Self-organized criticality, Causality and Correlation of Probability of Recurrence between Daily Mean Temperature and Dew Point across India

Rajdeep Ray
Department of Electronics and Communication Engineering,
Dr. B. C. Roy Engineering College
Durgapur 713206, India
ray.rajdeep78@gmail.com

Payel Majumder
Department of Computer Science and Engineering
NSHM Knowledge Campus,
Durgapur 713212, India
cse2002.payel@gmail.com

Mofazzal Hossain Khondekar
Department of Applied Electronics and Instrumentation Engineering
Dr. B. C. Roy Engineering College,
Durgapur 713206, India
hossainkm_1976@yahoo.co.in

Koushik Ghosh
Department of Mathematics,
University Institute of Technology, University of Burdwan,
Burdwan 713104, India
koushikg123@yahoo.co.uk

Anup Kumar Bhattacharjee
Department of Electronics and Communication Engineering
National Institute of Technology
Durgapur 713209, India
akbece12@yahoo.com

Abstract — The temperature and dew point records of seven weather stations, located in India have been scrutinized under the self-organized criticality regime. The data models an almost ideal scaling behaviour as of the type of $1/f^\alpha$ noise; with $\alpha \approx 1$. This scaling behaviour strongly suggests the presence of self-organized criticality (SOC) behind both the signals. To draw more insight into the detailed dynamics and to strengthen the explanation of such behaviour, causal relationship exploiting singularity spectrum analysis (SSA), multivariate singularity spectrum analysis (MSSA) and Correlation of Probability of Recurrence (CPR) based on recurrence plot have been studied in between the two signals for all the stations. The results reveal sufficient causal relationship and high correlation of probability of recurrence for strong support behind such critical dynamical systems.

Keywords — Time series analysis; Indian Climate; Self-organized criticality (SOC); Multivariate singularity spectrum analysis (MSSA); Correlation of probability of recurrence (CPR)

I. INTRODUCTION

The climate as well as weather parameters such as temperature, dew point can be considered as complex multi-component dynamical system. To study such complex systems handful of procedures and models have been applied by the researchers. Theory of multifractality is applied for studying the variations in temperature in Greenland [1] and Antarctica [2]. In 2004 (RIAL, et al., 2004) stated the significance of observing the Earth’s atmospheric condition as a nonlinear, complicated, dynamic system, but there were no references made to the earlier publications [1]. In [2] atmospheric condition was introduced as a complicated, nonlinear, energetic system without any models such as conceptual or mathematical. A. Andronov [3] explained the bonding in between the limit cycle and auto oscillation. Any non-linear system constrained to a constant, non-oscillating, energy flow shows an auto-oscillation or self-organization. Thus, dynamical systems can be viewed as a system which is dissipative, self-organized and not linear. There are many geophysical phenomena such as sand pile, land slide, earthquake whose time structure has a power spectrum of the type $1/f^\alpha$. These processes are specifically known as "noise" although $\alpha$ is not exactly equal to one. In such scenario, the inception of the power law behaviour has not been understood in depth. A plausible explanation of such behaviors is stated by the theory of Self-Organized Criticality (SOC). This theory, which is inspired in the phenomenology of sand piles, was devised by [4] [5]. SOC can explained how certain spatially extended natural (or simplified) systems spontaneously evolve to a critical state of catastrophically unstable and then after dissipation of some energy again become stable. The understanding of the dynamics of the process is responsible for determining SOC enactment. In this work investigation has been done to find out the corroboration of SOC behaviour in long-term temperature and dew point data sets (from 9th October, 1996 to 1st February, 2013) in deferent regions (Kolkata, Chennai, New Delhi, Mumbai, Bhopal, Agartala and Ahmedabad) of India [6] [7]. Specifically, we explore for hyperbolic or power law distributions of time series of daily mean temperature and dew point. However absolutely not certain about the manifestation of such a scaling impression which shall reinforce the connection between temperature and dew point dynamics and SOC, To study the detailed dynamics and to support the explanation of such behaviour, causal relationship with the help of SSA, MSSA [8] [9] and Correlation of Probability of Recurrence (CPR) depending on recurrence plot [10] have been studied in between the two signals for all the stations.
II. PROBABLE SOC STRUCTURE IN TEMPERATURE AND DEW POINT

