A wide nonlinear analysis of reactive power time series related to electric arc furnaces

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Abstract

Prediction of electric arc furnace (EAF) reactive power is an appropriate solution to compensate for static VAR compensator delay and improve its performance in flicker reduction. A linear autoregressive moving average (ARMA) can only pull out the linear deterministic (LD) component of EAF reactive power time series. For the prediction to be made through both nonlinear deterministic (ND) and LD components, employing nonlinear models is necessary. However, before developing the nonlinear models for prediction, the necessity of the employing them should be verified by investigating the significance of the ND components in the process. This paper presents a novel approach for wide analysis of nonlinear behavior of EAFs reactive power time series related to eight ac EAFs installed in Mobarakeh steel industry, Isfahan, Iran to answer the question about the importance of their ND components. In the approach, a suitable linear autoregressive moving average (ARMA) model with order (4,4) is used for the time series to extract the residual time series. Then, a number of well established nonlinear analysis techniques such as time delay reconstruction, surrogate data, delay vector variance and recurrence plot methods are applied to the original and residual time series. To describe the nonlinear characteristics of the time series, some new indices are defined. They quantify the significance of the ND component in compare with LD component.

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1. Introduction

Electric arc furnaces (EAFs) are widely used in today’s industry because of their productivity, precision, flexibility and some advanced applications (such alloy) [1].

The time varying nature of arc furnace gives rise to voltage fluctuations with frequencies in the range of 0.5–25 Hz which produces the effect known as flicker [2]. Accurate modeling of the EAF is essential to solve flicker problem through assessing voltage fluctuations [3–5] and studying the effects of compensation techniques on flicker. There are many methods for EAF load modeling in the both time and frequency domains. These methods use empirical formulas that relate arc length/radius, arc voltage and current [6–9], voltage source models [10–12], deterministic chaotic systems [13], non linear and chaotic time varying resistor models [8,9] and stochastic processes [1,6,14]. Since the melting process is a non-stationary stochastic process, a stochastic model could be an appropriate choice for voltage flicker analysis [1]. In [6,15], it was shown that voltage fluctuations as well as reactive powers variations at the point of common coupling (PCC) and the arc furnace bus behave as a band limited white noise with time varying amplitude. Based on this, for investigation of compensation techniques performance through EAF simulation and calculation of flicker indices on 10 min observations, random variation law was attributed to arc length model. In [14], the time dependent variation of electric arc voltage was modeled by a second order autoregressive (AR) model through applying a parameters estimation algorithm to a limited data obtained from two arc furnaces. Because of structure and order of the model, an appropriate full-band fitting may not be obtained. In [1], an approach including application of the Markov method to time series corresponding to each frequency component of the EAF current was proposed to predict the components for the next one cycle.

A widely used method for flicker reduction is to employ static VAR compensators (SVCs) [16]. The SVC is installed at the primary side of arc furnace transformers and reduces the variation of reactive power drawn from the source by injecting a reactive power opposite to that of EAF. However, the ability of SVC in flicker compensation is limited by delays in reactive power measurements and thyristor ignition [17]. An appropriate solution for improving the performance of SVC in flicker reduction is developing a technique to compensate for its delay. This technique is based on the prediction of the reactive power of EAF for a half-cycle ahead [17,18]. For this purpose, a procedure for the development of a
stochastic model for reactive power of EAF at the SVC bus was presented in [18]. The procedure uses a huge data set collected for a considerable number of melting processes, including currents and voltages of eight ac EAFs installed in Mobarakeh steel industry in Iran. The reactive power time series are produced from the recorded currents and voltages based on the fundamental reactive power calculated by one-period integration and updated at successive half-cycles. Appropriate structures of autoregressive moving average (ARMA) models for EAF reactive power are obtained by analyzing these time series. These models are valid for most of the cases that can be recommended as common models in reflecting behavior of EAFs reactive power for different melting processes and their different stages. However, the coefficients of the ARMA models are time varying, so updating the coefficients is necessary as time passes. The methods used in adaptive filters were employed in [19] for online determining the models coefficients. Also in [20] the genetic algorithm is used to online calculation of the ARMA model coefficients. The models used for prediction of the EAF reactive power in [18–20], are linear ARMA models. On the other hand, nonlinearity analysis of variation of the arc furnace properties was briefly carried out in [13,21].

