UNDERSTANDING COUPLING AND SYNCHRONIZATION IN EEG OF EPILEPTIC DISCHARGE USING RECURRENCE PLOTS WITH VARYING THRESHOLD

Ashish Kaul Sahib, N. Pradhan
Department of Psychopharmacology
National Institute of Mental Health and Neurosciences, NIMHANS
Bangalore, India

Abstract — The emergence of epileptic seizure is not clearly understood and the phenomenon is characterized by a synchronous discharge of large population of neurons. The property of recurrence of dynamical systems is utilized in understanding the synchronicity of epileptic discharge. The synchronization index is evaluated from the recurrence distribution of phase space. This study presents the variations in the recurrence dynamics by varying the threshold. The results of synchronization index indicate that the enhanced synchronicity is observed during seizure and decreases to near baseline level following seizure. Therefore the recurrence can be used in measuring the state of synchronicity in neuronal ensemble in various traits of brain function.

Keywords- Seizure, Recurrence, Synchronization, EEG.

I. INTRODUCTION

Understanding synchronization between various natural time-series is actually a fundamental problem. At present it is clear that real-world processes are generally nonlinear by nature. Thus applying linear approach for their analysis will not give a clear understanding about the underlying dynamics of the process or system. Since the frequency contents of EEG signals are of importance, spectral estimates by Fourier analysis are widely used to study the pattern features of EEG signals and in the analysis of background EEG activity [8]. However, spectral methods are heavily dependent on the statistical predictability of the process. Non-periodic signals such as transients, brief seizure, etc, get superimposed on EEG and severely delimit the spectral methods of analysis. Moreover, spectral methods may fail to detect occasional spikes of low amplitude. With the development of the theory of nonlinear dynamics (NLD), there has been a tangible shift towards NLD invariant measures in EEG classification and diagnosis. The statistical characteristics of EEG keep on changing with varying period lengths, indicating altered mechanisms of generation, in an unpredictable fashion. In nonlinear methods there is a tacit assumption of quasi-stationarity in EEG dynamics [1]. The experimental data or the time series is presumed to be stationary in a physiological context. It is hardly a surprise that EEG is not random-noise but possesses some definite characteristic structure for a given brain state. However, the problem remains as to how to detect unambiguously its structure patterns and formulate specific physical rules for the emergence of such patterns in EEG. It may not be an overemphasis that a thorough inquiry of the non-stationary transitions may be crucial in understanding the meaning of EEG in a given brain state. The emergence of epileptic seizure is characterized by a synchronous discharge of a large population of neurons. One approach to nonlinear time series analysis consists of reconstructing, from the EEG time series an attractor of the underlying dynamics. The shape of the attractor gives hints about the system i.e. whether it is periodic, chaotic etc. Eckmann et al. (1987) have introduced the tool called recurrence plots which can visualize the recurrence of states in phase space [5]. The recurrence plot gives an idea about the transition of the system at various instances of time. We have taken the cross recurrence plot (CRP) across various pairs of channels of the brain to determine the strength of coupling during a seizure.

II. METHODOLOGY

A. Phase Space Reconstruction

Biomedical signals are usually observed in one-dimensional form, and are represented discretely in the form of a time-domain vector, s(n). It can be inferred that the one-dimensional time-domain vector, s(n), is a projection of the signal generator source, represented by an unknown but underlying multidimensional dynamic state vector x(n). The multidimensional dynamic state vector is composed of an unknown number of variables, represented through its dimension d. The transition from a sampled one-dimensional time-domain signal s(n), to the corresponding sampled d-dimensional state space requires the application of Takens Theorem [9]. Takens Theorem represents a technique to reconstruct an approximation of the unknown dynamic state vector x(n) in d-dimensional state space by lagging and embedding the observed time series s(n). This reconstructed approximation is the state vector

\[ y(n) = [s(n), s(n+T), s(n+2T), \ldots, s(n+T(d-1))] \]  \hspace{1cm} (1)

Composed of time-delayed samples of s(n), where T is the time delay and d is the embedding dimension of the system.
Time delay or lag (T) is determined from the average mutual information (AMI) criteria.

\[ \text{AMI}(T) = \sum_{s(n),s(n+T)} P[s(n),s(n+T)] \log \left( \frac{P[s(n),s(n+T)]}{P[s(n)]P[s(n+T)]} \right) \]

The delay at which the mutual information reaches its first minimum is chosen as the optimum delay T for embedding. The maximum embedding dimension d is determined using false nearest neighbor approach. The dimension d is increased in steps until the amount of false nearest neighbours approach zero. For some natural processes the amount of false neighbours may not be zero. In that case the value of d at which the amount of false neighbours remains constant even if the dimension is increased is chosen as the maximum embedding dimension.