Due to rise in temperature, concentrations of water vapour in the surrounding become higher. The processes, the rising temperature and concentration of water vapour boost the overall surrounding temperature resulting in atmosphere warming, which in turn father unstable deportation. The effect of this deportation can be stated in two different ways. Firstly, the atmosphere will start cooling as temperature decays exponentially. Alternatively at the same instant, the volume of atmospheric dust also rises. These two processes amalgamate as the temperature of the atmosphere starts cooling and water vapour starts to condense on the dust particles at dew point temperature. Latent heat will be released resulting an increase in surface temperature of the Earth, simultaneously, the atmospheric dust will be lowered, and a new cycle will begin. Hence it is apparent that the system might organise itself in a critical state with avalanches of change at all sizes via which dissipation manifests itself through cycles. These avalanches may be considered to be similar with many actual systems and phenomena including sand pile model earthquake [11], WWW (World Wide Web) [12], and spreading of family names [13] which are claimed to be fitted into the SOC structure. This statistically steady-state behaviour is a signature of SOC. It can be distinguished by hyperbolic or power law distributions of various quantities [14]: that of the relative frequency $R$ of avalanches releasing energy (or mass) $\xi$, i.e.

$$R(\xi) \sim (1/\xi)^\beta.$$  \hspace{1cm} (1)

These distribution laws for that particular dynamics indicate the unavailability of any characteristic length of time scale, and are called as the celebrated scale invariance of SOC systems.

Distributions $D(x)$ of temperature or dew point events with property $x$ that behave like $D(x) \sim x^{-\alpha}$ are most effectively analyzed by taking the help of the integrated (cumulative) distribution

$$D(x) = \int_0^x \, dD(x) = \int_0^x \, D(x) \, dx.$$  \hspace{1cm} (2)

$M$ is the best possible event found in the data set.

To stay away from data fluctuations in the low (high) value regime derived by the choice of logarithmic (linear) bins, the integrated representation is used instead of histograms.

If we consider $M \rightarrow \infty$ and $\alpha > 1$ then it can be written that

$$D(x) \sim x^{-\alpha + 1}.$$  \hspace{1cm} (3)

Our temperature or dew point data sets are normally within the range $1 < t < 5961$ days and $1 < x < 98 \, ^{\circ}F$. Hence, $M$ cannot be replaced by $\infty$ in Eq. (2) and obtain

$$D(x) := \frac{1}{x^\alpha} \left[ 1 - \left( \frac{x}{M} \right)^{(\alpha - 1)} \right].$$  \hspace{1cm} (4)

Hence, the log-log plot of $D(x)$ versus $x$ absolutely deviates from a straight line as $x$ comes nearer to $M$ and it is seen that the distribution of all temperature or dew point data sets follow power law behaviour.

III. CAUSALITY TEST BASED ON THE SINGULAR SPECTRUM ANALYSIS

As stated above that the distribution of all temperature or dew point data sets follow probable SOC structure, a frequently asked question one has to encounter in analysis of time series is whether one climatic variable is predictable based on the another. However [15] has proposed one method to address this question. The limitations of Criteria for Granger causality are manifold [16]. Though there are some nonlinear [17] as well as some nonparametric [18] extensions in the literature, these method also has many disadvantages [19] [20] [21]. Moreover there are also many other alternative methods but they are hardly used. By the singular spectrum analysis (SSA) technique one can overcome many of these difficulties.

A. A brief description of the Basic SSA

A detailed discussion about the SSA can be found in [9] [8]. A short discussion on the modus operandi of the SSA technique is as follows:

Consider a time series $X_r = (x_1, ..., x_r)$ . Fix the window length $L(\leq T/2)$ such that $K = T - L + 1$.

