Investigation and quantification of Phase coherence index for different types of forcing in DC glow discharge plasma

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**A B S T R A C T**

The evidence of finite nonlinear interaction in a DC glow discharge plasma has been demonstrated by estimating phase coherence index for different types external forcing techniques likewise noise, sinusoidal square etc. The existence of finite phase coherence index i.e finite correlation prompts us to carry out nonlinearity analysis using delay vector variance (DVV). Finite nonlinear interaction obtained from phase coherence index values is observed to be predominant at a particular amplitude of square forcing which corroborates our nonlinearity analysis using DVV. Existence of phase coherence index has been demonstrated introducing continuous wavelet transform (CWT). Characterization of the difference in the phase distribution by the difference in the waveform in real space instead of dealing in Fourier space has been facilitated by introducing structure function or path length for different orders to study and identify the dynamical system. The expression of path length eventually enables us to evaluate the phase coherence index. The transition in the dynamics is observed through recurrence plot techniques which is an efficient method to observe the critical regime transitions in dynamics.

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1. Introduction

The theory of nonlinear, deterministic dynamical systems provides powerful theoretical tools to characterize geometrical and dynamical properties of the attractors of such systems [1]. Substantial work has been carried out to understand and reveal different nonlinear processes and the hidden dynamics in order to gain relevant information about the system of interest. Numerous scientific disciplines such as astrophysics, biology, geoscience uses data analysis techniques to get an insight into the complex systems observed in nature [2] which show generally a non-stationary and complex behaviour although almost all methods of time series analysis [12], traditional linear or nonlinear, must assume some kind of stationarity [3]. As the complex systems are characterized by different transitions between regular, laminar and chaotic behaviors, the knowledge of these transitions is essential to understand underlying mechanism behind a complex system. Linear approaches are insufficient to study the changes in the dynamics during the measurement period that usually constitute an undesired complication of the analysis. Nonlinear approach such as recurrence plot analysis will be suitable to graphically detect different patterns and structural changes in time series data which exhibit characteristic large and small scale patterns caused by the typical dynamical behaviour [13]. In this report an attempt has been made to understand the hidden dynamics by adopting recurrence plot (RP) [27], along with structure function or path length which turns out to be a powerful tool in analysing a nonlinear, complex signal. Plasma effects are finding ever increasing applications in astrophysics, solid state physics, physics of gas discharge and thermonuclear fusion [14]. Plasmas are intrinsically nonlinear whose effects manifest in the form of various exotic structures such as double layers [15], solitons, vortices, different types of waves, instabilities and turbulence [18]. Glow discharge plasma being rich in high energy, electrons and ions are capable of exhibiting many such nonlinear phenomenon [16,17]. So much effort has been endeavoured to study the intricacies involving the topics like finite nonlinear interactions and its associated phase coherence index. The investigation of the nonlinear wave interaction is based on the decomposition of a signal into its amplitude and phase part albeit we have to assume implicitly weak nonlinearity. From this point of view the amplitude along with the phase information obtained from the Fourier transform is convenient for our analysis permitting us wave number/frequency decomposition. Structure function or path length analysis [6–8] bears a significant aspect in this regard. Depiction of structure function has been executed for original (ORG) as well as for phase randomised (PRS), phase constant surrogate data (PCS) requiring detailed knowledge about the generation of surrogate time series. Quantification measure of the structure
function has been adopted by estimating phase coherence index [6,9]. In fact the purpose of this paper is to discuss the nonlinear interaction and matters pertaining to phase information [4], nonlinearity [10,11] etc with emphasis on the calculation of the phase coherence index. This is the first time that the distribution of wave phases for two different kinds of surrogate data namely phase randomised, phase constant along with the original one has been executed by applying some external means of forcing. In order to avoid the influence of the choice of origin we make two surrogate data sets from the original FPF and characterise difference in real space by evaluating the path length or equivalently structure function. The nonlinear interaction not only contributes to the exchange of energy among the wave modes [4], but also to the synchronisation of the wave phases.

The paper is organized as follows. In Section 2, we present a brief schematic of the experimental setup, followed by the results of the analysis of recurrence plot, structure function, phase coherence index respectively in Sections 3 and 4. Section 5 represent a comprehensive analysis of nonlinearity with surrogate data as well as with delay vector variance method. Conclusions are presented in Section 6.

