Numerical investigation and dynamical analysis of mixed convection in a vented cavity with pulsating flow

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A B S T R A C T

In the present study, numerical investigation of pulsating mixed convection in a multiple vented cavity is carried out for the range of parameters; Reynolds number (500 ≤ Re ≤ 2000), Grashof number (106 ≤ Gr ≤ 108), Strouhal number (0 ≤ St ≤ 2). The governing equations are solved with a general purpose finite volume based solver. The effects of various parameters on the fluid flow and heat transfer characteristics are numerically studied. It is observed that the flow field and heat transfer rate are influenced by the variations of Reynolds, Grashof and Strouhal numbers. Furthermore, recurrence plot analysis is applied for the analysis of the time series (spatial averaged Nusselt number along the vertical wall of the cavity) and for a combination of different parameters, the systems are identified using recurrence quantification analysis parameters including recurrence rate, laminarity, determinism, trapping time and entropy.

1. Introduction

Mixed convection has various engineering application areas such as design of heat exchangers, nuclear reactors, solar collectors, cooling of electronic equipments and modern buildings. A vast amount of literature is dedicated to the mixed convection in cavities [1–7]. The unsteadiness of flow and instability mechanism inside vented cavities have attracted much attention due to the need for efficient design of these systems with maximum heat transfer and minimum power requirements. Laminar transient mixed convection in a vertical cylindrical cavity is studied numerically by [1]. They studied the dynamic field for different Richardson numbers and different geometrical parameters such as the inlet and outlet positions of the fluid. They showed that the most efficient configuration with respect to thermal storage efficiency is obtained for the cylindrical cavity. [2] have numerically studied the periodic laminar flow and heat transfer in a lid-driven square cavity due to an oscillating thin fin for Reynolds number of 100 and 1000. They examined the dynamical system for periodic flow and thermal fields for a range of Strouhal numbers between 0.005 and 5. Unsteady flow and thermal field around a thin fin on a sidewall of a differentially heated cavity is studied experimentally by [8]. They have observed the transition to the quasi-steady state, separation and oscillations of the thermal flow above the fin and these oscillations trigger instability of the downstream thermal boundary layer flow which enhances the convection. [9] have investigated the pulsating flow of 2D laminar flow in a heated rectangle for Reynolds number of 100 and Strouhal numbers between 0 and 0.4. They showed that the prescribed pulsation enhances heat transfer in the cavity due to the periodic change in the recirculation flow pattern generated by the pulsation. [10] have numerically studied the laminar pulsating flow in a backward facing step with an upper wall mounted adiabatic thin fin. They have investigated the effects of Reynolds number, pulsating amplitude and frequency on the fluid flow and heat transfer. They have reported that compared to steady flow with no-fin case, adding a fin is not advantageous for heat transfer enhancement in pulsating flow. [3] have numerically studied the fluid mixing in cavity with time periodic lid velocity using finite element method. They showed that for the best mixing an optimum frequency exists and oscillation amplitude and the geometric aspect ratio have also influence on the mixing. [6] have studied the flow and heat transfer in a cavity with double sided oscillating lids. They studied the effects of oscillating frequency of the lid motion and Reynolds number. They observed that the oscillating frequency change the flow pattern at very low Reynolds number significantly. [4] have studied the unsteady laminar flow with an square enclosure with two ventilation ports. They showed that for Strouhal number of 0.1, the mean Nusselt numbers on the four walls exhibit large amplitudes of oscillation, but at Strouhal number of 10, the amplitudes of oscillation on various walls are generally degraded. They also showed that, heat transfer enhancement is observed for the range of considered


Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>Gr</td>
<td>Grashof number, $\frac{g \beta H^3}{V^2}$</td>
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<tr>
<td>$H$</td>
<td>cavity length</td>
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<tr>
<td>$h$</td>
<td>local heat transfer coefficient</td>
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<tr>
<td>$k$</td>
<td>thermal conductivity</td>
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<tr>
<td>$n$</td>
<td>unit normal vector</td>
</tr>
<tr>
<td>Nu</td>
<td>local Nusselt number, $hH/k$</td>
</tr>
<tr>
<td>$p$</td>
<td>pressure</td>
</tr>
<tr>
<td>Pr</td>
<td>Prandtl number, $\frac{\mu H}{k}$</td>
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<tr>
<td>Re</td>
<td>Reynolds number, $\frac{u_0 H}{v}$</td>
</tr>
<tr>
<td>Ri</td>
<td>Richardson number, $\frac{Gr}{Re}$</td>
</tr>
<tr>
<td>St</td>
<td>Strouhal number, $\frac{f H}{u_0}$</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature</td>
</tr>
<tr>
<td>$u$, $v$</td>
<td>x-y velocity components</td>
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<tr>
<td>$x$, $y$</td>
<td>Cartesian coordinates</td>
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Greek symbols

<table>
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<tr>
<td>$\alpha$</td>
<td>thermal diffusivity</td>
</tr>
<tr>
<td>$\beta$</td>
<td>thermal expansion coefficient</td>
</tr>
<tr>
<td>$\phi$</td>
<td>non-dimensional temperature, $\frac{T - T_1}{T_1 - T_c}$</td>
</tr>
<tr>
<td>$\nu$</td>
<td>kinematic viscosity</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density of the fluid</td>
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<tr>
<td>$\tau$</td>
<td>non-dimensional time, $\frac{u_0 t}{H}$</td>
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Subscripts

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<thead>
<tr>
<th>Symbol</th>
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<tr>
<td>$c$</td>
<td>cold</td>
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<tr>
<td>$h$</td>
<td>hot</td>
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<td>$m$</td>
<td>mean</td>
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Strouhal numbers, [11] have investigated the flow field and heat transfer characteristics in a square cavity with two ventilation ports for the mixed convection case. At the inlet port, pulsating velocities for a range of Reynolds numbers and Richardson numbers are imposed. They reported that optimum Strouhal number is between 0.5 and 1 for the best thermal performance and minimum pressure drop. [12] have studied the characteristic of mixed convection in a multiple ventilated cavity and its transition from laminar to chaotic state for the Reynolds numbers between 1000 and 2500. They have observed that as Ri increases the solution may exhibit a change from steady-state to periodic oscillation, and then to non-periodic oscillatory state. They used non-linear time series analysis tools to compute the correlation dimension, Kolmogorov entropy and Lyapunov exponents in order to detect chaos.

In this article, we have studied the pulsating flow in a multiple vented cavity for a range of Reynolds, Grashof and Strouhal numbers. The effects of these parameters on the flow field and heat transfer are numerically investigated. Furthermore, time series data obtained from spatial averaged Nusselt number along the walls of the cavity is analyzed using recurrence plots. Recurrence quantification analysis parameters including recurrence rate, laminarity, determinism, trapping time and entropy are also provided to quantify the non-linear time series of Nusselt numbers for different combinations of Reynolds, Grashof and Strouhal numbers.

2. Numerical simulation

A schematic diagram of the physical problem considered in this study is shown in Fig.1. A square cavity (height H) with multiple ventilation ports is considered. At the inlet ports which are located at left and right vertical walls of the cavity, uniform velocity with a sinusoidal time dependent part $(u = u_0(1 + 0.75 \sin(2\pi ft)))$ and uniform temperature $(T_c)$ are imposed. The widths of each inlet and outlet ports are set to 0.1H. The vertical walls are kept at constant temperature $T_1$ while the top and bottom walls are assumed to be adiabatic. Working fluid is air with a Prandtl number of Pr = 0.71. It is assumed that thermo-physical properties of the fluid is temperature independent. The flow is assumed to be two dimensional, Newtonian, incompressible and in laminar flow regime. By using the dimensionless parameters,

$$\left( \frac{U}{V}, \frac{X}{H} \right) = \left( \frac{u}{u_0}, \frac{x}{H} \right), \quad \left( \frac{P}{\rho u_0^2}, \frac{P}{\rho u_0^2} \right) = \theta = \frac{T - T_c}{T_1 - T_c}, \quad (1)$$

for a two dimensional, incompressible, laminar and unsteady case, the continuity, momentum and energy equations can be expressed in the non-dimensional form as in the following:

$$\frac{\partial U}{\partial \tau} + \frac{\partial V}{\partial X} = 0, \quad (2)$$

$$\frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = - \frac{\partial P}{\partial X} + \frac{1}{Re} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right), \quad (3)$$

$$\frac{\partial V}{\partial \tau} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = - \frac{\partial P}{\partial Y} + \frac{1}{Re} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \frac{Gr}{Re^2} \Theta, \quad (4)$$

$$\frac{\partial \Theta}{\partial \tau} + U \frac{\partial \Theta}{\partial X} + V \frac{\partial \Theta}{\partial Y} = \frac{1}{RePr} \left( \frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2} \right), \quad (5)$$

where the relevant physical non-dimensional numbers are Reynolds number (Re), Grashof number (Gr) and Strouhal number (St) are defined as

$$Re = \frac{u_0 H}{V}, \quad Gr = \frac{g \beta (T_1 - T_c) H^3}{V^2}, \quad St = \frac{f H}{u_0} \quad (6)$$

The boundary conditions for the considered problem in non-dimensional form can be expressed as:

- At the inlet ports, velocity is unidirectional sinusoidal, temperature and velocity are uniform, $(U = 1 + A \sin(2\pi St), V = 0, \theta = 0)$
- At the bottom wall, downstream of the step, temperature is constant, $(\theta = 1)$
- At the exit ports, gradients of all variables in the x-direction are set to zero, $(\frac{\partial u}{\partial X} = 0, \frac{\partial v}{\partial X} = 0, \frac{\partial \Theta}{\partial X} = 0)$
- On the top and bottom walls, adiabatic wall with no-slip boundary conditions are assumed, $(U = 0, V = 0, \frac{\partial \Theta}{\partial X} = 0)$
- On the left and right vertical walls, constant temperature with no-slip boundary conditions are assumed, $(U = 0, V = 0, \theta = 1)$

Local Nusselt number is defined as
where \( h_x \) represent the local heat transfer coefficient and \( k \) denote the thermal conductivity of air. Spatial averaged Nusselt number is obtained after integrating the local Nusselt number along the left and right vertical walls of the cavity as

\[
\text{Nu}_m = \frac{1}{L} \int_0^L \text{Nu}_x \, dx.
\]

Time and spatial averaged Nusselt number is obtained after integrating spatial averaged Nusselt number for one period of the oscillation \( \tau \) as

\[
\text{Nu}_m = \frac{1}{\tau} \int_0^\tau \text{Nu}_x \, dt.
\]

Eqs. (2)–(5) along with the boundary and initial conditions are solved with Fluent (a general purpose finite volume solver [13]). The convective terms in the momentum and energy equations are solved using QUICK scheme and SIMPLE algorithm is used for velocity–pressure coupling. The system of algebraic equations are solved with Gauss–Siedel point by point iterative method and algebraic multi-grid method. The convergence criteria for continuity, momentum and energy equations are set to \( 10^{-5}, 10^{-6} \) and \( 10^{-7} \), respectively. The unstructured body-adapted mesh of appropriate size consists of only triangular elements. Mesh independence of the solutions has been confirmed it is seen that the solution for grid size with 43,934 triangular elements is close to the solution of 32,234 triangular elements. The code is validated against the result of [14]. The comparison is made for the case at \( Re = 500 \) (based on the width of the inlet port) and \( Ri = 0 \). Fig. 2 shows the local Nusselt number distributions along the walls of the cavity for this flow condition computed with the FLUENT code (current settings) and computed in [14]. To present the local Nusselt number on the three walls of the cavity (left, bottom and right), S coordinate system was utilized [14]. The comparison results show good overall agreement.