Step1. Formation of the trajectory matrix: the one-dimensional time series $X_r = (x_1, ..., x_r)$ is transformed into the multi-dimensional $L$-lagged vectors or, simply, lagged vectors $(Y_1, ..., Y_K)$ where $Y_i = (x_{i-L+1}, ..., x_i) \in \mathbb{R}^L$. The sole parameter of the imbedding is $L$, where $L$ is the length of window ,an integer such that, $2 \leq L \leq N$. The output of this step is the trajectory matrix $Y = (Y_1, ..., Y_K) = [y_{ij}]_{i=1}^{K}$. \hspace{1cm}


Step3. Singular Value Decomposition (SVD): Eigen values and eigenvectors of the matrix $YY^T$ are computed and are represented in the form $YY^T = \Lambda \Lambda^T$. Where $\Lambda = \text{diag}(\lambda_1, ..., \lambda_L)$ is the diagonal matrix of eigen values of $YY^T$ are put in order so that $\lambda_1 \geq \lambda_2 \geq ... \lambda_L \geq 0$ and $H = (H_1, H_2, ..., H_L)$ is the corresponding orthogonal matrix of eigenvectors of $YY^T$.

Step4. Selection of eigenvectors: select a group of $l (1 \leq l \leq L)$ eigenvectors $H_{i_1}, H_{i_2}, ..., H_{i_l}$. The grouping is done on the basis of dividing the elementary matrices $Y_i$ into many classes and calculating the sum of the matrices within each $i_1, ..., i_l$. Then the matrix $Y_i$ corresponding to the group $I$ is defined by $y_{i_k} = y_{i_1} + ... + y_{i_l}$.

Step5. Reconstruction: compute the matrix $\tilde{Y} = \sum_{i=1}^{l} H_{i_k} H_{i_k}^T Y$ as an approximation to $Y$. By
averaging over the diagonals of the matrix \( \bar{Y} \) transition to the one-dimensional series can now be achieved.

Define abbreviations and acronyms the first time they are used in the text, even after they have been defined in the abstract. Abbreviations such as IEEE, SI, MKS, CGS, sc, dc, and rms do not have to be defined. Do not use abbreviations in the title or heads unless they are unavoidable.

B. Causality Test

Causality test implements Multivariate Singular Spectral Analysis (MSSA) [22] [23] which is an annex of the basic SSA applied to multivariate time series. Let SSA applied to multivariate time series.

Analysis (MSSA) \[22\] \[23\] which is an annex of the basic SSA, as called Multivariate Singular Spectral Analysis (MSSA) \[22\] \[23\].

Let \( \Delta = \max \left( \tilde{X}_{SSA}^{R} \right) \) and \( \Delta = \max \left( \tilde{X}_{MSSA}^{R} \right) \). So now \( \tilde{X}_{SSA}^{R} = \tilde{X}_{SSA}^{R} \) \( \tilde{X}_{SSA}^{R} \) and \( \tilde{X}_{MSSA}^{R} = \tilde{X}_{MSSA}^{R} \) \( \tilde{X}_{MSSA}^{R} \) which are of length \( N - n + 1 \).

Step 3: SSA is performed on the new \( \tilde{X}_{SSA}^{R} = \tilde{X}_{SSA}^{R} \) \( \tilde{X}_{SSA}^{R} \) and \( \tilde{X}_{MSSA}^{R} = \tilde{X}_{MSSA}^{R} \) \( \tilde{X}_{MSSA}^{R} \) to obtain the \( \tilde{X}_{SSA}^{R} = \tilde{X}_{SSA}^{R} \) \( \tilde{X}_{SSA}^{R} \) and \( \tilde{X}_{MSSA}^{R} = \tilde{X}_{MSSA}^{R} \) \( \tilde{X}_{MSSA}^{R} \) which are also of length \( N - n + 1 \).