2. Experimental setup

The experiments were carried out in a cylindrical hollow cathode DC glow discharge argon plasma with a typical density and temperature of $\sim 10^7$ cm$^{-3}$ and 2–6 eV respectively. The chamber was evacuated by rotary pump to attain a base pressure of 0.001 mbar. Experiments were performed under different forcing condition like sinusoidal, square, noise keeping operating neutral pressure fixed at 0.12 mbar with discharge voltage (DV) being fixed at 435 V. An unbiased Langmuir probe was used to obtain the floating potential fluctuations acquired with a sampling time of $10^{-6}$ s. A signal generator was coupled with the DV through a capacitor for observing fluctuations in presence of forcing as shown in the schematic diagram of Fig. 1.

3. Floating potential fluctuation, recurrence plot

The sequential change in floating potential fluctuations (FPF) acquired by applying noise, sinusoidal and square forcing is presented in Fig. 2. The use of three external forcing, i.e. sinusoidal, square and noise is very important from the practical point of view in any forced dynamical system. Phenomena like phase locking, which is very important in the chaos control can be achieved using the sinusoidal forcing. Similarly, other phenomenon like period pulling, resonance, etc. require the use of sinusoidal and square forcing. As far as forcing is concerned noise is omnipresent in any experimental system. Also, the phenomenon like stochastic resonance (enhancement of sub-threshold signal in the presence of external noise forcing) and coherence resonance can be achieved using noise forcing. Due to all these physical applications, we tried to explore the nonlinearity and other feature of plasma dynamics in the presence of three different forcing. The dynamical change in the FPF’s has been detected with recurrence plots (RP) and discussed in the framework of RP.

The recurrence plot (RP) is a relatively new technique of time series signals to understand the hidden insights involving the intricacies of the interplay between nature of different periodicity of the system and was introduced by Eckman and Kamphorst [19]. The RP expressed as a two dimensional square matrix (in Eq. (1)) represents the occurrence with ones and zeroes for states $X_i$ and $X_j$ and find the hidden periodicity in a time series signal which is not observable by naked eye.

$$R_{ij} = H(\epsilon - ||X_i - X_j||); i, j = 1, \ldots, M$$

where $M$ is the number of data points of the signal, $H$ is the heaviside function and $||.||$ is the norm (Euclidean norm), $\epsilon$ is the choice of the threshold. A crucial parameter of RP is the threshold $\epsilon$. Therefore, special attention has to be required for its choice. If $\epsilon$ is chosen too small, there may be almost no recurrence points and we cannot learn anything about the recurrence structure of the underlying system. On the other hand, if $\epsilon$ is chosen too large, almost every point is a neighbour of every other point, which leads to a lot of artefacts. For this case it was proposed to choose such that the recurrence point density is selected to be approximately 1% [20] in our case.

According to Takens embedding theorem, for a time series data $X_i$, an embedding can be made using the vector $Y_i = X_i, X_{i+1}, \ldots, X_{i+\tau-1}$, which represent the original time series embedded into d dimensional phase space with $\tau$ being the delay. RPs are graphical, two dimensional representations showing the instants of time at which a phase space trajectory returns approximately to the same regions of phase space. A recurrence is said to occur whenever a trajectory visits approximately the same region of phase space indicating $R_{ij} = 1$, whereas if the state does not recur with itself we are left with $R_{ij} = 0$.

RP’s of FPF’s are depicted in Fig. 3 for different types of external perturbation for the qualitative analysis and visualisation of the recurrences of dynamical system. Starting with the Fig. 3a and b (corresponding to the time series fluctuation of first and last subplot in the left panel of Fig. 2a) without any external forcing and for the noise forcing amplitude of $A=4$ V we can hardly observe any point indicating almost zero recurrence followed by the appearance of distinct diagonal lines with scattered points in between long diagonal lines for square forcing of $A=1$ V. The arrangement of the scattered point for square forcing of $A=2$ V (Fig. 3d) occupies a large region in between the bold diagonal lines in recurrence plot with increasing number of diagonal lines. The RP’s in Fig. 3e shows a long diagonal line with some faint signature of non-diagonal lines which becomes more ordered and prominent for increasing value of sinusoidal forcing depicted in Fig. 3f indicating deterministic behaviour. We have clearly delineated the ordered behaviour in Fig. 4 through RP plots in presence of increasing sinusoidal forcing. Initially in Fig. 4b we are left with a main diagonal line with some other faint diagonal lines. The arrangement of the broken diagonal lines along with the scattered points within main long diagonal lines are seen to become more ordered in Fig. 4c and d at forcing amplitudes of $A=2$, $2.4$ V. At higher forcing amplitudes of $5$ V, $6$ V, RP plots exhibit some prominent arrangement of long diagonal lines. Diagonal lines in the plots are indicative of periodic, deterministic behaviour and represent similar evolution of states at different times.

