a diesel engine and measured the internal pressure in the cylinder over many cycles. The purpose of this paper is to present these experimental results and analyze the cycle-to-cycle variations of the mean indicated pressure (MIP) using recurrence plots (RPs), recurrence quantification analysis (RQA) and continuous wavelet transform (CWT). In particular, we investigate the cyclic variations in MIP for six different rotational speeds of the crankshaft of the diesel engine. We find that depending on the magnitude of the speed of rotation, the pressure variations may have a strongly periodic component and/or be intermittent. The strong periodicities appear in low-frequency bands and may persist over many cycles, whereas the intermittency is present at higher frequencies.

Our presentation is organized as follows. In Section 2 we describe the experimental facility for pressure measurement in a diesel engine. This is followed by an analysis of the MIP time series using autocorrelation in Section 3, using RP and RQA in Section 4, and with CWT in Section 5. In the final section of the paper we summarize the results with a few concluding remarks.

2. Experimental facility

Fig. 1 is a schematic diagram of the experimental setup used in our study. The diesel engine was fuelled by standard diesel fuel and the in-cylinder pressure was measured in one of the three cylinders under steady-state conditions, using a piezoelectric pressure sensor. Pressure data were collected over 978 cycles for six different rotational speeds of the crankshaft: \( n = 1000, 1200, 1400, 1600, 1800 \text{ and } 2000 \text{ rpm} \) under full loading. The specific technical details of the engine are provided in Table 1.

A measurement path starts inside the combustion chamber where a piezoelectric crystal sensor of the type 8Qp500c (manufactured by the AVL Company) is mounted (No. 8 in Fig. 1). From the sensor the signal is transferred through connecting wires to a charge amplifier (No. 7 in Fig. 1), and then decoded by a computer with a card made by Keithley Instruments (No. 5 in Fig. 1). Consecutive measurements with a sampling frequency of 1024 times per crankshaft revolution are realized automatically and controlled by the crankshaft position sensor (No. 9 in Fig. 1). The loading of the engine was controlled by an eddy-current brake coupled to the crankshaft. The MIP is defined as a constant alternative pressure which acting on the engine piston during the whole expansion stroke performs the same amount of work as the real variable pressure in the cylinder [16]. The MIP can be expressed as \( MIP = \frac{Li}{Vs} \), where \( Li \) is the amount work indicated in the cylinder, and
Vs is the piston displacement volume of the cylinder. The work Li is estimated numerically by integration of the measured pressure [16].

3. Time series analysis: autocorrelation

The time series of MIP for six rotational speeds of the crankshaft (n = 1000, 1200, 1400, 1600, 1800 and 2000 rpm) are depicted in Fig. 2. As a first step to understand the dynamics of the MIP fluctuations, we calculate the autocorrelation of their time series. We denote the MIP time series as pci(i), i = 1, 2, 3, ..., N. The discrete autocorrelation (AC) of this time series at lag j is given by

\[ AC(j) = \sum_{i} pci(i)pci(i-j), \]  

(1)

with suitable normalization to 1 (for j = 0).

The AC describes the degree of correlation between the points in the time series. Fig. 3 depicts the AC plotted against time delay, for three crankshaft speeds of 1200, 1600 and 2000 rpm. It is apparent from this figure that depending on the speed, the AC decays at a different rate. In particular, we find that at speeds of 1200 and 2000 rpm, respectively, the MIP time series has a rather low and high degree of autocorrelation between adjacent and near-adjacent observations; at the intermediate speed of 1600 rpm, on the other hand, the AC undergoes fast modulation. The slow decay of the autocorrelation plots indicates presence of nonstationarity in the MIP time series. Among the various methods available for the analysis of nonstationary signals, RP, RQA and CWT have been used extensively by researchers. We will use these techniques below to reveal important features of the MIP data.

In order to construct a RP or perform a RQA for the MIP time series, it is customary to embed each time series on to a high dimensional phase space of dimension m using the time-delayed vectors [40]:

\[ pci(i) = [pci(i), pci(i-\delta), pci(i-2\delta), pci(i-3\delta), \ldots, pci(i-(m-i)\delta)]. \]  

(2)

Here \( \delta \) denotes the time delay. The optimum value of \( \delta \) may be estimated by calculating the average mutual information (AMI). For an MIP time series, the AMI is given by the expression [1,31]:

\[ AMI(\delta) = \sum_{k,l} P_{k,l} \ln \frac{P_{k,l}}{P_k P_l} , \]  

(3)

where for some partition of the pressure: \( pci(\delta) \in [pcimin, pcimax] \), \( P_k \) and \( P_l \) are the probabilities of finding the pressure, respectively, in the kth and lth intervals separated by the time delay \( \delta \), and \( P_{k,l} \) is the joint probability. The optimum value of \( \delta \) corresponds to the first minimum of AMI.