Step 4: If \( \left| \tilde{X}_{SSA}^{R} - \tilde{X}_{MSSA}^{R} \right| > 0 \), Step 2 to step 4 is repeated.

Step 5: The \( \tilde{X}_{SSA}^{R} \) and \( \tilde{X}_{MSSA}^{R} \) of step 2 are now modified into two parts \( X_{SSA}^{R} \) and \( X_{MSSA}^{R} \) where \( X_{SSA}^{R} = x_{1}, \ldots, x_{N-n} \) and \( X_{MSSA}^{R} = x_{1}, \ldots, x_{N-n} \). Similarly \( Y_{SSA}^{R} \) and \( Y_{MSSA}^{R} \) can be obtained.

Let \( X_{SSA}^{R} \) and \( X_{MSSA}^{R} \) are the trajectory matrices of \( X_{SSA}^{R} = x_{1}, \ldots, x_{N-n} \) and \( X_{MSSA}^{R} = x_{1}, \ldots, x_{N-n} \). \( X_{SSA}^{R} \) is used to reconstruct noise free \( \tilde{X}_{SSA}^{R} \) using SSA along with trajectory matrix \( X_{SSA}^{R} \). Since \( X_{SSA}^{R} \) and \( Y_{SSA}^{R} \) are the trajectory matrices of \( X_{SSA}^{R} \) and \( Y_{SSA}^{R} \) then the joint trajectory matrix \( H_{SSA}^{R} \) will be given as

\[
H_{SSA}^{R} = \begin{bmatrix} X_{SSA}^{R} \\ Y_{SSA}^{R} \end{bmatrix}
\]

\( X_{SSA}^{R} \) and \( Y_{SSA}^{R} \) are used to reconstruct \( \tilde{X}_{SSA}^{MSSA} \) using MSSA along with the trajectory matrix \( H_{SSA}^{R} \). \( \tilde{X}_{SSA}^{R} \) is used to forecast the next \( n \) data points \( \tilde{X}_{SSA}^{R} \). Similarly \( \tilde{X}_{SSA}^{MSSA} \) is used to forecast the next \( n \) data points \( \tilde{X}_{MSSA}^{R} \). Iterated forecasting algorithm is used for estimating the \( \tilde{X}_{SSA}^{F} = \tilde{X}_{SSA}^{R} \), \( \tilde{X}_{SSA}^{R} \), \( \tilde{X}_{SSA}^{F} \), \( \tilde{X}_{SSA}^{R} \) which is as follows:

Step 1: \( \tilde{X}_{SSA}^{F} = \tilde{X}_{SSA}^{R} \) \( \tilde{X}_{SSA}^{R} \) and \( \tilde{X}_{MSSA}^{F} = \tilde{X}_{MSSA}^{R} \) \( \tilde{X}_{MSSA}^{R} \) which are of length \( N - n + 1 \).

Step 2: Let \( \tilde{X}_{SSA}^{F} = \tilde{X}_{SSA}^{F} \) \( \tilde{X}_{SSA}^{F} \) and \( \tilde{X}_{MSSA}^{F} = \tilde{X}_{MSSA}^{F} \) \( \tilde{X}_{MSSA}^{F} \) which are also of length \( N - n + 1 \).

Step 3: SSA is performed on the new \( \tilde{X}_{SSA}^{F} = \tilde{X}_{SSA}^{F} \) \( \tilde{X}_{SSA}^{F} \) and \( \tilde{X}_{MSSA}^{F} = \tilde{X}_{MSSA}^{F} \) \( \tilde{X}_{MSSA}^{F} \) to obtain the \( \tilde{X}_{SSA}^{F} = \tilde{X}_{SSA}^{F} \) \( \tilde{X}_{SSA}^{F} \) and \( \tilde{X}_{MSSA}^{F} = \tilde{X}_{MSSA}^{F} \) \( \tilde{X}_{MSSA}^{F} \) which are also of length \( N - n + 1 \).

Step 4: If \( \left| \tilde{X}_{SSA}^{R} - \tilde{X}_{MSSA}^{R} \right| > 0 \), Step 2 to step 4 is repeated.

