Nonlinear Local Projection Technique for ECG

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Abstract— The electrical activity of the heart shows complex dynamical behavior. This paper presents a nonlinear local projective noise reduction filter for electrocardiograms. In order to automatically detect the best neighborhood size of the projection, recurrence plots are used. The minimum gives a clear indication of the best size of the neighborhood in the embedding space. We also use the box-assisted algorithm to efficiently find nearest neighbors in m-dimensional space. The method is successfully applied to ECG signals.

Keywords— ECG; noise reduction; local projection technology; recurrent plot;

I. INTRODUCTION

As we all know, Electrocardiogram(ECG) has been the most commonly used biomedical test for diagnosis of various heart diseases. Because of the feebleness of ECG, it will be disturbed by all kinds of noise. The noise will degrade the rightness of computer’s automatic diagnosis. So the signal must be preprocessed.

In recent years, biologists have discovered that the electrical activity of the heart shows nonlinear dynamical behavior and chaos phenomenon. Linear filters cannot remove the noise without also distorting the signal. This is due to the fact that already pure signals from chaotic systems show broad band spectra, and no Fourier-based method is able to distinguish this from random noise. For the latter, one has somehow to use the fact that deterministically chaotic motion takes place on attractors which are smooth submanifolds of the total available phase space. This implies that state vector constructed from delay variables are constrained to fall onto geometrical objection which are locally linear[1].

We will suppress measurement errors in an ECG using a nonlinear local projection method developed for chaotic signals, which was applied to many different types of time series such biomedical signal[2], human speech[3,4], to cite a few examples. The method was detailed in Ref. [2]. But the parameter, namely, neighborhood size was determined through a visual inspection of the sample. Here we make use of the recurrence plots to optimize the parameter [3]. Also in order to decreasing time, we present box-assisted algorithm, which has heuristically developed in the context of time series analysis[5]. In order to show that the program for noise reduction is correct, a noise reduction process is first implemented for x-axial time series of Lorenz equation contaminated by 50% Gaussian white noise. and then applied to ECG signal which is contaminated by 25% Gaussian white noise.

II. LOCAL PROJECTIVE NOISE REDUCTION SCHEME

A. Local Projection Scheme

Let us consider a dynamical system

\[ x_{n+1} = F(x_n) \]

\[ x_n = h(z_n) \]

\( \{x_n\} \) is observable signal, measurement noise is zero.

Delay vectors are constructed as follows:

\[ x_n = \{x_{n-(m-1)\tau}, x_{n-(m-2)\tau}, \ldots, x_{n-\tau}, x_n\} \]

Where \( m \) is the dimension of the vector and \( \tau \) a delay. If \( m \geq 2d + 1 \), the m-dimensional delay embedding space is equivalent to the original unobserved phase space of the dynamical system. For more details, see [10]. Then the original dynamic can be written as:

\[ x_{n+1} = f(x_{n-(m-1)\tau}, \ldots, x_{n-\tau}, x_n) \]

Or

\[ \tilde{F}(x_n) = \tilde{F}(x_{n-(m-1)\tau}, \ldots, x_n, x_{n+1}) = 0 \]

linearized (5) as

\[ a^{(\sigma)} \cdot R(x_n - \overline{x}^{(n)}) = O(\|x_n - \overline{x}^{(n)}\|^2) \]

Here, \( \overline{x}^{(n)} = \frac{1}{|u^{(n)}|} \sum_{x_i \in u^{(n)}} x_i \) is the centre of mass of the delay vectors in a small neighborhood, \( R \) a diagonal weight matrix which will allow us to focus the noise reduction on the most stable middle coordinates of the delay vectors. This is achieved by choosing \( R_{11} \) and \( R_{mm} \) large and all other diagonal entries \( R_{ii} = 1 \).

When the measurement noise is not zero, the \( (6) \) will not be valid exactly but only up to some error related to the noise :

\[ a^{(\sigma)} \cdot R(y_n - \overline{y}^{(n)}) = \varepsilon_n \]

The local projection technology is the projection of \( z_n = R(y_n - \overline{y}^{(n)}) \) onto the nullspace, that is
\[ Z_0 = \sum_{q=1}^{Q} (z_n, a^q)^t \], where \( Q \) is the dimension of the nullspace. In order to find the nullspace, we require the nullspace and space of useful signal be as orthogonal as possible. We require \( \sum_{n \in u(t)} \left| \sum_{q=1}^{Q} (z_n, a^q)^t \right| \) to be minimal for the correct choice of the set of \( a^q \), Also we have to find \( Q \) orthonormal vectors \( a^q \). In order to satisfy the above restrictions, we have:

\[
L = \sum_{n \in u(t)} \left| \sum_{q=1}^{Q} (z_n, a^q)^t \right| - \sum_{q=1}^{Q} \lambda^q (a^q, a^q) = 0
\]

or simplified as

\[
L = \sum_{n \in u(t)} \left| \sum_{q=1}^{Q} (z_n, a^q)^t \right| - \sum_{q=1}^{Q} \lambda^q (a^q, a^q) = 0
\]

with respect to \( a^q \) and \( \lambda^q \), get

\[
Ca^q - \lambda^q a^q = 0
\]

\[
C_{ij} = \sum_{n \in u(t)} (z_n)^t (z_n)
\]