The minimal sufficient embedding dimension m can be calculated from an analysis of the false nearest neighbors (FNNs). The concept of FNNs was introduced by Kennel et al. [20], with improvements made in [15,34] to account for noise effects. The FNN analysis may be briefly described as follows. For each point i in
the time series, look for its nearest neighbor \( j \) in the \( m \)-dimensional space and compute the ratio of the distances between these two points in both \( m \) and \( m+1 \) dimensions:

\[
\rho_{i,m} = \frac{d_{\text{euclid}}(\mathbf{p}_c(i), \mathbf{p}_c(j))_{m+1}}{d_{\text{euclid}}(\mathbf{p}_c(i), \mathbf{p}_c(j))_{m}}.
\] (4)

Note that \( p_c(i) \) and \( p_c(j) \) denote the state vectors at points \( i \) and \( j \), respectively, and the Euclidean distance is used. If the ratio \( \rho_{i,m} \) is larger than a chosen threshold value \( \rho_0 \), then the neighbor is false. The value of \( m \) for which the number of FNNs is close to zero is the proper embedding dimension. For each of the six rotational speeds of the crankshaft, we have estimated the time delay (\( \delta \)) and embedding dimension (\( m \)), using AMI and FNN, respectively. These values are listed in Table 2.
4. Recurrence plots and recurrence quantification analysis

The RP was introduced by Eckmann et al. [10] as a graphical device to describe the dynamical properties of a time series in a qualitative manner (see also [5]). For an MIP time series, a RP may be constructed on the basis of a so-called recurrence matrix $R$ whose $ij$th element $R_{ij}$ is given by

$$R_{ij} = \Theta[\varepsilon - ||p_i(i) - p_i(j)||], \quad i, j = 1, 2, 3, \ldots, N. \quad (5)$$

Here the second term inside the brackets denotes the distance between two state vectors in the $m$-dimensional phase space (see above), $\varepsilon$ is a chosen (small) threshold distance, and $\Theta$ represents a Heaviside function. Depending on the value of the element $R_{ij}$ being 0 or 1, respectively, a blank space is left or a black dot is drawn in an RP. Information about the dynamics of a time series can be obtained from the line structure and point density in a RP. For example, a homogeneous RP with no structure is typical of a stationary or autonomous process such as white noise. RPs of oscillating systems, on the other hand, have diagonally oriented or periodic recurrent structures (i.e., diagonal lines or checkerboard patterns). Vertical or horizontal lines in a RP signify the presence of laminarity or intermittency in the time series, whereas abrupt changes in dynamics as well as extreme events are characterized by white areas or bands.

Figs. 4a–f depict the RPs of the MIP time series shown in Figs. 2a–f, for the crankshaft speeds of 1000, 1200, 1400, 1600, 1800 and 2000 rpm, respectively. These RPs as well as the RQA that follows are carried out using the CRPTOOL software developed by Marwan [30]. Notice that in Figs. 4a, c, d and e, there are several vertical lines identifying the presence of intermittency in the pressure time series. On the other hand, in Fig. 4b which applies for the speed of 1200 rpm, we observe a series of lines with unit slope parallel to the main diagonal. These structures indicate a more regular oscillatory behavior. Interestingly, at the maximum speed of 2000 rpm, the RP has a checkerboard structure suggesting a regular oscillatory behavior also, similar to Fig. 4b. These results will be corroborated by wavelet analysis in the following section.

Table 2

<table>
<thead>
<tr>
<th>$n$ (rpm)</th>
<th>$\delta$</th>
<th>$m$</th>
<th>DET</th>
<th>LAM</th>
<th>$L_{\text{ENTR}}$</th>
<th>$V_{\text{ENTR}}$</th>
<th>TT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>1</td>
<td>3</td>
<td>0.805089</td>
<td>0.589715</td>
<td>1.60121</td>
<td>1.45827</td>
<td>3.13967</td>
</tr>
<tr>
<td>1200</td>
<td>5</td>
<td>4</td>
<td>0.436374</td>
<td>0.577993</td>
<td>0.86625</td>
<td>1.27346</td>
<td>2.86414</td>
</tr>
<tr>
<td>1400</td>
<td>1</td>
<td>3</td>
<td>0.796848</td>
<td>0.509972</td>
<td>1.59503</td>
<td>1.26297</td>
<td>2.86798</td>
</tr>
<tr>
<td>1600</td>
<td>1</td>
<td>3</td>
<td>0.813068</td>
<td>0.565695</td>
<td>1.65988</td>
<td>1.39722</td>
<td>3.07883</td>
</tr>
<tr>
<td>1800</td>
<td>2</td>
<td>4</td>
<td>0.422264</td>
<td>0.231198</td>
<td>2.17219</td>
<td>0.76089</td>
<td>2.39303</td>
</tr>
<tr>
<td>2000</td>
<td>2</td>
<td>3</td>
<td>0.358780</td>
<td>0.532995</td>
<td>1.74248</td>
<td>1.45962</td>
<td>3.18304</td>
</tr>
</tbody>
</table>