Step 5: The \( \tilde{X}_{SSA}^{F} \) and \( \tilde{X}_{MSSA}^{F} \) of step 2 are now modified into two parts \( X_{SSA}^{F} \) and \( X_{MSSA}^{F} \) where \( X_{SSA}^{F} = x_{1}, \ldots, x_{N-n} \) and \( X_{MSSA}^{F} = x_{1}, \ldots, x_{N-n} \). Similarly \( Y_{SSA}^{F} \) and \( Y_{MSSA}^{F} \) can be obtained.

Let \( X_{SSA}^{F} \) and \( X_{MSSA}^{F} \) are the trajectory matrices of \( X_{SSA}^{R} = x_{1}, \ldots, x_{N-n} \) and \( X_{SSA}^{R} = x_{1}, \ldots, x_{N-n} \). \( X_{SSA}^{F} \) is used to reconstruct noise free \( \tilde{X}_{SSA}^{F} \) using SSA along with trajectory matrix \( X_{SSA}^{R} \). Since \( X_{SSA}^{F} \) and \( Y_{SSA}^{F} \) are the trajectory matrices of \( X_{SSA}^{F} \) and \( Y_{SSA}^{F} \) then the joint trajectory matrix \( H_{SSA}^{F} \) will be given as

\[
H_{SSA}^{F} = \begin{bmatrix} X_{SSA}^{F} \\ Y_{SSA}^{F} \end{bmatrix}
\]

\( X_{SSA}^{F} \) and \( Y_{SSA}^{F} \) are used to reconstruct \( \tilde{X}_{SSA}^{MSSA} \) using MSSA along with the trajectory matrix \( H_{SSA}^{F} \). \( \tilde{X}_{SSA}^{R} \) is used to forecast the next \( n \) data points \( \tilde{X}_{SSA}^{R} \). Similarly \( \tilde{X}_{SSA}^{MSSA} \) is used to forecast the next \( n \) data points \( \tilde{X}_{MSSA}^{R} \). Iterated forecasting
If $F_{x \to y} < 1$ and $F_{y \to x} < 1$ it can be concluded that a feedback system exist between $X$ and $Y$ which is called F-feedback (forecasting feedback), and $X$, $Y$ are mutually supportive for an F-feedback system.

IV. Correlation of Probability of Recurrence

In this study the connections between temperature and dew point data sets have been estimated from the repetitiveness of systems which is dynamical in nature. The correlation probability of recurrence also known as CPR [10] is used, which is based on recurrence plots (RP) [24] [25] and was initially implemented to determine phase synchronization between non-stationary and non-phase-coherent time series data set. The detailed descriptions of RP are available in a considerable number of literatures. In brief, Eckmann [26] launched a graphical tool called Recurrence Plot (RP) with a motive to obtain qualitative feature of a time series data set. It shows all the instances when the phase space trajectory [10] [27] of the system traverse almost the same region in the phase space. At a different instant of time $j$, the recurring of $i$ state is depicted in a two dimensional squared matrix with black and white dots. A recurrence is represented by the black dots and time is depicted by both the axes. This type of a recurrence plot (RP) can be mathematically demonstrated by

$$R_{i,j} = H(x_i - s_j), \quad x_i \in \mathbb{R}^d, i,j = 1, \ldots, N$$  \hspace{1cm} (8)$$

$R_{i,j}$ is the Recurrence Plot (RP), and $N$ denotes the number of considered states $s_i$. Threshold distance $\|s_i - s_j\|$ norm is represented by $H$, $H(\ast)$ is the Heaviside step function.

Using the one dimensional time series $x_i$ of a single observable variable (temperature or dew point), a phase space trajectory $S_i$ can be reconstructed by a method called Taken’s Time Delay method [28] as, $S_i = (x_i, x_{i+d}, \ldots, x_{i+(d-1)d})$

where $d$ denotes embedding dimension and $\tau$ is the time delay. A method based on False Nearest Neighbours [29], Mutual Information [30] respectively is used to estimate the embedding dimension $d$ and the delay or lag $\tau$.

