Estimating peak dominant frequency of chaotic fluid-bed systems using recurrence rate

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Abstract—Recurrence plot (RP) and recurrence quantification analysis (RQA), are applied for visualizing and quantifying recurrence states (fundamental features) of dynamical systems, respectively. Recurrence rate (RR), the simplest variable of RQA measures density of recurrence points throughout RP. In this work, variance of RR against different epoch lengths (time windows) for three different systems; chaotic Lorenz system, completely periodic time series and pressure fluctuations signal of fluidized bed (a chaotic system) was calculated. The results showed for all systems, the curve of variance of RR versus epoch length, after a descending area suffered a break and then tended to a constant value. It was found that the break point of the curves for all systems had a physical concept representing the peak dominant frequency (PDF) of the systems. The values of PDFs were estimated by inversion of the values of epoch lengths in break points. Additionally, comparison of the present method and fast Fourier transform (FFT) confirmed the results entirely.

INTRODUCTION

Fluidization phenomenon refers to a process in which solid particles are suspended in a gas or liquid and are fluidized alike to state of a liquid. Nowadays, the fluidization phenomenon progressively has been more developed and is considered as a suitable alternative for many industrial processes. By nature, gas-solid fluidized beds have great advantages such as: high heat and mass transfer, high effective surface contact, good particle mixing, low pressure drop, and thermal homogeneity, bringing about its multi-faceted applications in various processes. Seeing as the performance of gas-solid fluidization depends on its hydrodynamics, hence, quantitative and detailed understanding of its hydrodynamics implicate a challenge both in science and engineering, and further investigations are still needed.

Interpreting of the gas-solid fluidized bed hydrodynamics can be achieved through measurement of a specific parameter of the system, for example pressure fluctuations. Pressure signals encompass the effects of numerous different dynamic phenomena taking place in the bed, such as gas turbulence, bubbles hydrodynamics and the bed operating conditions. Whereas the measurement method is crucial, careful evaluation of measured parameter is more substantial. Various nonlinear analysis methods, such as time delay embedding theory, have been used for analyzing the dynamic changes in hydrodynamics of fluidized beds. All methods of nonlinear time series analysis are based on the attractor reconstruction of the system in the state space; these methods are accompanied by limitations such as: uncertainty through attractor reconstruction methods. Additionally, since two-phase structure of the fluidized bed has a low-dimensional chaotic behavior (typically more than 3 and less than 5) in the state space, thus, attractors with dimensions more than three can not be figured unless by projection into the two or three-dimensional spaces. On the other hand, long term data sampling, which is required for typical nonlinear evaluation of the pressure fluctuations signal in bubbling fluidized beds, usually is involved with some difficulties during experimental
measurements, e.g., data saving and data acquisition. These troubles have involved the search for more powerful mathematical techniques to overcome limitations.

Recently techniques of recurrence plot (RP) and recurrence quantification analysis (RQA) have been introduced to examine non-linear dynamical systems. The main advantages of the RP and RQA are to provide useful information for non-stationary and short-term time series. Accordingly, RP and RQA can reduce troublesomes such as need for time consuming and difficult long-term data sampling required in typical methods of non-linear analysis. The aim of this work is to apply recurrence rate, the simplest variable of RQA, to estimate the peak dominant frequency for three different behaviour of dynamical systems chaotic Lorenz, fluidized beds, and completely periodic time series. Furthermore, more validations are done by comparing the results of proposed method and the results achieved through fast Furrier transform.

**THEORY**

Recurrence Plots enable the visualization of n-dimensional phase space trajectory as a 2-dimensional plot of the recurrence states. Both the axes of the recurrence plots represent the time series that is under consideration. Recurrence plots are based on the recurrence:

$$R_{i,j} = \Theta(r - | | x_i - x_j ||) \quad i, j = 1, 2, \ldots, N$$

where $N$ is the number of states considered for analysis, $r$ is a radius threshold, $|| \ldots ||$ is the norm, and $\Theta$ is the Heaviside function. For given point to be recurrent, it must fall within the threshold distance, $r$.

To achieve quantitative information regarding RP, recurrence quantification analysis (RQA) was invented. RQA quantifies different structures founded in the recurrence plot. RQA is a powerful tool for evaluation non-stationary and short length time series as well as stationary ones. Thus it could be a very effective tool in study of different dynamical systems by providing variables that are simple to treat by numerical algorithms. In addition, RQA eliminates eliminating necessity for time consuming and difficult long-term data sampling required in classic methods of nonlinear analysis. Furthermore, the application of non-linear time series analyses on experimental data containing noise are commonly troublesome, while the effects of noise may be minimized through the input parameters manipulation.

Recurrence rate is the only RQA parameter which is used in this paper. Recurrence rate ($RR$) is mathematically defined as:

$$RR = \frac{1}{N^2} \sum_{i,j=1}^{N} R_{i,j}$$

$\sum R_{i,j}$ is total number of recurrence points and $N^2$ gives all possible number of $a_{i,j}$, hence, $RR$ expresses the fraction (or density) of repeated states throughout the time series. Obviously, radius threshold plays an important role in number of recurrence, therefore suitable selection of $r$ certainly influences on proper understanding of the given time series. Too small selection of radius threshold make almost no recurrence points, while too large value of radius induces, almost every point as neighbour of every other point, which may leads to misleading information. There are several number of researches on the optimal selection of radius. One of the best and straightforward methods is selection a radius corresponding to $RR$ less than 10%.13, 15

**Fast Fourier transform**

Fourier analysis is a helpful implement designed for data analysis in frequency domain. Fourier analysis decomposes a signal into constituent sinusoids of different frequencies and is performed by means of discrete Fourier transform (DFT). Estimated Fourier transform, $X(f)$, of the given time series $x(i)$ including $N$ point is equal to:

$$X(f) = \sum_{i=1}^{N} x(i) \exp\left(-j2\pi if\right)$$
where \( f \) and \( j \) stand for frequency and the complex number, respectively. If \( N \) is a power of 2, above equation becomes to fast Fourier algorithm, an efficient algorithm for computing the DFT of a sequence.