Fig. 3. Autocorrelations of the MIP time series for speeds ($n = 1200, 1600$ and 2000 rpm) of the crankshaft.
It should be emphasized that a RP provides only a qualitative description of the dynamics of a time series. As a means of quantifying the various features of a RP, Webber and Zbilut [43] formulated a procedure called RQA, by introducing several parameters based on the line structure and point density in a RP. The RQA approach was further extended by Marwan et al. [27–29], with introduction of additional parameters. For the analysis of the MIP time series, we will use the following RQA parameters: recurrence rate ($RR$), determinism ($DET$), laminarity ($LAM$), trapping time ($TT$) and entropies ($L_{ENTR}$ and $V_{ENTR}$). The $RR$ is defined as

$$RR = \frac{1}{N^2} \sum_{i,j=1}^{N} R_{ij}^{m}.$$

(6)
Using the distribution of the lengths of diagonal lines $P(l)$ or vertical lines $P(v)$, the other RQA parameters can be defined as follows (see [28] for details):

$$DET = \frac{\sum_{l=l_{\text{min}}}^{N} l P(l)}{\sum_{l=1}^{N} R_{l}^{n,e}},$$

$$LAM = \frac{\sum_{v=v_{\text{min}}}^{N} v P(v)}{\sum_{v=1}^{N} v P(v)},$$

$$TT = \frac{\sum_{v=v_{\text{min}}}^{N} v P(v)}{\sum_{v=1}^{N} P(v)},$$

$$L_{\text{ENTR}} = -\sum_{l=l_{\text{min}}}^{N} \text{prob}(l) \ln \text{prob}(l),$$

$$V_{\text{ENTR}} = -\sum_{v=v_{\text{min}}}^{N} \text{prob}(v) \ln \text{prob}(v),$$

where $\text{prob}(l)$ and $\text{prob}(v)$ represent probabilities defined by

$$\text{prob}(x) = \frac{P(x)}{\sum_{x=x_{\text{min}}}^{N} P(x)}.$$

Here $x$ stands for $l$ or $v$, and $l_{\text{min}}$ and $v_{\text{min}}$ denote the minimum values of the diagonal and vertical line lengths, respectively, which should be chosen for a specific dynamical system. In our calculations we have taken $l_{\text{min}} = v_{\text{min}} = 2$.

The $RR$ is a measure of the density of recurrence points in an RP; it is simply a total count of the black dots in the RP and corresponds to the probability that a specific state will occur. $DET$ is given by the ratio of recurrence points that form diagonal structures to all recurrence points, and is a measure of predictability of the time series. The amount of recurrence points which form vertical lines in an RP is estimated by the $LAM$ parameter. This parameter indicates the extent of laminar phases or intermittency in the time series. The $TT$ quantifies the average length of the vertical lines and is thus related to $LAM$; in other words, $TT$ describes how long the system remains in a specific laminar phase. The parameter $L_{\text{ENTR}}$ is the Shannon entropy based on the frequency distribution of the diagonal line lengths, and may be considered as a measure of complexity of a deterministic structure in a dynamical system. The larger the value of $L_{\text{ENTR}}$, the more complex is the deterministic structure. An analogous measure based on the frequency distribution of the vertical line lengths in an RP is given by the Shannon entropy $V_{\text{ENTR}}$. We have calculated these RQA parameters for each of the six MIP time series shown in Fig. 2. Their values are listed in Table 2. The results of the RPs analysis can be compared to each other as we have used the same $RR$ parameter ($RR = 0.2$).

In Fig. 5, we have displayed the results presented in Table 2 by plotting their variations with crankshaft speed. Among other things, we observe that $L_{\text{ENTR}}$ is largest at the speed $n = 1800$, revealing higher structural complexity. Note also that at this speed ($n = 1800$ rpm), the parameters $LAM$ and $TT$ have their minimum values while $DET$ reaches its minimum value at $n = 2000$ rpm. The small value of $DET$ means low predictability, whereas the small values of $LAM$ and $TT$, respectively, indicate a dominance of large fluctuations, and a short duration of time spent in a laminar phase in the intermittent dynamics.