A. Probability of Recurrence

From Probability of recurrence the rate of dark points ($R_{i,j} = 1$) per line parallel to the main diagonal are simply obtained and can be given by

$$P(\tau) = \frac{1}{N - \tau} \sum_{i=1}^{N-\tau} R_{i,i+\tau}$$  \hspace{1cm} (9)$$

where the number of the states is represented by $N$. In other words, an estimation of the probability is given by $P(\tau)$ which states that a system will return to a certain state after a delay of $\tau$.

B. Correlation of Probability of Recurrence(CPR)

The Correlation of Probability of Recurrence (CPR) [31] is defined as the cross-correlation coefficient between the probabilities of recurrence of two trajectories $\bar{x}$ and $\bar{y}$:

$$CPR = \langle \bar{P}_x(\tau) \bar{P}_y(\tau) \rangle$$  \hspace{1cm} (10)$$

$\bar{x}$ is the series $x$ normalized to zero mean and standard deviation one and $\langle \rangle$ represents the expectation value. To calculate CPR correctly, only $p(\tau)$ have to be considered in such a way that $p(\tau)$ for such $\tau$ larger than the auto correlation time $\tau_c$ of the system at which the auto correlation function of the system falls to $1/e$. Hence Eq. (10) becomes

$$CPR = \langle \bar{P}_x(\tau > \tau_c) \bar{P}_y(\tau > \tau_c) \rangle$$  \hspace{1cm} (11)$$

Where,

$$\tau_c = \max [\tau_c(\bar{x}), \tau_c(\bar{y})]$$  \hspace{1cm} (12)$$

The CPR values are binned in three categories: (a) strong connectedness ($|CPR| > 0.8$), (b) moderate connectedness $(0.5 < |CPR| < 0.8)$, and (c) weak connectedness ($|CPR| < 0.5$).

V. Result and Discussion

The temperature and dew point distribution has been devised in equation (4). Then equation (4) has been solved for $\alpha$. Values of $\alpha$ for Temperature and Dew Point series for seven different stations are illustrated in Table 1. The data shows an almost perfect scaling behaviour as explained by Eq. (4), with $\alpha \approx 1$. This result should be considered as a strong affirmation towards possible presence of SOC dynamics in the signals. This theory has been proposed as a description for power-law scaling noticed in the signals under test. Now it is very much necessary to conceptualize the extension of the contribution of theory of SOC in understanding the concealed causes of the observed power-law behaviour in complex dynamical systems like these signals.

<table>
<thead>
<tr>
<th>Station</th>
<th>$\alpha$ (Temperature)</th>
<th>$\alpha$ (Dew point)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kolkata</td>
<td>1.225 ± 0.0002</td>
<td>1.233 ± 0.0003</td>
</tr>
<tr>
<td>Chennai</td>
<td>1.223 ± 0.0002</td>
<td>1.232 ± 0.0002</td>
</tr>
<tr>
<td>New Delhi</td>
<td>1.225 ± 0.0002</td>
<td>1.237 ± 0.0002</td>
</tr>
<tr>
<td>Mumbai</td>
<td>1.225 ± 0.0002</td>
<td>1.234 ± 0.0002</td>
</tr>
<tr>
<td>Bhopal</td>
<td>1.224 ± 0.0002</td>
<td>1.240 ± 0.0002</td>
</tr>
<tr>
<td>Agartala</td>
<td>1.227 ± 0.0002</td>
<td>1.234 ± 0.0002</td>
</tr>
<tr>
<td>Ahmedabad</td>
<td>1.224 ± 0.0003</td>
<td>1.237 ± 0.0002</td>
</tr>
</tbody>
</table>