The Wold decomposition theorem [22] states that any discrete, stationary signal can be decomposed into a deterministic (predictable) and an unpredictable component, which are uncorrelated. The deterministic component can be used for prediction of the future of the corresponding signal and can be described precisely by a set of linear or nonlinear equations. The unpredictable component is same as a white noise signal has not useful data for prediction of the future of the corresponding signal. In this study, the signal $x_k$ (here it is the calculated EAF reactive power) is decomposed as

$$x_k = f(x_{k-1}, x_{k-2}, ... , x_{k-p}, a_{k-1}, a_{k-2}, ... , a_{k-q}) + a_k$$

(1)

where $x_k$ is the value of the signal at time $k$, $a_k$ represents the unpredictable component of signal $x$ and $f(\cdot)$ represents the deterministic component of signal that is a function of past values of $x$ and $a$ that are all known.

Here the deterministic component is divided to linear and nonlinear components. Thus each signal can be broken to three components: linear deterministic (LD), nonlinear deterministic (ND) and unpredictable (U) components. In this case, Eq. (1) can be rewritten as

$$x_k = g(x_{k-1}, x_{k-2}, ... , x_{k-p}, a_{k-1}, a_{k-2}, ... , a_{k-q})$$

$$+ h(x_{k-1}, x_{k-2}, ... , x_{k-p}, a_{k-1}, a_{k-2}, ... , a_{k-q}) + a_k$$

(2)

where $g(\cdot)$ and $h(\cdot)$ are linear and net nonlinear functions that represent the deterministic components (Here we use “net nonlinear” phrase for a function which its Taylor series don’t include any linear component).

An appropriate linear ARMA model considers only the LD component ($f(\cdot)$ in Eq. (2)) that causes the residual series contains ND and U components ($h(\cdot) + a_k$). Hence linear ARMA models do not use the ND component for prediction. To make predictions through both ND and LD components, employing nonlinear models is necessary. However, before developing the nonlinear models for prediction, the necessity of the employing these models should be verified by investigating the significance of the ND components in the process.

This paper presents a comprehensive nonlinear analysis of EAFs reactive power time series to answer the question about the importance of their ND components. For this purpose, the recorded data from eight ac EAFs installed in Mobarakeh steel industry, described in [18,19] are analyzed. The reactive power time series are decomposed to the linear parts and residual signals. We use ARMA(4,4) model, found in [18] as one of the most suitable models for EAF reactive power time series. The high order linear model captures entire LD component of the EAF reactive power time series. The residual series will be approximately free of LD component and only contains ND and U components. The residual signals can be used to calculate the ratio of ND to total deterministic (TD) components of EAF reactive power time series. In this study, time delay reconstruction [23], surrogate data [24], delay vector variance (DVV) [25] and recurrence plot (RP) [26,27] methods are applied to the original and residual time series to obtain the significance of ND component. Section 2 describes the EAFs reactive power data used for the nonlinear analysis. Section 3 presents application of time delay reconstruction method to obtain the embedding dimension (ED) of the EAFs reactive power time series. Section 4 uses surrogate data method along with several nonlinear measures to determine the nonlinear time series. The largest Lyapunov exponent (LLE) is used to obtain the chaotic nature of the time series. Delay vector variance (DVV) method is applied to the EAFs reactive power time series in Section 5 to provide a nonlinear measure for surrogate method that is used to indicate the nonlinear series. In addition, ability of DVV in quantifying the predictability of the signals is used to calculate the ratio of ND to TD components of the signals. Finally, recurrence plot (RP) method is applied to the EAFs reactive power time series in Section 6.

2. EAF reactive power time series and their residuals

The analysis is carried out on the EAFs reactive power time series calculated from actual input voltage and current waveform data related to eight ac EAFs installed in Mobarakeh steel industry, Isfahan, Iran. A single line diagram of the EAFs system is as given in Fig. 1. The EAFs system is served by two 400 kV transmission lines. Three transformers are used to step down the transmission voltage to 63 kV for the arc furnace transformers and 33 kV for two SVCs that are used to resolve the voltage flicker problem caused by

<table>
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<th>Nomenclature</th>
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<tr>
<td>$x_k$</td>
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<tr>
<td>$a_k$</td>
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<tr>
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<td>$M$</td>
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<tr>
<td>$e$</td>
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<tr>
<td>$p(l)$</td>
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</table>
the arc furnaces. Eight 70 MW ac arc furnaces are connected through EAF transformers and local feeders to the 63 kV bus. The arc furnace transformers have the following characteristics: 90–108 MVA, 63 kV/230–720 V, \( Y_{/d} \) with \( x = 6.5\% \) based on voltage 720 V.