3. Non-linear time series analysis and recurrence plots

Non-linear time series analysis can be used to study the complex non-linear dynamical systems. The time sequence of the spatial averaged Nusselt number \((u_0, \ldots, u_n)\) is obtained from the numerical simulation. According to [15], the reconstructed attractor of the original system is given by

\[
U(i) = (u_i, u_{i-\tau}, \ldots, u_{i-(m-1)\tau}),
\]

with \( \tau \) and \( m \) representing the embedding delay and embedding dimension, respectively. The attractor constructed using the above equation will have the same mathematical features of the original system, such as dimension, Lyapunov exponents, etc. The delayed variable \( u_{i\tau} \) carries information about the influences of all other variables during time \( \tau \). One can introduce the third \( u_{i\tau+2} \) and \( m \)th \( u_{i\tau+(m-1)\tau} \) variable and obtain the whole \( m \)-dimensional phase space where the variables incorporate all the influences of the original system provided that \( m \) is large enough.

To get an estimate for delay term \( \tau \) autocorrelation function or average mutual information function can be used. The latter takes into account the non-linear correlations as well. In the mutual information function method \( \tau \) is the first minimum of the function [16],

\[
l(\tau) = -\sum_{k=1}^N \sum_{i=1}^N P_k(\tau) \ln \frac{P_k(\tau)}{P_k^2},
\]

where \( P_k \) and \( P_k^2 \) represent the probabilities that the variables assume a value inside the \( k \)th and \( k \)th bins, respectively. \( P_k(\tau) \) denote the joint probability that \( x_i \) is in bin \( k \) and \( x_{i\tau} \) is in bin \( k \).

To obtain an estimate for the proper embedding dimension \( m \), the method of false nearest neighbor (FNN) can be used [17]. In this method, for each point \( i \) in the time series, look for its nearest neighbor in the \( m \) dimensional space and compute the ratio of the distances between these points in \( m \) and \( m + 1 \) dimension as

\[
\rho_{lm} = \frac{|U(i) - U(j)|_m}{|U(i) - U(j)|_{l+1}},
\]

where \( U(i) \) and \( U(j) \) denote the state vectors at points \( i \) and \( j \), and Euclidean norm is used. If this ratio is larger than a given threshold, then \( U_i \) is marked as having a false neighbor. The proper embedding dimension \( m \) is the value where FNN is close to zero.

The recurrence plots introduced by [18] is a graphical tool used for analyzing the dynamical properties of a time series obtained either from numerical simulation or experimental test rig. A recurrence plot (RP) is obtained from the recurrence matrix \( R \) whose entries can be given as

\[
R_{ij} = \theta(\epsilon - \|y_i - y_j\|), \quad i, j = 1, \ldots, N,
\]

where \( \| \cdot \| \) denote the distance between the two state vectors, \( \epsilon \) is the predefined threshold value and \( \theta \) is the Heaviside function. Depending on the value of the entries of \( R \), either a black dot is
drawn or a blank space is left in the RP. For computing the distance $\| \cdot \|$, different norms can be used (2-norm, max-norm). Dynamical features of the time series data can be extracted from the RPs [19–21]. For example, a periodic signal with mono-frequency are represented as equally spaced parallel lines (parallel to the main diagonal line) in RP. The RP of white noise is a homogeneous RP with no structure. Abrupt changes in the dynamics, laminarity of the time series and short term dynamics can be obtained from RPs [20,22,23]. The deterministic patterns of RPs based on the statistics of the vertical and diagonal lines can be quantified using numbers with the method of Recurrence Quantification Analysis (RQA). The dynamics of the system can be identified using RQA method. In order to quantify the complexity of the RPs, we will use the following RQA parameters [22,23]:

![Fig. 3. Streamlines (top) for isotherms (bottom) for different Reynolds numbers at $Gr = 10^5$.](image)

![Fig. 4. Time evolution of the length averaged Nusselt number along the left wall of the cavity at Reynolds number of 1000 for different Strouhal numbers and Grashof numbers.](image)
Recurrence rate ($RR$):

$$RR = \frac{1}{N^2} \sum_{i,j=1}^{N} R_{ij}.$$  

Determinism ($DET$):

$$DET = \frac{\sum_{l=l_{\text{min}}}^{N} l p(l)}{\sum_{l=1}^{N} l p(l)}.$$  

Entropy ($ENT$):

$$ENT = - \sum_{l=l_{\text{min}}}^{N} p(l) \ln(p(l)).$$

Laminarity ($LAM$):