The power spectrum of a signal which represents the contribution of every frequency of the spectrum to the power of the overall signal can be estimated through the magnitude of \( X(f) \) squared. The variance of such estimation for the power spectrum does not decrease by an increase in \( N \). In order to decrease the variance, the signal is repeatedly divided into windows and an average of the power spectrum within the windows is used to acquire an estimate of the power spectrum (the Welsh method of power spectrum estimation). However, decrease of the samples within the windows gives poor frequency resolution. Hence, an appropriate window width should be chosen to obtain a satisfactory trade off between frequency resolution and variance. Using the Hanning window and without any overlap between the windows, the averaged power spectrum becomes:

\[
P_{xx}(f) = \frac{1}{L} \sum_{n=1}^{L} P_{xx}^n(f)
\]

where \( L \) is number of windows and \( P_{xx}^n(f) \) is the power spectrum estimate of each window.

**EXPERIMENTAL PROCEDURE**

Experiments were done in a gas-solid fluidized bed made of a Plexiglas-pipe. The column was 15 cm inner diameter and 2 m height. Air at ambient temperature introduced to column through a perforated plate distributor with 435 holes of 7 mm arranged, in a triangular pitch. A cyclone was applied to separate particles from air at high superficial gas velocities. Sand particles (Geldart B) with different mean sizes of 150 \( \mu \)m and also particle density of 2640 kg/m\(^3\) were used in the experiments. The bed was operated with loaded sand heights \( L/D = 2 \), at gas velocity \( U = 0.5 \) m/s.

Absolute pressure fluctuations were recorded via a probe of 50 mm length and diameter of 4 mm through a fine mesh net at the side facing of the fluidized bed. The Piezoresistive transducer (Kobold, SEN-3248 B075) used in the experiments contained a response time of less than 1 millisecond. Van Ommen et al.\(^{17}\) demonstrates that the model of Bergh and Tijdeman\(^{18}\) provides reliable predictions of the frequency response characteristics of probe-transducer system over a wide variety of probe length and diameter. The model for transducer with assumed inner space volume of 1500 mm\(^3\) predicts the first resonance frequency of 679 Hz and the amplitude ratio of 1.1 at 200 Hz (Nyquist frequency) and lowers at lower frequencies. This confirms the measuring technique is appropriate for gathering dynamic information from pressure signals in the expected range of frequencies, characteristically less than 20 Hz in a fluidized bed\(^{11}\). The measured signals were band-pass (hardware) filtered at lower cut-off frequency of 0.1 Hz to eliminate the bias value of the pressure fluctuations and upper cut-off Nyquist frequency (200 Hz).

**RESULTS AND DISCUSSION**

Figures 1a–c show pressure fluctuations signals for different dynamical systems, completely periodic, complex Lorenz, and fluidized bed system. Principally, there are differences between dynamic behavior of these systems, the periodic system contains regular patterns (deterministic behavior), whereas Lorenz and fluidized bed systems have more complex patterns as well as chaotic feature.\(^{1,2}\)

In this section, RR is implemented to find a straightforward method for estimation of PDFs of different dynamical behaviors. Initially, time series is divided into non-overlapped sub-series (epoch), then for each epochs RR is calculated separately and finally variance of whole RRs is considered.

Figures 2a–c represent variance of recurrence rate \( (\sigma^2_{RR}) \) versus epoch length for the three systems. As can be observed, diagrams of \( \sigma^2_{RR} \) for all periodic, Lorenz and fluidized bed systems involve two distinct regions, a descending region with high changes of \( \sigma^2_{RR} \) and after suffering a break, steady region wherein \( \sigma^2_{RR} \) changes very slightly with epoch length.
Figure 1. Time series of different dynamical systems; a) Completely periodic; b) Chaotic Lorenz; c) Fluidized bed.
a) Periodic System, N=1000, r=0.05, fs=1000 Hz

b) Lorenz system, N=50,000, r=0.1, fs=72 Hz
The steady region for periodic, Lorenz and fluidized bed systems approximately take place at epoch lengths of 20, 80 and 240 epochs length, respectively. Given their sampling frequencies, frequencies of the breaks are 1000Hz/20, 72Hz/80, and 400Hz/240, respectively, equal to 50, 0.9, and 1.7 Hz, respectively. These values correspond to PDFs of these systems, however to facilitate additional validation, FFT analysis could be employed. Figures 3a–c show power spectrum versus frequency for these systems. The achieved PDFs through FFT analysis is in agreement with achieved results of the present technique. Hence, the present method seems to be a reliable technique to PDF estimation of different dynamical systems.
a) 

b)
Figure 3. Power spectrum versus frequency for three different systems

CONCLUSIONS

In this work, a new application for RR, the simplest variable of RQA was presented to estimate the peak dominant frequencies of different systems. It was observed; regardless the different nature of dynamical features, diagrams of variance of RR versus can reveal peak dominant frequency different systems. Furthermore, to verify this claim a validation was done through comparing the results of proposed method and the results achieved through fast furrier transform.

REFERENCES