5. Wavelet analysis

In this section we perform a wavelet analysis of the MIP time series shown in Fig. 2, using a CWT. Wavelets have been used for signal analysis in a wide variety of applications [3,12,21,33]. They are particularly useful for the analysis of signals with time-varying spectra and intermittent signals. Using a variable-size window in a time–frequency plane, wavelet analysis provides an efficient approach by which both time and frequency
resolutions can be adjusted in an adaptive fashion. A wavelet transform uses a window that narrows when focusing on small-scale or high-frequency features of the signal and widens on large-scale or low-frequency features, analogous to a zoom lens \cite{21}. In an earlier paper, we used a CWT to study cycle-to-cycle pressure variations in a SI engine \cite{39}.

A wavelet is a small wave with a compact support. In order to be classified as a wavelet, a function $\psi(t)$ should have zero mean and finite energy:

$$\int_{-\infty}^{\infty} \psi(t) \, dt = 0, \quad \int_{-\infty}^{\infty} |\psi(t)|^2 \, dt < \infty.$$  \hspace{1cm} (13)

The CWT of a function $x(t)$ with respect to a wavelet $\psi(t)$ is defined as a convolution of the function with a scaled and translated version of $\psi(t)$. The wavelet $\psi(t)$ is referred to as an analyzing wavelet or a mother wavelet. The convolution is expressed by the integral \cite{21}:

$$W(s, \tau) = \int_{-\infty}^{\infty} x(t) \psi^*_s(\frac{t - \tau}{s}) \, dt,$$  \hspace{1cm} (14)

where

$$\psi_{s,\tau}(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t - \tau}{s}\right)$$  \hspace{1cm} (15)

is a scaled and translated version of the mother wavelet $\psi(t)$, and an asterisk on $\psi$ denotes its complex conjugate. The symbols $s$ and $\tau$ are called a scale parameter and a translation parameter, respectively. The scale parameter controls the dilation ($s > 1$) and contraction ($s < 1$) of the mother wavelet. The factor $1/\sqrt{s}$ is introduced in Eq. (15) so that the function $\psi_{s,\tau}(t)$ has unit energy at every scale. The translation parameter $\tau$.

Fig. 5. Variations of (a) $DET$ and $LAM$, (b) $L_{\text{ENTR}}$ and $V_{\text{ENTR}}$, and (c) $TT$ with crankshaft speed, for the same value of $RR = 0.2$. 


369
indicates the location of the wavelet in time; in other words, as \( \tau \) varies, the signal is analyzed in the vicinity of this point. The amount of signal energy contained at a specific scale \( s \) and location \( \tau \) is given by the squared modulus of the CWT:

\[
P(s, \tau) = |W(s, \tau)|^2.
\]

This energy density function is called the wavelet power spectrum (WPS) of the signal. It is also referred to as a scalogram, analogous to the spectrogram of a short-time Fourier transform. The WPS which depends on both scale and time is represented by a surface. By taking contours of this surface and plotting them on a plane, a time-scale representation of the wavelet power spectrum may be derived. From a time-scale representation, the various periodicities and intermittency can be identified by visual inspection.

The integral formulation given in Eq. (14) applies to the CWT of a signal \( x(t) \) that is a continuous function of \( t \). In order to use it for a discrete signal such as the MIP signal, this integral representation must be discretized in an appropriate fashion. Consider a discrete signal given by a time series \( \{x_n\} \) with \( n = 1, 2, 3, \ldots, N \). For such a time series, Eq. (14) may be discretized as [42]:

\[
W_n(s) = \sum_{n'=1}^{N} \left( \frac{\delta \tau}{s} \right)^{1/2} x_{n'} \psi^* \left( \frac{(n' - n) \delta \tau}{s} \right).
\]

Here \( n \) is the time index, and \( \delta \tau \) is the sampling interval. The factor \( (\delta \tau/s)^{1/2} \) preserves the unit energy property referred to earlier. In order to calculate the CWT using Eq. (17), the convolution procedure given by this equation should be performed \( N \) times for each scale. However, as shown in [42], it is possible to carry out all \( N \) convolutions simultaneously in Fourier space by means of a discrete Fourier transform (DFT).