Each SVC is composed of a 108 MVAr \( \Delta \)-connected three-phase thyristor-controlled reactor (TCR) and 97.2 MVAr filter capacitors. Shunt capacitor banks have been arranged in the second, third, fourth and fifth harmonic filters.

Data for the EAFs including three-phase supply voltages and currents, measured at the primary side of the arc furnace transformers, was collected for scrapping, melting and refining stages of several melting processes during three months. Data records cover 10, 20, 50 and 100 s of real time furnace operation. Each data set includes a number of samples with 128 \( \mu s \) sampling time (the sample frequency is 7812.5 Hz). By dividing 20, 50 and some of 100 s data to 10-s data sets, there will be 675 data sets in total with durations of 10-s and thirty 100-s data sets for achieving the analyses.

Based on the explanation in [18], two different reactive power signals are analyzed.

Signal \( q_1 \): The fundamental reactive power calculated by one period samples (0.02 s) that is updated in each half cycle (0.01 s).

Signal \( q_2 \): The reactive power approximated from half-cycle samples (0.01 s) that is updated in each half cycle. Employing a half-cycle integration interval is an attempt to optimize the flicker suppression capability. Although if the current signal includes the LD component, an ARMA(4,4) model is applied to time series based on \( x \), and their residuals \( e_1 \) and \( e_2 \). Table 1 shows the percentage of 10-s time series with \( d \) specified in the first row. According to Table 1, 70%, 60% and 62% of 10 s time series have \( d \) equal to 5 for \( q_1 \) and \( e_2 \) signals respectively. Also for \( q_1 \), 98% of time series have \( d \leq 5 \). So it seems selecting \( d = 5 \) for all signals is a suitable choice. Table 2 gives the mean and standard deviation (SD) values of parameter \( d \) related to all 10-s and 100-s signals. According to this table, the average of \( d \) for the 10 s and 100 s data records for all signals is near to 5. So this table same to Table 1 offers \( d = 5 \) for all signals. These results lead us to choose \( d = 5 \) for all signals. In Table 2, DT(%) denotes the percentage of time series that are counted as deterministic in Cao’s method. A deterministic time series has useful data that makes it predictable. Results show that almost all signals \( e_1 \) and \( e_2 \) are unpredictable. Since the only deterministic component in these signals is ND component, hence the analysis results for time delay reconstruction method indicate that ND components of EAF reactive power time series are not considerable.

### 4. Surrogate data method

The method of surrogate data [24] provides a rigorous statistical test for the null hypothesis that the data have been generated by a linear stochastic process [30]. In this method, a number of surrogate time series are generated with the same linear properties such as mean, variance, autocorrelation and power spectrum of the original time series. Then, the value of some nonlinear measures are computed and compared for the original and the generated surrogate time series. If the measures for original time series and the surrogates are significantly different, the null hypothesis is not satisfied.

#### Table 1

<table>
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<th>( d )</th>
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<th>4</th>
<th>5</th>
<th>6</th>
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<td>( q_1 )</td>
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<td>56.74</td>
<td>38.52</td>
<td>2.67</td>
<td>0</td>
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<tr>
<td>( q_2 )</td>
<td>0.89</td>
<td>24</td>
<td>70.22</td>
<td>4.89</td>
<td>0</td>
</tr>
<tr>
<td>( e_1 )</td>
<td>0</td>
<td>10.96</td>
<td>60.44</td>
<td>28.59</td>
<td>0</td>
</tr>
<tr>
<td>( e_2 )</td>
<td>0</td>
<td>10.52</td>
<td>61.93</td>
<td>27.56</td>
<td>0</td>
</tr>
</tbody>
</table>
rejected and the original time series is considered as a nonlinear time series.

There are several methods for producing surrogates. In this paper, we use two methods, phase randomizing (PR) [24] and iterative amplitude adjusted Fourier transform (iAAFT) [31].