![Fig. 1. Experimental setup for glow discharge plasma.](image-url)
Fig. 2. Raw signal with increase in amplitude of the (a) noise (0–8 V) (b) sinusoidal (0–6 V) (c) square forcing (0–6 V).

Fig. 3. Recurrence plots for different types of forcing: (a) No forcing (b) Noise (A=4 V) (c) Square (A=1 V) (d) Square (A=3 V) (e) Sinusoidal (A=0.8 V) (f) Sinusoidal A=5 V.

4. Method of analysis: structure function, phase coherence index

After visualising the recurrence of the dynamical system we proceed to concern ourselves with the phase information. When we attempt to obtain phase information from data, the Fourier transformation has been the starting point for this purpose. The Fourier transformation of a time series $X(t)$ is defined as

$$X(\omega) = \int_{-\infty}^{\infty} X(t) e^{-i\omega t} dt$$

(2)

where $\omega$ is the angular frequency which provides information on amplitude and phase distribution. The decomposition of a time se-
ries into its amplitude spectrum and phase part is shown in Fig. 5.
In space plasma research in the context of geomagnetic pulsation and
the power law type spectrum of magnetic field turbulence [21] amplitude spectrum has always been in discussion in the literature for many years. The phase distribution on the other hand has not achieved much attention in space plasma application. A possible reason may be that the phase distribution in Fourier space appears to be completely random as seen from the phase part depicted in the right panel of Fig. 5.

\[ \phi(\omega) = \arctan \frac{\text{Im}X(\omega)}{\text{Re}X(\omega)} \]  

(3)

From a given FPF we generate two surrogate data sets. Firstly, we decompose the original data (ORG) into the power spectrum and the phases using the Fourier transform. We then randomly scramble the phases keeping the power spectrum unchanged, and from these two pieces of information in the Fourier space, we perform the inverse Fourier transform to create the phase-randomized surrogate (PRS). Phase-correlated surrogates (PCS) are generated by making the phases equal without shuffling the Fourier phases. The three data, ORG, PRS and PCS share exactly the same power spectrum, while their phase distributions are all different. The distribution of phases of the ORG, looks almost as random as that of the PRS in Fig. 6, due to the arbitrary choice of the coordinate origin and so the phase coherence even if it exists is easily obscured. So, we can characterize the difference in the phase distribution by the difference in the waveform in real space, instead of dealing with the variables in Fourier space. The difference can be captured by the structure function or path length, the expression of which is
given below in Eq. (4) where $m$ is order of the path length and $\tau$ is the measure characterising the magnification level of the curve. Path lengths of the curves are shorter when the phases are correlated (for PCS) and that for ORG and PRS appears to be almost similar as clearly depicted in Fig. 7c. Similarly with the increase in order (Fig. 7a and b) magnitude of the path length also goes on increasing irrespective of the nature of all types of forcing. Distribution of phases extracted for a time series along with its surrogates are depicted in Fig. 6. The distribution looks scattered in both the cases of ORG and PRS while that is seen to remain constant for PCS.

$$S(\tau) = \langle |x(t_i + \tau) - x(\tau)|^m \rangle$$

(4)

Shown in Fig. 8a–c are the path lengths by applying different amplitudes of noise, sinusoidal and square forcing respectively. The path length portrayed in all of these figures after increasing settles to a saturation value or displays some fluctuation similar to the one showed by Koga et al. [4]. This range of $\tau$ is often used by the plasma physicist to detect the region of stationarity where all the analysis are carried out. In case of external forcing applied in the form of noise (Fig. 8a) the right portion of the profile of $S(\tau)$ seems to be completely flat whereas that for sinusoidal forcing (Fig. 8b) exhibits an oscillatory trend and for square forcing (Fig. 8c) it appears to bear a different shape. The shape of structure function helps us to identify the characteristic nature of the forcing applied as the rising part of the path length for different noise amplitude in Fig. 8a have a value of slope less than that of square and sinusoidal forcing and rest of the parts are almost flat. Amplitude of the forcing have effect in increasing the slope of the rising part of structure function.