For a time series of finite length, computation of CWT using DFT requires that the time series is cyclic. To satisfy this requirement, the time series is often padded with zeros. Zero padding leads to errors at the ends of the wavelet power spectrum. The region in which these edge effects become significant is described by the cone of influence (COI). The results outside the COI may be unreliable and should be interpreted with caution. In the figures presented here, the COI is indicated by a thin solid curve.

Using Eq. (17), the WPS for each scale can be evaluated as \( |W_n(s)|^2 \). In our analysis we used a complex Morlet wavelet as the mother wavelet. A Morlet wavelet consists of a plane wave modulated by a Gaussian function and is described by

\[
\psi(\eta) = \pi^{-1/4} e^{i o_0 \eta} e^{-\eta^2/2},
\]

where \( o_0 \) is the center frequency, also referred to as the center of the wavelet. Strictly speaking, the Morlet wavelet as defined above does not satisfy the zero-mean condition given in Eq. (1), and a correction factor is needed to fulfill the zero-mean condition. However, this correction factor is negligible if a value of \( o_0 > 5 \) is chosen. Morlet wavelets have been used successfully in a wide variety of applications for feature extraction in time series data. In our analysis we have used a Morlet wavelet with \( o_0 = 6 \). This value of \( o_0 \) provides a good balance between time and frequency localizations. For \( o_0 = 6 \), the Fourier period is approximately equal to the scale \( s \). In other words, the terms scale and period may be used interchangeably for interpretation of the results. Note that in Eq. (18), the coefficient \( \pi^{-1/4} \) is used as a normalization factor to ensure that the Morlet wavelet has unit energy.

With the aid of Eqs. (17) and (18), we have calculated the WPS for the time series of the MIP. Figs. 6a–f illustrate the time-period plots of the WPS for crankshaft speeds: \( n = 1000, 1200, 1400, 1600, 1800 \) and \( 2000 \) rpm, respectively. From these figures, the following observations can be readily made.

Fig. 6a applies for the smallest crankshaft speed examined: \( n = 1000 \) rpm, and indicates a sporadic variability between the 16-cycle period and 32-cycle period. Among these periods, the power in the wavelet spectrum is distinctly higher around the 32-cycle period that persists over a brief interval between 150 and 275 cycles and also between 800 and 900 cycles. As the crankshaft speed is increased to \( n = 1200 \) rpm, we observe a strong 12–32-cycle periodic band extending continuously between 70 and 600 cycles and then over the remaining cycles. These results are consistent with those found in the RP of Fig. 4b. As the crankshaft speed is further increased (\( n = 1400, 1600 \) and 1800 rpm—Figs. 6c, d and e), high-frequency bands are seen to appear in an intermittent fashion. In Figs. 6c and d, they appear around the 4-cycle period, whereas in Fig. 6e, the
intermittent band is around the 2-cycle period. Finally, when the speed is 2000 rpm, we see from Fig. 6f that there is a strong periodic band around the 90-cycle period, which ranges continuously from approximately 100 cycles to 825 cycles.

6. Concluding remarks

Using RPs, RQA and CWT, we have analyzed the cycle-to-cycle variations of the MIP in a diesel engine for six rotational speeds of the crankshaft. Our results indicate that depending on engine speed, the pressure variations in the cylinder exhibit different types of behavior ranging from strongly periodic oscillations (for 1200 and 2000 rpm) to intermittent fluctuations (at other speeds). The various periodicities and the number of cycles over which they persist are determined from a time-period representation of the wavelet power spectrum. The intermittency patterns are also readily discerned from the wavelet spectra. The results obtained by wavelet analysis are found to be consistent with those derived from RPs and RQA. For example, the periodic behavior observed in Figs. 6b and f reflects the strong periodicities seen in the RPs of Figs. 4b and f, respectively. In addition, the intermittent patterns seen in Figs. 6a, c, d are consistent with the intermittency revealed respectively by the vertical line structures in the RPs shown in Figs. 4a, c, d. This type of intermittency is referred to in the literature as on-off intermittency and has been observed in other applications [37,38,41].

The combustion process in a diesel engine is influenced by the rate of fuel injection and fuel–air mixture composition. At higher crankshaft speeds, the time interval between fuel injection and self-ignition is smaller. In this case, the combustion process becomes more ordered as the ignition time is determined primarily by the geometrical position of the piston. Simultaneously the process of fuel–air mixing is enhanced because higher
piston velocities make the air swirling more efficient. As a consequence, the combustion process in a specific engine cycle is related to that in the preceding cycle. This mixing phenomenon is hardly present at low rotational speeds of the crankshaft. Furthermore, in this regime random variations in the amount of fuel fed to the engine may occur.

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References