Assume the value of a nonlinear measure for the original time series \( \{x_i^s\} \) and \( i \) th surrogate \( \{x_i^f\} \) are indicated as \( t_0 = t(\{x_i^s\}) \) and \( t_f = t(\{x_i^f\}) \), \( i = 1, \ldots, M \), respectively where \( M \) is the total number of surrogates. By sorting \( t_0, t_1, \ldots, t_M \) in an increasing manner, the smallest and largest values will take ranks 1 and \( M + 1 \), respectively. There are two kinds of nonlinear measures: one sided and two sided measures. In the case of one sided measure, the null hypothesis (that assumes the process is linear) is rejected if

\[
(M + 1)(1 - \alpha) < \text{rank}(t_0)
\]

(4)

where \( \alpha \) is the significance level or the probability that the null hypothesis is rejected and \( \text{rank}(t_0) \) denotes the rank of \( t_0 \) among \( t_s \).

When two sided nonlinear measure is used, the null hypothesis is rejected if

\[
(M + 1)(1 - \alpha/2) < \text{rank}(t_0) \quad \text{OR} \quad \text{rank}(t_0) \leq (M + 1)\alpha/2
\]

(5)

In the analysis of the EAFs reactive power time series by surrogate data method, we select parameters \( \alpha \) and \( M \) as 0.1 and 19 respectively. In addition, four nonlinearity measures: third order auto covariance (C3), Time reversibility (REV), correlation dimension and largest lyapunov exponent (LLE) are employed for the analysis.

- The third order auto covariance is a higher-order extension of the traditional auto covariance and is given by [30]:

\[
T_{\text{C3}}(\tau) = \frac{\langle x_k x_{k-\tau} x_{k-2\tau} \rangle}{\langle x_k x_{k-\tau} \rangle^3/3}
\]

(6)

where \( \tau \) is a time lag and \( \langle \cdots \rangle \) is the mean function over all \( k \).

- Time reversibility function is a measure for the asymmetry due to time reversal [30].

\[
T_{\text{REV}}(\tau) = \frac{\langle (x_k - x_{k-\tau})^2 \rangle}{\langle x_k - x_{k-\tau} \rangle^2/2}
\]

(7)

- Correlation dimension [32] is calculated by Taken’s estimator for the original and surrogate series.

- A more common technique to determine the presence of chaotic behavior in the system is the largest lyapunov exponent (LLE) [33], which measures the divergence of nearby trajectories. It can also be used as a nonlinear measure for the original series and its surrogates.

Tables 3–6 show the results of applying the surrogate method to the EAF reactive power time series for determining the percentage of nonlinear signals. The analysis is achieved by different measures with the aid of TSTOOL package. A big difference in the

| Table 2 | Mean and std of d related to signals and percentage of deterministic time series (DT). |
|---------------------|---------------------|---------------------|---------------------|
| Time series | Mean (d) | SD (d) | DT (%) |
| 10-s q1s | 4.178 | 0.581 | 77.19 |
| 10-s e1s | 5.176 | 0.604 | 0.44 |
| 100-s q1s | 4.73 | 0.58 | 96.67 |
| 100-s e1s | 5.7 | 0.7 | 0 |
| 10-s q2s | 4.7911 | 0.5303 | 33.63 |
| 100-s e2s | 5.1704 | 0.5935 | 0.3 |
| 100-s q2s | 5.2 | 0.48 | 40 |
| 100-s e2s | 5.5 | 0.82 | 0 |

results related to various nonlinear measures is observed. The results related to measures C3 and REV show low percentages of nonlinear signals among signals \( e1 \) and \( e2 \) so that they are lower than the percentage of nonlinear signals of \( q1s \) and \( q2s \). This result can be interpreted by the high unpredictable component compare to the deterministic components in the residual signals (\( e1 \) and \( e2 \)). In other words, in these methods the ND component is getting lost in the presence of the high unpredictable component. As the result, with these methods the signals have been shown linear because of the linear unpredictable component. As a consequence, these measures are not suitable for nonlinear analysis of signals that have strong noise compared to deterministic components.

The mean and standard deviation of LLE values for the EAFs reactive power time series are shown in Table 7. Since these values
are positive and very near to zero, we can say that EAF reactive power time series are not chaotic, and the small positive value of LLEs arises from the existence of strong noise component in the signals.