Now we define the phase coherence index to evaluate the degree of phase coherence $C_p(\tau)$ in Eq. (5) between the Fourier modes. If the original data has random phase then $C_p(\tau)$ would
be 0 whereas \( C_\phi(\tau) \approx 1 \) if the phases are completely correlated. The profile of phase coherence index with variation of \( \tau \) is shown in Fig. 9a–d for zero forcing, noise, sinusoidal, square forcing respectively. We found that \( C_\phi \) remains around the value from 0 to 0.25 for almost a wide range of the values of \( \tau \) in case of noise forcing (Fig. 9a) whereas the use of sinusoidal as well as square forcing results in enhancing the values of \( C_\phi \) up to 0.4, 0.5 respectively indicating more correlation between the wave phases. The plot of phase coherence index displayed in Fig. 10 with the application of three different forcing reflects the same thing. \( C_\phi \) reaches its maximum value at an intermediate value of forcing amplitude (A) of 2 V for both sinusoidal and square forcing whereas that is seen to remain at a very low value (almost 0) for a wide range of values of \( \tau \) for noise type forcing indicating almost zero correlation. Similarly (Fig. 10) the variation of \( C_\phi \) with the increase in the frequency of sinusoidal and square signal follows no particular trend but rather a zigzag nature with the values of \( C_\phi \) being greater in case of square forcing indicating more correlation between the wave phases. So it seems plausible not to draw any particular conclusion regarding the values of \( C_\phi(\tau) \) with the frequency (f) for a particular type of external forcing kept at fixed amplitude.

\[
C_\phi(\tau) = \frac{S_{\text{PRF}}(\tau) - S_{\text{ORC}}(\tau)}{S_{\text{PRF}}(\tau) - S_{\text{ORC}}(\tau)} \tag{5}
\]

The origin of phase coherence index has been explained in the framework of continuous wavelet transformation (CWT) \cite{22,23} as explained below. One particular wavelet, the Morlet \cite{24,25}, is defined as

\[
\psi(s) = \frac{\pi}{2} e^{i \omega_0 s} e^{-\frac{s^2}{2}} \tag{6}
\]

where \( \omega_0 \) is the dimensionless frequency and \( s \) is the dimensionless time. We restrict ourselves in using only Morlet wavelet due to its benefit of giving good balance of time frequency localization. The CWT of the time series \( d(t) \) with respect to the wavelet \( \psi(s) \) is defined as

\[
W_{d,\psi}(s,t) = d(t) \ast \psi(t) \tag{7}
\]

\( \psi(s) \) is the wavelet at the scale \( s \) (which is linearly related to the characteristic period of the wavelet). The wavelet power is defined as \( |W_{d,\psi}|^2 \). As the wavelet is not completely localized in time
CWT has edge artifact. Following [25], it is therefore useful to introduce a Cone of Influence (COI) in which edge effects cannot be ignored. Using the aforementioned transformation Fig. 11a depicts the accumulation of power scattered in large frequency band (especially from several hundred Hz to 2 kHz and also some portion in very low frequency range for the fluctuation containing maximum phase coherence index value corresponding to RP plot in Fig. 3d). The ordinate in this figure is expressed as wavelet period related to inverse of the frequency with the power being denoted by color axis. The CWT portrayed in the Fig. 11b with \( C_p \sim 0.08 \) displays the accumulation of power in one frequency value not actually distributed in the band of frequency corresponding to the RP portrayed in Fig. 4f. So it is evident that for maximum \( C_p \) values the energy or the power concentration occupies a larger region of the frequency band. Absence of energy concentration indicates the considerable decrease in \( C_p \) index values. Thus the results prove that the values of phase coherence is indicative of the accumulation of power/energy in a band of frequency and the results of CWT also validate the visualisation through recurrence plots.

5. Nonlinearity analysis with delay vector variance for different types of forcing

The correlation of the wave phases are indicative of the nonlinear interactions amongst them. As we have explored the finite interaction between the correlation of phases by estimating phase coherence index for square and sinusoidal forcing, it seems plausible to carry out the aspect of nonlinearity for them which is addressed by DVV analysis. DVV method [5] endeavours to characterise a time series based upon its predictability and thrives to compare the result to those obtained for the linearised versions of the signal (surrogates). Due to the standardisation within the algorithm, the method is robust to the presence of noise. The approach for the calculation of DVV is that for every set of original time series the corresponding surrogates are generated. The averaged over all sets normalized by the variance of the time series yield the measure of nonlinearity given in Eq. (8).

\[
\sigma^2 = \frac{1}{N} \sum_{k=1}^{2N} \sigma_k^2
\]

The linear or nonlinear nature of the time series is examined by performing DVV analyses on both the original and a number of surrogate time series, using the optimal embedding dimension of the original time series. Increase in the value of amplitude of noise level results in upward shifting of the variance of the surrogate over the bisector line [26].

