RECURRENT QUANTIFICATION ANALYSIS TO COMPARE THE
MACHINABILITY OF STEELS

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ABSTRACT
Machinability, though is a simple term, is difficult to generalize. But nevertheless, it can be realized as the ease or
difficulty with which a material can be machined. Assessing the machinability of various materials before they are used in
commercial manufacturing is very demanding, as the machinability affects the material removal rate, surface finish of the
workpiece, cutting power consumption and tool wear rate. The present work aims at establishing Recurrence
Quantification Analysis, a relatively new technique in the study of chaotic systems, as a potential tool to establish and
compare the machinability of steels. The technique has its roots in quantifying the Recurrence Plots obtained by the phase
space reconstruction of time domain signals. Variation in Determinism, one of the variables of the technique, is used as a
mean to establish the comparison of machinability.

Keywords: machinability, recurrence plot, recurrence quantification analysis, turning.

1. INTRODUCTION
The term machinability is often applied to work materials to describe their machining properties; it can
have several meanings depending on the cutting process under consideration. Any statement regarding
machinability may only apply under the particular set of circumstances existing when the observation was made. It
is still a dilemma as to which criterion or effect is to be considered to assess machinability; may it be tool wear,
surface finish or power requirements. However, experiences suggest that in finishing processes, tool wear
and surface finish are the most important considerations; in the case of roughing operations, tool wear and power
consumption are important [1].

Steel and its variants form the most widely used alloys today. Being so, establishing the machinability of
different steel is very essential. Many attempts have been made to obtain a quantitative measure of machinability- a
machinability index or number [1]. A method for determining such an index would be most helpful, particularly to steel manufacturers who must check the
machinability properties of their work materials. But however, no method of finding machinability index is
universally accepted, as machinability is not that simple an issue.

The present work proposes Recurrence Quantification Analysis, a relatively new technique in the study of nonlinear dynamic systems, as a potential tool to
establish the machinability of materials. In doing so, the materials considered were four varieties of steel, each
having different proportions of alloying elements. The cutting force signals in the feed direction were considered
for the analyses. To establish the credibility of the technique, surface finishes of the workpieces and power
drawn by the spindle motor during machining were also measured and compared.

1.1 Recurrence plots
Recurrence Plots (RPs) were first described by J. P. Eckmann, S. O. Kamphorst and D. Ruelle in
"Recurrence plots of dynamical systems" in 1987 [10]. RP is a technique by which we can qualitatively assess a time
series signal embedded in phase space. A recurrence plot can be represented as:

\[ R_{ij} = \Theta(\varepsilon_j - ||x_i - x_j||) ; i, j = 1…N \]

Where \( x_i \) stands for the point in phase space at which the system is situated at time \( i \), and \( \varepsilon_j \) is a predefined threshold
for whose selection are there criteria aplenty. \( || \cdot || \) is a norm used to calculate the distances between points in
phase space. \( \Theta \) is the heavyside function. The matrix corresponding to \( R_{ij} \) consists of values of only 1 and 0. RP
will ultimately be a black and white plot with time on both the axes. A black point in a RP means that the system
returns to an \( \varepsilon_j \)-neighbourhood of the corresponding point in phase space [2, 3]. This recurrence gives the name to
the method. There are a lot of variations of RP and one needs to look at the application in question before deciding
on the variant of RP to be used [2, 4]. Distance Plot (DP) is a common variant, where, instead of a black and white
plot one gets a coloured plot by coding the distances between the points in phase space to fall into different
ranges of distances which are suitably colour coded [2]. Figures 1, 2, 3 and 4 demonstrate obtaining DP and RP
from time series data.