$$LAM = \frac{\sum_{v=v_{\text{min}}}^{N} v p(v)}{\sum_{v=1}^{N} v p(v)}.$$  

Averaged diagonal line length ($L$)

$$L = \frac{\sum_{l=l_{\text{min}}}^{N} l p(l)}{\sum_{l=1}^{N} l p(l)}.$$  

Trapping time ($TT$)

$$TT = \frac{\sum_{v=v_{\text{min}}}^{N} v^2 p(v)}{\sum_{v=1}^{N} v^2 p(v)}.$$  

Fig. 5. Time evolution of the length averaged Nusselt number along the left wall of the cavity at Strouhal number of 0.5 for different Reynolds numbers and Grashof numbers.

Fig. 6. Variation of the normalized averaged Nusselt number along the left and right walls with Strouhal numbers at different Reynolds numbers and Grashof numbers.
\( P(l) \), \( P(v) \) represent the distribution of the lengths of diagonal lines and vertical lines, respectively. \( l_{\text{min}}, v_{\text{min}} \) denote the minimum values of the diagonal and vertical line lengths, respectively. \( RR \) denotes the fraction of recurrence points in RPs. \( DET \) represents the fraction of recurrence points that form the diagonal lines. It gives a measure for predictability of the time series data. \( ENT \) is the Shannon entropy based the distribution of the lengths of diagonal lines. It captures the complexity of diagonal lines in RPs. For higher values of \( ENT \), the deterministic structure is more complex. \( LAM \) is the fraction of the points which form the vertical lines and represents the occurrence of laminar states in the system. \( TT \) is the trapping time which shows the average length of vertical lines. This gives an estimate for the mean time that the system will be trapped at a specific state.

4. Results and discussion

In the current study of pulsating mixed convection flow in a multiple vented cavity, numerical simulation is performed for the

Fig. 7. Frequency spectrum of the time series data (averaged Nusselt number along the left wall of the cavity) for different Grashof numbers and Strouhal numbers at Reynolds number of 500.

Fig. 8. Frequency spectrum of the time series data (averaged Nusselt number along the left wall of the cavity) for different Grashof numbers and Strouhal numbers at Reynolds number of 2000.
range of parameters $500 \leq \text{Re} \leq 2000$, $10^4 \leq \text{Gr} \leq 10^6$ and $0 \leq \text{St} \leq 2$. Amplitude of the forcing at the inlet is set to 0.75.

**Table 1**

Optimal embedding dimensions ($m$) and embedding delays ($\tau$) for different combination of parameters.

<table>
<thead>
<tr>
<th>Re</th>
<th>Gr</th>
<th>St</th>
<th>$m$</th>
<th>$\tau$</th>
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<tbody>
<tr>
<td>500</td>
<td>$10^4$</td>
<td>0.25</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>500</td>
<td>$10^5$</td>
<td>0.5</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>500</td>
<td>$10^6$</td>
<td>2</td>
<td>3</td>
<td>16</td>
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<tr>
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<td>2</td>
<td>8</td>
</tr>
<tr>
<td>500</td>
<td>$10^6$</td>
<td>0.5</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>500</td>
<td>$10^6$</td>
<td>2</td>
<td>4</td>
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<tr>
<td>2000</td>
<td>$10^6$</td>
<td>2</td>
<td>4</td>
<td>9</td>
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</table>

Fig. 3 shows the streamlines and isotherm plots for varying Reynolds number at $\text{Gr} = 10^5$. These flow conditions correspond to Richardson number of 0.4, 0.1 and 0.025, respectively. For all of the considered cases, a symmetrical structure in the streamlines are observed. With the increasing values of Reynolds number (lower values of Richardson number), inertia forces dominate over buoyancy forces and under the inlet ports in the vicinity of the left and right vertical cavities streamlines are distorted. On the mid-top wall of the cavity symmetrical cells are formed with increasing values of Reynolds number. A close inspection on the isotherms reveals that maximum temperature gradients appear near the inlet and outlet ports. With increasing values of Richardson number, isotherms spread more towards the mid of the cavity because of the buoyancy effects.