The current experimental evidence still demands a through experiment with respect to a possible causal relation of the emergent power laws to an underlying self-organized critical state. With this notion for seven different stations the causality between the pair of temperature-dew point time series to be quantitatively and qualitatively estimated with the
help of MSSA considering SSA as the basic background. The proper selection of the window length \((L)\) and the number of significant Eigen triples \((I)\) is essential to test the causality between two signals using SSA. These selections differ for different signals and the purpose of the analysis. The most appropriate value of the window length is that value of \(L\) for which the points of different lagged vectors \(X_m\) and \(X_n\) \((m \neq n)\) are linearly independent. It is obtained from the correlogram as the value of the lag for which the autocorrelation function (ACF) crosses for the first time the standard Gaussian confidence interval (95% CI) \([32]\). The optimized Eigen triple grouping is done employing a ratio term \((\mathcal{R}_i)\) given by:

\[
\mathcal{R}_i = \frac{\lambda_i}{\sum_{j=1}^{I} \lambda_j} \times 100
\]  

(13)

where \(\mathcal{R}_i\) is an estimation of the percentage of the energy contribution of the \(i\)th principal component of the signal under investigation. The number of components, \(I_{\text{max}}\) up to which high content of energy is available, is considered as the optimized number of elements in the group and the first \(I_{\text{max}}\) Eigen triples are chosen as the most significant Eigen triples \((i.e. I = 1: I_{\text{max}})\). As per the above criteria the window lengths \((L)\) and the Eigen-triples \((I)\) chosen for the different signals are as represented in Table 2.

Taking \(X\) as the temperature signal and \(Y\) as the dew point signal the values of \(F_{X/Y}\) and \(F_{Y/X}\) are obtained choosing the above window lengths and the corresponding Eigen triples taking 65% of the total dataset of each signals to forecast the rest 35% of the data set of the corresponding signals. The values of \(F_{X/Y}\) and \(F_{Y/X}\) calculated and the corresponding remarks on causality are given in Table 3.

Table 2. Window length and Eigen triples of the different signals

<table>
<thead>
<tr>
<th>Station</th>
<th>Signal</th>
<th>Window length ((L))</th>
<th>Eigen triples ((I))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kolkata</td>
<td>Temperature</td>
<td>61</td>
<td>1:7</td>
</tr>
<tr>
<td></td>
<td>Dew point</td>
<td>87</td>
<td>1:8</td>
</tr>
<tr>
<td>Chennai</td>
<td>Temperature</td>
<td>74</td>
<td>1:4</td>
</tr>
<tr>
<td></td>
<td>Dew point</td>
<td>82</td>
<td>1:5</td>
</tr>
<tr>
<td>New Delhi</td>
<td>Temperature</td>
<td>86</td>
<td>1:5</td>
</tr>
<tr>
<td></td>
<td>Dew point</td>
<td>65</td>
<td>1:10</td>
</tr>
<tr>
<td>Mumbai</td>
<td>Temperature</td>
<td>83</td>
<td>1:3</td>
</tr>
<tr>
<td></td>
<td>Dew point</td>
<td>86</td>
<td>1:5</td>
</tr>
<tr>
<td>Bhopal</td>
<td>Temperature</td>
<td>71</td>
<td>1:4</td>
</tr>
<tr>
<td></td>
<td>Dew point</td>
<td>88</td>
<td>1:7</td>
</tr>
<tr>
<td>Agartala</td>
<td>Temperature</td>
<td>75</td>
<td>1:4</td>
</tr>
<tr>
<td></td>
<td>Dew point</td>
<td>82</td>
<td>1:5</td>
</tr>
<tr>
<td>Ahmedabad</td>
<td>Temperature</td>
<td>75</td>
<td>1:4</td>
</tr>
<tr>
<td></td>
<td>Dew point</td>
<td>90</td>
<td>1:7</td>
</tr>
</tbody>
</table>