While Figures 1 to 4 have sine wave as the underlying time series, random noise with a standard
deviation of 1 is the underlying time series for Figures 5 and 6. One can notice easily the difference in RPs of the
two cases. While it is the characteristic of deterministic signals to show diagonal lines in RP, scattered points in
RP is exhibited by random signals [2, 3, 5]. These are just
only two typical types of plots to pick from a large pool of
Recurrence Plots. RPs require quite a few criteria and
input parameters to be set carefully. The time delay (\( \tau \)) and
embedding dimension (m) required for state space embedding are obtained correspondingly by Mutual Information method [2, 6] and False Nearest Neighbour algorithm [6, 7]. Threshold for the signal in Figure-1 is 10% of the mean phase space diameter [2] whereas that for the signal in Figure-5 is 25% of the mean phase space diameter, only to get more number of points in the RP. There are numerous criteria to select a proper threshold for a given application [2, 3, 8, 9] even as there lies no fixed single method to select an appropriate threshold.

Figure-1. Time series of sine wave (frequency 4 Hz).

Figure-2. Phase space plot of sine wave.

Figure-3. Distance plot of sine wave (embedding dimension= 2, time delay= 6, Maximum norm).

Figure-4. Recurrence plot of sine wave (embedding dimension= 2, time delay= 6, threshold= 0.1, Maximum norm).

Figure-5. Random noise (standard deviation of 1).
It is possible to look at RPs and qualitatively assess the dynamics of the underlying system. But, it demands practice. There are some general guidelines present to interpret the behaviour of a system by observing its corresponding Recurrence Plot [2, 4, 9].

1.2 Recurrence quantification analysis

It’s always difficult to judge the status of a system just by observing the corresponding RP. Some means of quantification of RPs would make understanding the behaviour easy. Charles L. Webber et al. came up with a technique called Recurrence Quantification Analysis in 1992 which was based on quantifying the diagonal line structures present in RPs. In 2003, Norbert Marwan et al. successfully added to that the quantifications based on vertical line structures [2]. Some of the important variables in RQA are listed below [2].

a) Recurrence Rate (RR): Percentage of recurrence points in RP.

\[
RR = \frac{1}{N^2} \sum_{i=1}^{N} R_{ij}
\]

d) Entropy (ENTR): Shannon entropy of the probability distribution of diagonal line lengths.

\[
ENTR = -\sum_{i=1}^{N} p(i) \log p(i)
\]

e) Laminarity (LAM): Percentage of recurrence points which form vertical lines.

\[
LAM = \frac{\sum_{ij} v P(v)}{\sum_{ij} P(v)}
\]

f) Trapping Time (TT): Average length of vertical lines.

\[
TT = \frac{\sum_{ij} v P(v)}{\sum_{ij} P(v)}
\]

Figure-6. RP of random noise (embedding dimension = 6, time delay = 1, threshold = 1.6322, Euclidean norm).

Figure-7. RQA of a sine series with frequency 4 Hz (m=2, \(\tau = 2\), \(\varepsilon_1 = 0.0155\), Euclidean norm).

Figure-8. RP and RQA of random noise (m=3, \(\tau = 3\), \(\varepsilon_1 = 0.4951\), Euclidean norm).

Figure-7 shows the above discussed RQA variables for a sine series with a frequency of 4 Hz. The threshold is set so that RR is 1%. A DET value of 98.07% shows that the signal is deterministic. Figure-8 shows RQA of a random noise signal with a standard deviation of 1. Again, the criterion selected for setting the threshold is same as that considered for Figure-7.

RP of the noise signal is very scattered as can be seen from Figure-6, thus contrasting in nature RP of a deterministic signal such as sine series which is shown in Figure-4, where one gets structured diagonal lines. An interesting comparison can be made between the RQA variables of the sine series and that of the random noise drawing conclusions about the system dynamics. Very low value of DET for random noise (2.06% in Figure-8)
confirms that the system hardly has some determinism in it. This in turn will imply that the plot has no diagonal lines of considerable length. Hence, one gets a lower value of $L$ in this case. As $\text{ENTR}$ is dependent on the probability of distribution of diagonal line lengths, it will be lower for noisy signals. In case of random noise, the system is very agile. Hence, the system will hardly be laminar. $\text{LAM}$ shows very low values because of this very reason. For the very same cause, $\text{TT}$ will also be low for random noise. These variables give a feel of systems’ dynamic behaviour. All these behaviours are exactly opposite to that of a deterministic system, as can be noted from Figure-7.