The global minimum is given by the eigenvectors to the \( Q \) smallest eigenvalues. The noise component of the vector \( y_n \) is thus removed by replacing it by

\[
\hat{y}_n = y_n - R^{-1} \sum_{q=1}^{Q} \langle R(y_n - y_a^q), a^q \rangle a^q
\]

Note that each scalar measurement \( y_a \) appears as a component in \( m \) embedding vectors. In order to translate \( \{ y_a \} \) back into a scalar signal, the corrected scalar time-series values.

**B. Identification of the Best Neighborhood Size**

From (11), we have to look for neighbors, whose size plays a crucial role for noise reduction. Too big neighborhood cannot project the point along the proper direction. Because too many point including false ones will deform the correct direction. A very small neighborhood will not produce a unique direction. We present an automatic way called recurrence plots to detect the best neighbor- hood size, the following procedure is used:

1) To construct the recurrence plot,

\[
r_{ij} = \Theta(\epsilon - |y_i - y_j|)
\]

Where \( \epsilon \) is a predefined tolerance level and \( \Theta() \) is the Heaviside step function, \( \Theta(x) = 1 \) if \( x > 0 \) and \( \Theta(x) = 0 \) elsewhere. The matrix elements are unity for all pairs of indices \( i, j \) whose corresponding delay vectors have a distance smaller than \( \epsilon \).

2) \( N_p(\epsilon) \): Compute the histogram along the main diagonal direction,

\[
h_i = \sum_{k \neq i} r_{ik}
\]

The number of \( N_p(\epsilon) \) is then given by the number of \( h_i \) such that \( h_i > h_{th} \) and \( h_{i-1} < h_{th} \). We can define \( h_{th} \) as the average height of the histogram plus three times the standard deviation.

3) \( N_\perp(\epsilon) \): The average number of neighbors that points have

\[
N_\perp(\epsilon) = \sum_{i,j \neq i} r_{ij} / N
\]

4) Find the value of \( \epsilon \) such that the following quantity is minimized:

\[
\beta(\epsilon) = \left[ N_\perp(\epsilon) - N_p(\epsilon) \right] / N_\perp(\epsilon)
\]

**C. Efficient Neighbour Searching**

After the determination of the neighborhood, we will encounter another problem that is how to find nearest neighbors in \( m \)-dimension efficiently. This step is the most time-consuming in local projection method. As long as only small sets (say \( N < 1000 \) points) are evaluated, neighbors can be found in a straightforward way by computing the \( N^2 / 2 \) distances between all pairs of points. So a fast algorithm to process the data set is very important. To deal with the problem, we make use of the box-assisted method. If the point are not clustered, the operation count will only be \( \propto N \) for \( N \) points.

**III. EXPERIMENTS AND RESULTS**

**A. Local Projection Algorithm for Lorenz**

Now we want to show some results of the application of the nonlinear local projection noise reduction technique. We first use the algorithm to deal with the \( x \)-axis of Lorenz signal, which is contaminated with 50% Gaussian white noise as in Fig.1. Here the parameter \((\sigma, r, b) = (10.28, 8/3)\), initial values \((x_0, y_0, z_0) = (15.34, 13.68, 37.91)\). Using 4-order Runge-Kutta, \( h = 0.01 \). About three iterations can get a good result. The size of neighborhood is 19, 12.4, 7.1 respectively. The result can also prove the correctness of the algorithm.

**B. Local Projection Algorithm for ECG Singnal**

The ECG signal is obtained from a publicly available database[8] of the Massachusetts Institute of Technology an Beth Israel Hospital (Boston), which was artificially contaminate with 25% Gaussian white noise as in Fig.2. Sample rate is 250 per seconds. the phase space of ECG is showed in Fig.3. After three iterations of local projection...
algorithm, getting a good result, as in Fig.4. We select $m = 25, Q = 2$.


FIG. 2 ECG signal Upper: original data; Lower: data artificially contaminated with 25% Gaussian white noise.

FIG. 3 Delay representation with delay time 0.02s of the ECG. Upper: representation of the original data; Lower: representation of data artificially contaminated with 25% Gaussian white noise.

FIG. 4 local projection noise reduction on ECG.

C. Results

ECG signals present some geometric structures in the time delay embedding space. When contaminated with noise, the attractor can be approximated by means of locally nonlinear projective filtering. We have introduced here the local projection method for ECG noise reduction. The size of neighborhood is determined by recurrence plots whose lines are neither fragmented nor fat. With such an approach it is no longer necessary to visually inspect the recorded noisy sample during the filtering. The results showed the efficiency of the algorithm.
REFERENCES