5. Delay vector variance method

The delay vector variance (DVV) method [25] analyzes the nature of a time series in view of the prevalence of deterministic or unpredictable components. This method can also judge the nonlinearity of the process. In this method, delay vectors (DVs) defined by $z_k = [x_{n-k}, x_{n-(k-1)}, \ldots, x_{n-1}]$ are associated with a corresponding target, namely, the next sample $x_n$. DVV method can be summarized as follows [25]:

- The mean, $\mu_d$, and standard deviation, $\sigma_d$, are computed over all pair wise distances between DVs.
- The spans, $r_d$, are taken from the interval $[\mu_d - n_d \sigma_d; \mu_d + n_d \sigma_d]$, e.g., uniformly spaced, where $n_d$ is a parameter controlling the span over which the DVV-plot is computed. The set $\Omega_d(m, r_d)$ consists of all DVs that lie within a distance to $z(k)$ equal to the span $r_d$.
- For every set $\Omega_d(m, r_d)$, the variance of the corresponding targets is computed. The average over all sets, divided by the variance of the time series, yields the inverse measure of predictability, namely, the ‘target variance’, $\sigma^2(m, r_d)$. We only compute the variance if $\Omega_d(m, r_d)$ contains at least $N_0 = 30$ DVs.

Then, DVV plot, the target variance $\sigma^2(m, r_d)$ as a function of standardized distance $n_d$ can be drawn. For instance, Fig. 2 shows the DVV plots for signals $q_1$ and $e_1$ corresponding to one data record. The solid curve is used for the original signal and the dashed curve gives the mean value of DVV plots for 19 surrogates. By investigating DVV plots, two important results are obtained.

Firstly, the presence of a deterministic component will lead to small target variance $\sigma^2(m, r_d)$ for small spans. Thus, the minimum target variance $\sigma^2_{\text{min}}$ is related to prevalence of the deterministic component over the stochastic one. In other words, $\sigma^2_{\text{min}}$ yields an inverse measure for the predictability or significance of deterministic component of the time series. Fig. 2 shows that $\sigma^2_{\text{min}}$ for signal $q_1$ is about 0.1. This is much lower than that of signal $e_1$ that is about 0.45. Hence, the deterministic component of signal $q_1$ is much more than that of signal $e_1$. Deterministic component of $e_1$ is equal to nonlinear deterministic component of $q_1$. Thus, we define a new index, nonlinear to total deterministic ratio (NTDR) as follows:

$$\text{NTDR} = \frac{\sigma^2_{\text{min}}}{\sigma^2_{\text{min}}} \quad (8)$$

where $\sigma^2_{\text{min}}$ and $\sigma^2_{\text{min}}$ are the minimum target variance values for the EAF reactive power time series ($q_1$ and $q_2$) and their corresponding residual time series ($e_1$ and $e_2$).

The question about usability of nonlinear models for EAF reactive power time series can be answered with the aid of NTDR and some other indices that will be defined later. These indices have values between 0 and 1. If the nonlinear deterministic (ND) component is low in comparison to total deterministic (TD) component, NTDR gets near to zero and vice versa. Table 8 shows the mean and standard deviation of $\sigma^2_{\text{min}}$ calculated for signals $q_1$, $q_2$, $e_1$ and $e_2$ related to all data records and also the mean and standard deviation of NTDR for signals $q_1$ and $q_2$.

The mean value of NTDRs calculated for signals $q_1$ and $q_2$ with 10-s length is 0.1566 and 0.2605, respectively. This value for signals $q_1$ and $q_2$ with 100-s length is 0.0476 and 0.0797 respectively. These relatively small values demonstrate that describing EAF reactive power time series by the linear models is enough and there is no need to use nonlinear models.

Secondly, another application of DVV method is demonstration of nonlinear properties in time series. If DVV plot for the original signal and the surrogates are significantly different, it can be said that the original time series has been generated by a nonlinear process. For example, Fig. 2b presents a large difference between the DVV curve of the signal $e_1$ and mean of DVV curves of its surrogates. This indicates a significant nonlinear property in the signal $e_1$.