With \(X\) as the temperature signals and \(Y\) as the dew point signals the values of \(F_{X/Y}\) are less than 1, the values of \(F_{Y/X}\) are greater than 1 and \(F_{X/Y} < F_{Y/X}\) for Kolkata, Chennai, Mumbai, Bhopal and Ahmedabad which implies that there is no feedback system between temperature signal and dew point signal and dew point is always more supportive for the forecasting the temperature. Whereas the values of \(F_{X/Y}\) and \(F_{Y/X}\) are both less than 1 and \(F_{X/Y} < F_{Y/X}\) for New Delhi and Agartala which implies that there is a feedback system between temperature signal and dew point signal, there is Mutual support between the two series for forecasting and dew point is always more supportive for the forecasting the temperature for these two stations. These causal relationships is quite supportive and analogous with the SOC dynamics in natural phenomena of cyclic turbulence in heat transfer and dissipation governed by moisture content in air and hence toggling between normal temperature and dew point.

Moreover, for real-world systems a satisfactory explanation would also need to be considered, for the measured inter-event correlations, which by themselves are key for improvement of predictions of catastrophic events. For this purpose the correlation of the probability of occurrence (CPR) in phase space has been studied. The CPR values between the pair of temperature-dew point time series for seven different stations are estimated with the help of recurrence plots (RP).

Table 3: Causality factors and causal relationship between Temperature and Dew Point series for seven Stations \((X: Temperature, Y: Dew point)\)

<table>
<thead>
<tr>
<th>Station</th>
<th>(F_{X/Y})</th>
<th>(F_{Y/X})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kolkata</td>
<td>0.5017</td>
<td>1.0004</td>
</tr>
<tr>
<td>Chennai</td>
<td>0.3162</td>
<td>1.0001</td>
</tr>
<tr>
<td>New Delhi</td>
<td>0.4423</td>
<td>0.9828</td>
</tr>
<tr>
<td>Mumbai</td>
<td>0.5203</td>
<td>1.0005</td>
</tr>
<tr>
<td>Bhopal</td>
<td>0.4603</td>
<td>1.0039</td>
</tr>
<tr>
<td>Agartala</td>
<td>0.4947</td>
<td>0.9948</td>
</tr>
<tr>
<td>Ahmedabad</td>
<td>0.4957</td>
<td>1.0088</td>
</tr>
</tbody>
</table>

From table3 we can state that for Kolkata, Chennai, Mumbai, Bhopal and Ahmedabad there is no feedback system between the two series but dew point is supportive for forecasting of temperature; in case of Delhi and Agartala feedback system exists with mutual support between the two series for forecasting and dew point is more supportive than Temperature. The results of CPR analysis are in Table 4.

Table 4: CPR values for Temperature and Dew Point series for seven Stations

<table>
<thead>
<tr>
<th>Station</th>
<th>CPR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kolkata</td>
<td>0.8067</td>
</tr>
<tr>
<td>Chennai</td>
<td>0.8226</td>
</tr>
<tr>
<td>New Delhi</td>
<td>0.8264</td>
</tr>
<tr>
<td>Mumbai</td>
<td>0.8261</td>
</tr>
</tbody>
</table>
The results clearly show tight connectedness (as | CPR | > 0.8) between the pair of temperature-dew point time series for all the major stations. In this study, the high values indicate that at similar times the two time series with a high CPR tend to recur, suggesting some similarity in their underlying dynamics. This again suggests a way to explain the dynamics of SOC systems which generally evolve through avalanches to a transitional critical state and then again recur to a previous configuration.

VI. CONCLUSION

The present finding that the daily mean temperature and dew point of seven geographically diverse locations may have a probable SOC structure in their distribution is quite supportive to some previous investigations done rigorously [33] [34] [35] [36]. The causality test on the signal pairs infer suitable knowledge about the interplay between the two signals in regards of thermodynamic turbulence or avalanches present in natural systems. The high values of correlation of probability of recurrence (CPR) in between the temperature-dew point pair suggest a strong correlation and similarity in the phase-space recurrence and quite informative about the state transition of the system toward criticality. The overall scenario is very much collaborative and suitable for short term (probably long term) prediction of such weather parameters.

REFERENCES