Due to standardization of the distance axis, DVV plots are easy to interpret. At the extreme right all DVV plots converges to unity as the variance \( \sigma^2 \) of the surrogate will be equal to that of original time series. If the DVV plots obtained from surrogate time series yield the same thing with that of the original series or in other way if the scatter diagram coincides with the bisector line, the original time series can be supposed to be linear. The deviation from the bisector line is thus an indication of nonlinearity.

The method being robust with the application of noise, we have shown below the DVV plots with increasing noise amplitude in Fig. 12. Application of square as well as sinusoidal forcing will lead to the following DVV plots depicted in Figs. 13 and 14. We observe that the small value of square forcing results in the appreciable deviation from the bisector line whereas sinusoidal forcing has very less effect in making deviation from the bisector line thus indicating less nonlinearity. The application of square forcing make the deviation maximum at the intermediate value of amplitude of forcing at \( A = 2 \) V (Fig. 13d). In contrary DVV scatter plots displayed in Fig. 14 is observed to produce very small deviation or almost nil.

6. Conclusion

We paramountly aim to explore the nonlinear interactions introducing different types of external forcing by estimating the phase coherence index to characterise the correlation of phases among Fourier modes in a given time series by employing surrogate data technique to consolidate the idea of nonlinearity. Results of the DVV scatter plots are corroborated by the estimate of \( C_p \) exhibiting maximum values for square forcing than that obtained using sinusoidal forcing. The correlation of the wave phases are of indicative of the nonlinear interactions amongst them. Origin of this type of correlation is illustrated using continuous wavelet transform with Morlet wavelet which is particularly regarded a good choice on feature extraction process. Existence of power/energy concentration in a large region of frequency band is thought to be attributed to the increase in phase coherence index values in case of square, sinusoidal forcings which can also be regarded as one of the physical reasoning for the variation in \( C_p \). We found that \( C_p \) is almost never equal to zero indicating that there are always some correlation among the wave phases. The positive correlation in the value of \( C_p \) are in agreement with the finite nonlinearity. Physically in terms of path length, the correlated phases result in shorter path length than the cases when the phases are random. The existence of finite phase coherence index is estimated to be predominant at a particular value of the square forcing amplitude in view of its composition as an infinite summation of sinusoidal waves. We further ensure this estimate by the statistical analysis of DVV. Display of structure function or path length \( (S(\tau)) \) has been executed with variation in the value of \( \tau \) for different types of forcing. The nature of the structure function obviously helps us to identify the characteristic nature of the forcing applied and to study the dynamical system under different types of forcing am-

![Fig. 11. Continuous wavelet transformation of that fluctuation containing (a) maximum \( C_p \) with square forcing in the left panel (b) small \( C_p \) for sinusoidal forcing.](image-url)
amplitudes. The results of CWT analysis are also seen to corroborate the results of recurrence plots. An understanding of the recurrence plot reveals the trend of the nature of the dynamics while applying different types of external forcing with variable amplitude. The arrangement of the points in these plots for square forcing seem to be scattered and occupied a large region in between the bold diagonal lines with increasing number of diagonal lines in comparison to the more ordered structure for sinusoidal forcing which implies the presence of more deterministic behaviour in case of sinusoidal waveform than the non-sinusoidal square waveform. So, increase in square forcing amplitude is conjectured to increase the correlation/ interaction between the Fourier modes which is revealed by estimating phase coherence index and performing DVV analysis. We have extracted the original time series into the amplitude and phase part which is particularly portrayed for three different surrogates obtained by performing shuffling of phases and keeping phases constant respectively. This type of study bears important implication in various transport processes of charged particle where it is conventional to employ random phase approxim-ation [28]. For discussions of the various transport processes of the charged particle it is fundamentally important to determine whether the phases are randomly distributed or they have finite coherence. In view of this the study of energy diffusion and acceleration of charged particles dealing particularly with the effects of phase correlation is very crucial. One assumptions is important for this i.e the amplitudes of the external perturbations are not too high.

Interpretation of any plasma phenomena is dependent on an understanding of nonlinear dynamics. As an potential application of our work we can suggest characterising the devices using various nonlinear methods and techniques which is extremely beneficial in improving their performance in different parametric regimes for the self and externally excited plasma. We think that implementation of the techniques to the data obtained from DC discharge plasma under the application of this types of external forcings will be of immense benefit to extract and understand the feature of nonlinearity, information about dynamical system theory of plasma oscillation of different plasma system like glow discharge,
double plasma, dusty plasma. In summary we have investigated the dynamical behaviour of a glow-discharge plasma under different types of external perturbation and the dynamics is observed to depend on both amplitude and frequency of the perturbation.

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References