There are few codes and softwares available for Recurrence Plots and Recurrence Quantification Analysis. Some are listed here:

- Visual Recurrence Analysis
- CRP toolbox for MATLAB
- Dataplore
- TISEAN
- Bios Analyzer

For all the RPs and RQA sighted in this work, CRP toolbox was used which is to be used with MATLAB. The toolbox was developed as part of the dissertation work of and by Dr. Norbert Marwan, University of Potsdam, Germany. The toolbox can be freely downloaded from the web location http://www.tocsy.agnld.uni-potsdam.de

2. EXPERIMENTAL SETUP

Figure-9 shows the scheme of the experimental setup for the present work. Machining was performed on a Panther 1530/1650 lathe. Kennametal, CNMG 120408 tool insert was used with standard tool holder PCLNR 2020 K12. Cutting forces were sensed using a Kistler dynamometer of type 9257 B on which the tool post was mounted. The dynamometer in turn was connected to Kisteler 9257 B charge amplifier. The force signals in three directions from the amplifier were acquired by National Instruments PXI-4472 eight channel data acquisition card at a sampling rate of 1000 Hz with Lab VIEW 8.0 being the software interface. Cutting force signals in the feed direction ($F_f$) were then taken as the inputs for the analyses with CRP toolbox. Surface finishes of the work pieces were measured using Mitutoyo SJ-301 surface testing machine. A conventional wattmeter was employed to measure the power drawn by the spindle motor during machining.

3. EXPERIMENTS

Four different types of steel rods, whose compositions are given in Table-1, were commercially available and procured.

It can be noted from Table-1 that the elements forming the four materials are same, but the proportions of the elements are different from material to material. The steels were named as A, B, C and D for convenience.

In order to maintain the same cutting conditions for all the experiments, the diameter of the workpieces for all set of experiments were kept 16mm and the length between the centres of the lathe was kept 180 mm. Four different combinations of spindle speed, feed and depth of cut were selected for the experiments. The different combinations are listed in Table-2.

The table also lists the material removal rate (MRR) for these combinations. The table indicates that the experiments constitute a range of MRRs. The cutting forces in the three mutually perpendicular directions were sensed by the dynamometer and acquired by the mentioned data acquisition system. For the analyses

![Figure-9. Scheme of experimental setup.](image-url)
carried out in this work, cutting force signals in the feed direction were considered, as they were having the predominant magnitude over the other two.

The cutting feed force signals acquired were taken into the CRP Toolbox for analyses. The criterion selected for choosing the threshold for RP and RQA was to keep the recurrence rate constant at 1% (fixed RR) for all the analyses. Surface finishes of the work materials were measured after every experiment. Spindle power drawn was also noted down for every trial. Among the available many variables of RQA, it was found that DET is the most consistent variable and gives a very good trend for all the analyses. Surface finishes of the work materials were measured after every experiment. Spindle power drawn was also noted down for every trial. Among the available many variables of RQA, it was found that DET is the most consistent variable and gives a very good trend with variations in cutting conditions. Hence, the results discussed here are dealing with the variation of only DET.

Table-1. Compositions of the four different steels.