The complex interaction between the buoyancy and inertial forces in the pulsating flow will lead to the variation of the Nusselt number with respect to a change in Grashof or Reynolds number. Time evolution of the length averaged Nusselt number along the left wall of the cavity at Reynolds number of 1000 for different

![Fig. 9](image)

**Fig. 9.** Proper embedding delay determination for ($\text{Re} = 500$, $\text{Gr} = 10^4$, $\text{St} = 0.25$). The first minimum of the mutual information is seen at $\tau = 7$.

![Fig. 10](image)

**Fig. 10.** Minimal required embedding dimension determination for ($\text{Re} = 500$, $\text{Gr} = 10^4$, $\text{St} = 0.25$) False nearest neighbors ($\text{fnn}$) drop to zero at $m = 4$. 
Fig. 11. Recurrence plots of the averaged Nusselt number (along the left wall of the cavity) time series at Re = 500 for different Strouhal numbers at Gr $= 10^4$ (top) and Gr $= 10^6$ (bottom), x and y axis denote the number of samples.
Fig. 12. Recurrence plots of the averaged Nusselt number (along the left wall of the cavity) time series at Re = 2000 for different Strouhal numbers at Gr = 10^4 (top) and Gr = 10^6 (bottom). x and y axis denote the number of samples.
Strouhal numbers and Grashof numbers are shown in Fig. 4. The systems reaches periodic states after the initial transients. It takes more cycles to reach periodic state with increasing the Strouhal number. The effect of the Grashof number (Gr = 10^4) is significant for the heat transfer augmentation with increasing values Strouhal number. The effect of Reynolds number on the time evolution of the length averaged Nusselt number along the left wall of the cavity at Strouhal number of 0.5 for different Grashof numbers are depicted in Fig. 5. In these plots, it is seen that when the Reynolds number increases, the number of cycles to reach the periodic state increases and the peak value of the Nusselt number decreases at the maximum Grashof number (which corresponds to Richardson number of 4 at Re = 500 and Richardson number of 0.25 at Re = 2000).

Variation of the averaged Nusselt number with respect to frequency of the oscillating flow for different Reynolds number and Grashof numbers are depicted both for the left and right vertical wall of the cavity in Fig. 6. At Re = 500, averaged Nusselt values are higher with increasing Grashof number which is the case when the buoyancy effects become important. The Nusselt values are the same for the left and right vertical walls for all cases at Re = 500 except for the case at St = 0.25, Gr = 10^4. Obviously, these cases correspond to symmetrical states. Non-zeros values of the difference between the left and right averaged Nusselt numbers correspond to non-symmetrical states. At this Reynolds number, the peak value of the Nusselt number is obtained for St = 1. At Re = 1000, the averaged Nusselt values are higher compared to the values at Re = 500. At St = 0.25, the non-symmetry state is seen at Gr = 10^4 and Gr = 10^6. The peak values of the averaged Nusselt numbers are achieved at different Strouhal number of 0.1 for all cases except the Nusselt value at the right vertical wall for Gr = 10^6. At Re = 2000, the peak values are attained for St = 0.25 and the non-symmetric state is observed at St = 0.5 for Gr = 10^6. In all cases considered, the minimum of the Nusselt values are attained at the highest frequency. This may be due to the adaptation of the system to its new flow condition with increasing frequency which decreases the heat transfer response of the system.

The FFT plots for the non-linear time series data of the averaged Nusselt numbers when the initial transients are removed (when the system reaches periodic steady states) are shown at different Grashof and Strouhal numbers in Fig. 7 for Reynolds number of 500 and in Fig. 8 for Reynolds number of 2000. In these plots, it is seen that higher harmonics which are at the multiple integers of the fundamental frequency with different amplitude levels are observed for all of these flow conditions. At the parameter set (Re = 500, Gr = 10^4, St = 0.25, first plot of Fig. 6), the level of the non-linearity is significant as the amplitude of the fundamental harmonic is much less compared to the second harmonic.