To judge the nonlinearity of a time series conveniently, a scalar nonlinear measure to quantify the difference between DVV plots of the original time series and its surrogates is defined in [25]. Assume vector $s$ contains $\sigma^2(m, r_d)$ of the original time series for different $n_d$. Vectors $s^1$ to $s^4$ are similar vectors for the surrogates.

| Signals | $q_1$ | | $q_2$ | | $e_1$ | | $e_2$ |
|---------|-------|-------|-------|-------|-------|-------|
| Mean    |       | Mean  |       | Mean  |       | Mean  |
| SD      |       | SD    |       | SD    |       | SD    |
| 10-s Signals | 0.00017 | 0.00045 | 0.0016 | 0.00039 | 0.00025 | 0.0017 |
| 100-s Signals | 0.000052 | 0.000065 | 0.00053 | 0.000051 | 0.00007 | 0.00014 |

Fig. 2. DVV plot of signals related to one data record (solid curve) and the mean value of DVV plots corresponding to 19 surrogates (a) signal $q_1$ (b) signal $e_1$. 

Table 7
The mean and standard deviation of LLEs.
Table 8
Mean and standard deviation of minimum target variance $\sigma^{2*}_{\text{min}}$ and NTDR of EAFs reactive power time series and their residuals.

<table>
<thead>
<tr>
<th>Signals</th>
<th>$q^1$</th>
<th>$e^1$</th>
<th>$q^2$</th>
<th>$e^2$</th>
<th>$q^3$</th>
<th>$e^3$</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Mean of $\sigma^{2*}_{\text{min}}$</td>
<td>SD of $\sigma^{2*}_{\text{min}}$</td>
<td>Mean of NTDR</td>
<td>SD of NTDR</td>
<td>Mean of $\sigma^{2*}_{\text{min}}$</td>
<td>SD of $\sigma^{2*}_{\text{min}}$</td>
</tr>
<tr>
<td>10-s Signals</td>
<td>0.0885</td>
<td>0.0793</td>
<td>0.6347</td>
<td>0.1876</td>
<td>0.1566</td>
<td>0.1613</td>
</tr>
<tr>
<td>100-s Signals</td>
<td>0.0275</td>
<td>0.0114</td>
<td>0.6184</td>
<td>0.1340</td>
<td>0.0476</td>
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Table 9
The percentage of nonlinear time series obtained by using DVV method.

<table>
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<tr>
<th>Signals</th>
<th>$q^1$</th>
<th>$q^2$</th>
<th>$e^1$</th>
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<td></td>
<td>Mean of $\sigma^{2*}_{\text{min}}$</td>
<td>SD of $\sigma^{2*}_{\text{min}}$</td>
<td>Mean of NTDR</td>
<td>SD of NTDR</td>
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<tr>
<td>10-s Signals</td>
<td>46.07</td>
<td>47.41</td>
<td>96.59</td>
<td>95.70</td>
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<td>PR alg</td>
<td>41.92</td>
<td>46.37</td>
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</tr>
<tr>
<td>10-s Signals</td>
<td>56.67</td>
<td>50</td>
<td>100</td>
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<td>aAFT alg</td>
<td>50</td>
<td>46.67</td>
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</table>

where $\varepsilon$ is a threshold distance and $||y_i - y_j||$ represents the distance between vectors $y_i$ and $y_j$. The graphical representation of $R^q_j$ is called recurrence plot (RP) and is used for encoding the 1s as black and 0s as white points. To quantify the structures found in RPs, the recurrence qualification analysis (RQA) was proposed [34]. There are several measures that can be considered in the RQA. One crucial point for these measures is the distribution of the length of the diagonal lines ($R_{i+1,k+i}=1$) where $l$ is the length of the diagonal line) existing in the plot.