<table>
<thead>
<tr>
<th>Composition</th>
<th>Steel A</th>
<th>Steel B</th>
<th>Steel C</th>
<th>Steel D</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.09</td>
<td>0.071</td>
<td>0.071</td>
<td>0.072</td>
</tr>
<tr>
<td>Mn</td>
<td>1.05</td>
<td>1.05</td>
<td>1.09</td>
<td>1.08</td>
</tr>
<tr>
<td>Si</td>
<td>0.03</td>
<td>0.024</td>
<td>0.039</td>
<td>0.02</td>
</tr>
<tr>
<td>P</td>
<td>0.49</td>
<td>0.052</td>
<td>0.044</td>
<td>0.049</td>
</tr>
<tr>
<td>S</td>
<td>0.288</td>
<td>0.293</td>
<td>0.288</td>
<td>0.266</td>
</tr>
<tr>
<td>Sn</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>Ni</td>
<td>0.005</td>
<td>0.007</td>
<td>0.007</td>
<td>0.006</td>
</tr>
<tr>
<td>Cr</td>
<td>0.018</td>
<td>0.041</td>
<td>0.041</td>
<td>0.032</td>
</tr>
<tr>
<td>Cu</td>
<td>0.084</td>
<td>0.086</td>
<td>0.086</td>
<td>0.084</td>
</tr>
<tr>
<td>Mo</td>
<td>0.001</td>
<td>0.001</td>
<td>0.003</td>
<td>0.002</td>
</tr>
<tr>
<td>Al</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.004</td>
</tr>
<tr>
<td>Pb</td>
<td>0.306</td>
<td>0.293</td>
<td>0.267</td>
<td>0.288</td>
</tr>
<tr>
<td>O (ppm)</td>
<td>85</td>
<td>84</td>
<td>60</td>
<td>95</td>
</tr>
<tr>
<td>Fe (Bal %)</td>
<td>97.634</td>
<td>98.078</td>
<td>98.06</td>
<td>98.096</td>
</tr>
</tbody>
</table>

Table-2. Combinations of cutting parameters.

<table>
<thead>
<tr>
<th>Speed (rpm)</th>
<th>Feed (mm/rev)</th>
<th>Depth of cut (mm)</th>
<th>MRR (mm³/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combination 1</td>
<td>1250</td>
<td>0.0281</td>
<td>1.25</td>
</tr>
<tr>
<td>Combination 2</td>
<td>1250</td>
<td>0.0421</td>
<td>1.00</td>
</tr>
<tr>
<td>Combination 3</td>
<td>1250</td>
<td>0.06</td>
<td>0.75</td>
</tr>
<tr>
<td>Combination 4</td>
<td>1250</td>
<td>0.075</td>
<td>0.50</td>
</tr>
</tbody>
</table>

4. RESULTS, DISCUSSIONS AND CONCLUSIONS

Tables 3 to 6 summarize the results for cutting parameters combinations 1, 2, 3 and 4, respectively. Every Table lists the DET values for all the types of steels for that particular cutting combination. Besides, the tables also list the values of surface finish of the work materials and power drawn by the spindle motor while machining.

Table-3. Results for combination 1 of cutting parameters.

<table>
<thead>
<tr>
<th>Steel</th>
<th>DET</th>
<th>Surface finish R₄ in microns</th>
<th>Spindle power drawn (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>65.31</td>
<td>3.97</td>
<td>350</td>
</tr>
<tr>
<td>B</td>
<td>72.26</td>
<td>2.86</td>
<td>290</td>
</tr>
<tr>
<td>C</td>
<td>73.21</td>
<td>2.45</td>
<td>280</td>
</tr>
<tr>
<td>D</td>
<td>71.16</td>
<td>3.04</td>
<td>285</td>
</tr>
</tbody>
</table>

Table-4. Results for combination 2 of cutting parameters.

<table>
<thead>
<tr>
<th>Steel</th>
<th>DET</th>
<th>Surface finish R₄ in microns</th>
<th>Spindle power drawn (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>65.96</td>
<td>3.18</td>
<td>315</td>
</tr>
<tr>
<td>B</td>
<td>66.74</td>
<td>2.58</td>
<td>295</td>
</tr>
<tr>
<td>C</td>
<td>72.76</td>
<td>1.86</td>
<td>280</td>
</tr>
<tr>
<td>D</td>
<td>66.51</td>
<td>2.60</td>
<td>310</td>
</tr>
</tbody>
</table>

Table-5. Results for combination 3 of cutting parameters.