B. Recurrence Plots

Most of the processes in nature exhibit a distinct recurrent behaviour. Given a sufficient amount of time each state of the system may repeat or may be arbitrary close to a previously observed state, this is a fundamental property of deterministic dynamical systems and is typical for nonlinear or chaotic systems.

In 1987, Eckmann et al. introduced the method of recurrence plots (RPs) to visualize the recurrences of dynamical systems. Suppose we have a trajectory \( \{x_i\}_{i=1}^{N} \) of a system in its phase space. The components of these vectors could be, e.g., the position and velocity of a pendulum or quantities such as temperature, air pressure, humidity and many others for the atmosphere. The development of the systems is then described by a series of these vectors, representing a trajectory in an abstract mathematical space. Then, the corresponding RP is based on the following recurrence matrix:

\[ R_{ij} = \begin{cases} 1 : & \tilde{x}_i \approx \tilde{x}_j \\ 0 : & \tilde{x}_i \not\approx \tilde{x}_j \end{cases} \]

(3)

Where N is the number of considered states and \( \tilde{x}_i \approx \tilde{x}_j \) means equality up to an error (or distance) \( \varepsilon \). Note that this \( \varepsilon \) is essential as systems often do not recur exactly to a formerly visited state but just approximately. Roughly speaking, the matrix compares the states of a system at times i and j. If the states are similar, this is indicated by a one in the matrix, i.e. \( R_{ij}=1 \). If on the other hand the states are rather different, the corresponding entry in the matrix is \( R_{ij}=0 \). So the matrix tells us when similar states of the underlying system occur. The RP is obtained by plotting the recurrence matrix, and using different colours for its binary entries, e.g., plotting a black dot at the coordinates (i, j), if \( R_{ij} = 1 \), and a white dot, if \( R_{ij} = 0 \). Both axes of the RP are time axes and show rightwards and upwards (convention).

Since \( R_{ij} = 1 \) for all i=1,...,N by definition, the RP has always a black main diagonal line, the line of identity (LOI) [2]. Furthermore, the RP is symmetric by definition with respect to the main diagonal, i.e. \( R_{ij} = R_{ji} \).

C. Cross Recurrence Plot

A cross recurrence plot (CRP) is a graph which shows all those times at which a state in one dynamical system occurs simultaneously in a second dynamical system. In other words, the CRP reveals all the times when the phase space trajectory of the first system visits roughly the same area in the phase space where the phase space trajectory of the second system is. The data length of both systems can differ, leading to a non-square CRP matrix.

\[ CR_{ij}(\varepsilon) = \Theta(\varepsilon \| x_i - y_j \|), i,j \in \mathbb{R}^d \]

(4)

Both systems are represented in the same phase space, because a CRP looks for those times when a state of the first system recurs to one of the other system. Using experimental data, it is often necessary to reconstruct the trajectories in phase space. If the embedding parameters are estimated from both time series, but are not equal, the higher embedding should be chosen. However, the data under consideration should be from the same (or a very comparable) process and, actually, should represent the same observable. Therefore, the reconstructed phase space should be the same [2].

D. Selection of Threshold

A crucial parameter of an RP is the threshold \( \varepsilon \). Therefore, special attention has to be required for its choice. If \( \varepsilon \) is chosen too small, there may be almost no recurrence points and we cannot learn anything about the recurrence structure of the underlying system. On the other hand, if \( \varepsilon \) is chosen too large, almost every point is a neighbour of every other point, which leads to a lot of artefacts. A too large \( \varepsilon \) includes also points into the neighbourhood which are simple consecutive points on the trajectory. This effect is called tangential motion and causes thicker and longer diagonal structures in the RP as they actually are. Hence, we have to find a compromise for the value of \( \varepsilon \). Moreover, the influence of noise can entail choosing a larger threshold, because noise would distort any existing structure in the RP.

At a higher threshold, this structure may be preserved. Several “rules of thumb” for the choice of the threshold \( \varepsilon \) have been advocated in the literature, e.g., a few percent of the maximum phase space diameter has been suggested. Furthermore, it should not exceed 10% of the mean or the maximum phase space diameter. A further possibility is to choose \( \varepsilon \) according to the recurrence point density of the RP by seeking a scaling region in the recurrence point density [6].
E. Recurrence quantification Analysis

In order to go beyond the visual impression yielded by RPs, several measures of complexity which quantify the small scale structures in RPs have been proposed and are known as recurrence quantification analysis (RQA). These measures are based on the recurrence point density and the diagonal and vertical line structures of the RP. A computation of these measures in small windows (sub-matrices) of the RP moving along the LOI yields the time dependent behaviour of these variables. Some studies based on RQA measures show that they are able to identify bifurcation points, especially chaos–order transitions. The vertical structures in the RP are related to intermittency and laminar states. Those measures quantifying the vertical structures enable also to detect chaos–chaos transitions. The simplest measure of the RQA is the recurrence rate (RR) or percent recurrences

\[ RR(\varepsilon) = \frac{1}{N^2} \sum_{i=1}^{N} R_{i,i}(\varepsilon) \]  (5)

RR quantifies the percentages of recurrent points falling within the specified radius (\(\varepsilon\)). This variable can range from 0% (no recurrent points) to 100% (all points recurrent). It simply counts the black dots in the RP, it is a measure of the density of recurrence points [3].