In the case of deterministic systems, the diagonal lines mean that trajectories in the phase space are close to each other on time scales that correspond to the length of the diagonals. So the diagonal lines can be used to calculate the rate of recurrence in the system and hence its deterministic nature. Here we use the measures that are based on the diagonal lines to obtain the deterministic nature of signals and the ratio of nonlinear to total deterministic components. These measures are as follows:

- **Determinism [27]**
  \[
  DET = \frac{\sum_{l=1}^{l_{\text{min}}} l \cdot P^l(l)}{\sum_{l=1}^{l_{\text{min}}} P^l(l)}
  \]  
  where $P^l(l)$ denotes the frequency distribution of the lengths $l$ of diagonal structure. We choose $l_{\text{min}} = 2$ in our analysis.
- **The average diagonal line length [27]**
  \[
  L_{\text{ave}} = \frac{\sum_{l=1}^{l_{\text{min}}} l \cdot P^l(l)}{\sum_{l=1}^{l_{\text{min}}} P^l(l)}
  \]
- **The longest diagonal line ($l_{\text{max}}$) [27]**
- **Entropy [27]**
  \[
  \text{ENTR} = -\sum_{l=l_{\text{min}}}^{l_{\text{max}}} w^l(l) \ln(w^l(l))
  \]

where
  \[
  w^l(l) = \frac{P^l(l)}{\sum_{l=1}^{l_{\text{max}}} P^l(l)}
  \]

$\text{ENTR}$ reflects the complexity of the RP in respect of the diagonal lines, e.g. for uncorrelated noise the value of $\text{ENTR}$ is rather small, indicating its low complexity.

Based on the above measures and measure NTDR defined in the previous section, we define four new indices that are proportional to the ratio of nonlinear deterministic component to totally deterministic component.

- **DTNT**
  \[
  DTNT = \frac{DET}{DET_q}
  \]

- **LANT**
  \[
  LANT = \frac{L_{\text{ave}}}{L_{\text{ave}}_q}
  \]

- **LNT**
  \[
  LNT = \frac{L_{\text{max}}}{L_{\text{max}}_q}
  \]
where notations $q$ and $e$ indicate EAF reactive power time series ($q_1$ and $q_2$), and their residuals ($e_1$ and $e_2$) respectively. Similar to $NTDR$, these indices have values between 0 and 1 (0 for a signal free of ND component and vice versa).

The first RQA measures are calculated by cross recurrence plot toolbox of MATLAB [35] with $\varepsilon = 0.1 \sigma$, where $\sigma$ is the standard deviation of time series. Then the newly defined indices are calculated from them. Table 10 shows the mean and standard deviation of RQA measures and the new indices for 10-s signals. Again, the results demonstrate the low ND component compared to TD or LD components.

7. Conclusion

In order to consider both linear and nonlinear deterministic components of EAFs reactive power time series, nonlinear models must be applied. However, before developing the nonlinear models for the time series, the necessity of the employing these models should be verified by investigating the significance of the nonlinear deterministic components in the process. For this purpose, this paper presents results of a wide analysis of nonlinear behavior of the EAFs reactive power time series related to eight ac EAFs installed in Mobarakeh steel industry, Isfahan, Iran. The analyses are based on a novel approach. The basic aspects of this approach are

1. Applying a suitable linear ARMA model to many EAFs reactive power time series to extract their residuals.
2. Employing a number of well established nonlinear analysis techniques such as time delay reconstruction, surrogate data, delay vector variance and recurrence plot methods.
3. Defining a number of new indices for describing the nonlinear characteristics of the time series.

Some important results obtained from analysis are as follows:

1. A deterministic time series has useful data that makes it predictable. Results of application of Cao’s method shows that almost all signals $e_1$ and $e_2$ are unpredictable. Since the only deterministic component in these signals is ND component, hence the analyses results from time delay reconstruction method indicate that ND component of the EAFs reactive power time series are not considerable.
2. The largest Lyapunov exponent values calculated for the EAFs reactive power time series demonstrate that the time series are not chaotic.
3. The relatively small values of index $NTDR$ in delay vector variance method demonstrate that describing the EAFs reactive power time series by the linear models is enough and there is no need to use nonlinear models.

4. Since DVV method considers the deterministic component of signals only, it is insensitive to stochastic components, and it can give the best results among other studied nonlinear measures. The percentage of the EAFs reactive power time series with nonlinear properties among all time series obtained by the DVV method shows that almost all signals $e_1$ and $e_2$ and about half of signals $q_1$ and $q_2$ are nonlinear.

5. RQA measures and the newly defined indices in the recurrence plots method calculated for 10-s reactive power time series demonstrate the low ND component compared to TD or LD components.

Generally, as a final result we may say that using nonlinear models for the EAFs reactive power is not necessary and employing appropriate linear ARMA is enough for the prediction purposes.

References