<table>
<thead>
<tr>
<th>Steel</th>
<th>DET</th>
<th>Surface finish R₄ in microns</th>
<th>Spindle power drawn (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>69.89</td>
<td>3.30</td>
<td>285</td>
</tr>
<tr>
<td>B</td>
<td>66.20</td>
<td>2.51</td>
<td>285</td>
</tr>
<tr>
<td>C</td>
<td>72.36</td>
<td>1.96</td>
<td>285</td>
</tr>
<tr>
<td>D</td>
<td>65.98</td>
<td>2.52</td>
<td>290</td>
</tr>
</tbody>
</table>

Table-6. Results for combination 4 of cutting parameters.

<table>
<thead>
<tr>
<th>Steel</th>
<th>DET</th>
<th>Surface finish R₄ in microns</th>
<th>Spindle power drawn (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>67.53</td>
<td>3.20</td>
<td>285</td>
</tr>
<tr>
<td>B</td>
<td>65.16</td>
<td>2.78</td>
<td>290</td>
</tr>
<tr>
<td>C</td>
<td>73.18</td>
<td>2.30</td>
<td>270</td>
</tr>
<tr>
<td>D</td>
<td>65.68</td>
<td>2.47</td>
<td>290</td>
</tr>
</tbody>
</table>

For the same cutting conditions, better machinability implies smooth progress of the machining process and this results in very regular cutting force signals being produced, whereas low machinability implies rough machining, resulting in irregular cutting force signals. This very nature of the force signals gets reflected in the DET values. For good machining conditions, DET attains higher values; for poor machining conditions, DET slumps to lower values. This behaviour can be seen from the results tables.

In Table-3, DET attains a highest value for Steel C and lowest value for Steel A. The order of machinability of the materials in the decreasing note can thus be put as CBDA. This result is well complimented by the R₄ values.

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as well. Even though the power-drawn values are not showing the exact order, the best is Steel C with lowest power requirement whereas Steel D sits at the worst end. DET and $R_a$ values show that the machinability of B and D are very close. In Table-4, the order of machinability as indicated by DET values are again CBDA and both $R_a$ power-drawn compliment this order. Again, B and D are very close. For cutting parameters’ combination 3 (Table-5), DET values show an order of CABD, which is complimented by the power-drawn values. Again, even $R_a$ values also agree that C is the best machinable material. And B and D are very close once more. For the combination 4 of cutting parameters (Table-6), DET values indicate the machinability order of CADB which is well complimented by the values of power-drawn. However, A, B and D are very close in this case of machinability, as indicated by all the three determinants, but supremacy of C is again nonpareil. $R_a$ once more confirms this.

It is an interesting observation that the first two combinations of cutting parameters yield lower MRR relatively and the latter two yield relatively higher values of MRR. In the first two cases, $R_a$ values have complimented the results shown by DET, whereas in the latter two cases, power-drawn values have complimented the DET vales. This is in line with the statements made in section 1.0 that for finishing operations (low MRR), surface finish is can be considered as the criterion for determining machinability whereas for roughing operations (high MRR), power drawn can be considered as the criterion for determining machinability [1]. However, it has to be noted that MRR did not vary over a really wide range in these sets of experiments. From all the four different sets of experiments, it is clear that Steel C has the highest machinability under different cutting conditions. DET, $R_a$ and spindle power-drawn values simultaneously agree upon this. But, the order of machinability of Steels A, B and D vary with cutting conditions employed and the machinability of B and D are very close to each other.

From the above results and discussions, it can be concluded that the technique of Recurrence Quantification Analysis is very sensitive to changes in machining conditions and can be comprehensively employed to establish and compare the machinability of steels of different types.

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Free license for CRP Toolbox was given by Dr. Norbert Marwan, University of Potsdam, Germany under GNU. Dr. Marco Thiel, Nonlinear Dynamics Group, University of Potsdam, Germany shared his knowledge, research works and publications with us. The experimental facilities were provided by Technical Education Quality Improvement Programme (TEQIP), National Institute of Technology Karnataka, Surathkal, India

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