The proper embedding dimension $m$ and embedding delay $\tau$ parameters are calculated as described in the previous sections for each flow condition and they are tabulated in Table 1. Fig. 9 depicts the mutual information function for parameter combination (Re = 500, Gr = 10^4, St = 0.25) with respect to delay parameter and as can be seen the first minimum of the function is seen at $\tau = 7$. The percentage of false neighbors (fnn) calculated for increasing reconstruction dimensions are shown in Fig. 10 for parameter set (Re = 500, Gr = 10^4, St = 0.25). This plot show for embedding dimension of 4, the percentage goes very close to zero and hence the dynamics of oscillations is restricted to a four-dimensional phase space.

Fig. 11 and Fig. 12 show the recurrence plots (RPs) of the non-linear time series for the averaged Nusselt number along the left vertical wall for different Grashof numbers and Strouhal numbers at Re = 500 and Re = 2000, respectively. In these plots, x and y axis denote the number of time samples. On top of these plots, proper embedding dimensions and delays are also shown. The RPs and RQA are carried out using the software developed by [24]. In the RPs, equally spaced diagonal lines represent the presence of a single frequency in the oscillation. In the figures, the RP structures are diagonally oriented indicating the periodicity of the signals form the time series of averaged Nusselt numbers. In Fig. 11(a), the existence of the higher order frequency content can be identified from the spacing between the diagonally oriented lines. Again in these plots, the time instance when the system reaches periodic states can be seen. In Fig. 12(c), the paling of the plot away from the diagonal during 1000 time samples indicates the non-stationarity of the data set and then diagonally oriented lines indicate the periodicity of the data set. It is seen that for different parameter combinations, different structures can be identified from the recurrence plots. Recurrence quantification analysis (RQA) can be used to express the structures in recurrence plots by numbers [23]. RQA analysis makes it possible to identify the dynamics of the system. Table 2 shows the RQA analysis results for different combinations of parameters. ENT attains its largest value at the parameter sets (Re = 500, Gr = 10^6, St = 0.25) and (Re = 2000, Gr = 10^4, St = 0.5) indicating higher dynamical complexity. This is also indicated in the averaged Nusselt number plots in Fig. 8. For these combination of parameters, the non-symmetry flow states are seen where the values of the Nusselt numbers on the left and right vertical walls of the cavity are different from each other. The smallest value of TT indicates the shortest time in the laminar phase in the intermittent dynamics which is obtained at the parameter set (Re = 500, Gr = 10^4, St = 0.25). The higher values of DET represents the higher predictability of the system.

5. Conclusions

In this study, pulsating mixed convection in a multiple vented cavity is numerically investigated for a range of Reynolds, Grashof and Strouhal numbers. Recurrence quantification method is utilized to analyze the time series data of the averaged-Nusselt number along the left vertical wall of the cavity. The obtained results can be summarized as follows:

- The complex interaction between the buoyancy and inertial forces in the pulsating flow will lead to the variation of the Nusselt number with respect to a change in Grashof or Reynolds number.
- It takes more cycles to reach periodic state with increasing the Strouhal number and Reynolds number.
- The effect of the Grashof number (Gr = 10^4) is significant for the heat transfer augmentation with increasing Strouhal number.
- Non-zeros value of the difference between the left and right averaged Nusselt numbers which correspond to non-symmetrical states is seen St = 0.25, Gr = 10^4 for Reynolds number of 500.

### Table 2

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References


The recurrence quantification analysis for the time series of Nusselt number reveals different parameters quantifying structural complexity, predictability and time in the laminar phase in the intermittent dynamic for different combinations of parameters.

The minimum of the Nusselt values are attained at the highest frequency due to the adaptation of the system to its new flow condition with increasing frequency which decreases the heat transfer response of the system.

The FFT plots of the spatial averaged Nusselt number time series data show that higher harmonics at the multiple integers of the fundamental frequency with different amplitude levels are observed when the systems reach periodic steady states.

The recurrence plots for time series data of averaged Nusselt number show different structures in the plots indicating different dynamic features of the systems.

Maximum values of the ENT are obtained at the parameter sets (Re = 500, Gr = 10^4, St = 0.25) and (Re = 2000, Gr = 10^4, St = 0.5) indicating higher dynamical complexity.

The smallest value of TT indicates the shortest time in the laminar phase in the intermittent dynamics which is obtained at at the parameter set (Re = 500, Gr = 10^4, St = 0.25).