III. EXPERIMENTAL SETUP

The study uses EEG signals from the archived database of the department of Psychopharmacology at National Institute of Mental Health and Neurosciences, Bangalore. The signals were digitized and achieved during routine clinical EEG recording of neurological patients. The electroshock induced seizure EEG of subjects undergoing treatment of psychiatric symptoms is routinely archived to database. The patients had received ECT as per the clinical decision of the treating psychiatrist and no patient underwent any of this procedure for the sake of this research. 16 EEG records with spontaneously occurring generalized seizure group were recorded. We also collected 16 EEG records of subjects having received bi-temporal ECT. The extracted pre-seizure phase contained approximately 20 seconds(s) data preceding epileptic discharge and it was followed by seizure phase. The post-seizure data was approximately 20 s data following seizure. The archived records of EEG signals had been acquired in digital form using a 32 channel Neuroscan SynAmps System at 128 samples/s for spontaneous seizure group under International 10-20 electrode placement scheme. The extracted EEG records data were ported to Alpha Server 4100 (Digital Equipment Corporation) for analysis. EEG records were filtered by 300 order FIR filter bi-directionally (0.5 – 32 Hz) and then screened to extract artifact free generalized seizure and ECT induced seizure for analysis. The filtered EEG signal obtained is divided into 12 non-overlapping windows .The signal from each window is reconstructed in phase space using the time delay embedding.

For estimating the coupling strength between the regions present on the right and left hemisphere of the brain during the seizure, the cross recurrence plot (CRP) is taken. The threshold \(\varepsilon\) is chosen as one tenth of the largest phase space diameter. The distribution of recurrence i.e. the Recurrence rate (RR) which is obtained after taking the CRP is used in evaluating the synchronization index (\(\rho\)). RR quantifies the percentage of recurrent points falling within the specified radius (\(\varepsilon\)).

The synchronization index (\(\rho\)) is given as:

\[ \rho = \frac{(S_{max} - S)}{S_{max}} \]  (6)

Where \(S_{max}\) is the window having the maximum recurrence rate in the channel and \(S\) is the recurrence rate of the window of that particular channel. The index \(\rho\) takes values between 0 and 1.Symbol \(\rho = 0\) corresponds to absence of synchronization with regard to the seizure discharge and \(\rho = 1\) corresponds to perfect synchronization. The synchronization index (\(\rho\)) is also evaluated at threshold \(\varepsilon/3\) and \(\varepsilon/6\).

IV. RESULTS

The phenomena of seizure activity can be physiologically understood as an enhanced coherence resulting from the simultaneous bursts of firing across a mass of neurons. The 16-channels of EEG show a fronto temporal discharge of seizure. The EEG segments contain a seizure discharge while the background is relatively uniform.

Figure1. CRP between Fp1-Fp2 having 8192 data points (~64 s)
V. CONCLUSION

Even though brain is a massive interconnected system, it appears that its dynamics may be simpler in certain behavioral states. The brain possibly functions by large scale synchronization and simultaneous independence may be restricted to a few neural subsystems. The nonlinear time series EEG signal is reconstructed in phase from which the recurrence plot is obtained, which gives an idea about the transition of the nonlinear system in phase space. The approach of nonlinear dynamics in analyzing biological systems is superior to the traditional method of time-domain or spectral methods in revealing hidden structures in time series. It has given rise to a fundamentally different approach to EEG analysis. The present study presents the variations in the recurrence dynamics by varying the threshold ($\epsilon$). The synchronization index ($\rho$) has been evaluated at $\epsilon$, $\epsilon/3$ and $\epsilon/6$. The results of synchronization index as a function of time shows that during seizure there is enhanced synchronicity due to coupling, which is to remain constant irrespective of the changes in $\epsilon$. Since it remains constant it can be considered as a robust technique in finding fixed pattern dynamics in recurrence. Thus the study has clinical implications for visual and quantitative EEG interpretation in normal and pathological state and gives an indication about the dynamics of the system (brain) during an epileptic seizure in the form of distribution of recurrences (RR) across different channels of the brain.

References


