Technical Note

Study of phase synchronization in multichannel seizure EEG using nonlinear recurrence measure

D. Rangaprakash a,*, N. Pradhan b

a Department of Electrical Communication Engineering, Indian Institute of Science, Bangalore 560012, India
b Department of Psychopharmacology, National Institute of Mental Health and Neurosciences, Bangalore 560029, India

A R T I C L E   I N F O

Article history:
Received 14 November 2012
Received in revised form
22 November 2013
Accepted 26 February 2014
Available online 4 April 2014

Keywords:
Recurrence plot
Phase synchronization
Correlation between probabilities of recurrence
Multichannel EEG
Linear correlation
Epileptic seizure
Brain headmap

A B S T R A C T

Complex biological systems such as the human brain can be expected to be inherently nonlinear and
difficult to model. Most of the previous studies on investigations of brain function have either used
linear models or parametric nonlinear models. In this paper, we propose a novel application of a nonlinear
measure of phase synchronization based on recurrences, correlation between probabilities of recurrence
(CPR), to study seizures in the brain. The advantage of this nonparametric method is that it makes very
few assumptions thus making it possible to investigate brain functioning in a data-driven way. We have
demonstrated the utility of CPR measure for the study of phase synchronization in multichannel seizure
EEG recorded from patients with global as well as focal epilepsy. For the case of global epilepsy, brain
synchronization using thresholded CPR matrix of multichannel EEG signals showed clear differences
in results obtained for epileptic seizure and pre-seizure. Brain headmaps obtained for seizure and pre-
seizure cases provide meaningful insights about synchronization in the brain in those states. The headmap
in the case of focal epilepsy clearly enables us to identify the focus of the epilepsy which provides certain
diagnostic value. Comparative studies with linear correlation have shown that the nonlinear measure
CPR outperforms the linear correlation measure.

© 2014 Elsevier Ltd. All rights reserved.

1. Introduction

EEG signals, a record of electrical potentials obtained from the
human brain, are complex and random looking signals, which can
be analyzed as output of a stochastic process, i.e. system driven by
unknown input. Based on this approach, spectral estimation and
several other techniques have been developed for analyzing these
signals. Several models have also been developed. However, one
limitation of this approach is that it bears little or no consideration
to the nonlinear process that generates the signal. Another limita-
tion is that the models developed for these sophisticated biological
domains do not represent the biological system to any reasonable
extent. Hence, there is a need to analyze these signals with a differ-
tent perceptive. In this work, we are concerned mainly in applying
certain nonlinear techniques, which are also nonparametric, for the
analysis of EEG signals.

One of the most common neurological disorders is epilepsy
which is characterized by sudden and large neuronal discharges
in the brain [1]. Epilepsy is an indication of excessive synchronous
activity of neurons in the brain [1] and it clinically manifests as
seizure. Generalized seizures involve almost entire brain, whereas
focal seizures are localized to a particular region of the brain (called
epileptic focus). Clinical neurophysiologists have to review the EEG
recordings and look for seizures that may have occurred during the
monitoring session. However, reviewing a continuous EEG recor-
ding for seizures is a time consuming task, which is also prone
to human error. Thus a computer based seizure detection system
can be of great value for automatic identification of seizure EEG
segments. In this paper, we deal with a novel application of a recur-
rence based synchronization measure for seizure detection and
characterization using EEG recordings.

It has been observed that biomedical signals could be generated
by a nonlinear dynamical system [2]. In our work EEG signals are
analyzed as the output of a chaotic system rather than stochas-
tic system particularly in view of new developments in nonlinear
dynamics and chaos. Chaos is the unpredictable long time behavior
of a deterministic dynamical system which is highly sensitivity to
initial conditions. In case of chaotic systems, it is observed that
the system’s trajectory starting from a specific initial condition in state
space is different from another trajectory starting from a slightly
different initial condition. In a chaotic system, these two trajecto-
ries will soon start to diverge.

* Corresponding author. Present address: Electrical and Computer Engineering
Department, Auburn University, Auburn, AL, USA.
Tel.: +91 9535021951/+13342460604
E-mail address: abhi88isc@gmail.com (D. Rangaprakash).

http://dx.doi.org/10.1016/j.bspc.2014.02.012
1746-8094/© 2014 Elsevier Ltd. All rights reserved.
Since EEG signals are generated by complex nonlinear dynamical biomedical systems like brain, the number of state variables required to define these systems are not small and hence the number of data points to be considered becomes very large. In order to overcome this problem, we have used recurrence plot [3] based synchronization measure for analyzing EEG signals which come under the category of nonlinear dynamical techniques. Since we compute a distance matrix between points of the trajectory of given time series in a RP based technique, the measure permits the utilization of any kind of data and hence very useful for applications where the data could be nonstationary, or nonlinear.

EEG signals recorded under various experimental conditions have been analyzed using many nonlinear techniques. In particular, measures such as correlation dimension and Lyapunov exponents [2], which are based on nonlinear dynamical systems and chaos theory, have been used to describe the complex EEG signal. For computing invariants from the reconstructed attractors, chaos based approaches make certain general assumptions. It is assumed that the signal possesses a non-evolving low-dimensional attractor and it is also assumed that the signal is long, stationary and noiseless. To relax these assumptions, the Recurrence Plot (RP) method has been employed in the literature which also helps in visualizing the behavior of trajectories of dynamical systems in phase space [3]. The advantage of this method is that RPs have an apparent simplicity of implementation and interpretation, and do not require an understanding of system’s dynamical attractor. In view of these facts, a recurrence based phase synchronization technique, which is both nonlinear and non-parametric, has been employed in this paper.

The technique of RPs for data analysis is not only a useful tool for visualization but also for quantifying even for data sets with short length [4]. Furthermore, a set of quantification measures obtained from what is known as recurrence quantification analysis (RQA) quantify systematically the different structures found in RP [5]. One of the advantages of RQA resides in its elegant independence from constraining assumptions and limitations plaguing other analyses. Since recurrence structures are simply tallied within the signal (or between signals), there is no need to pre-condition the data. Recurrence analysis is not limited by signal non-stationarity and transients. Hence RQA has proven to be ideally suited for the study of numerous real-world systems encompassing a multitude of disciplines. RQA has found numerous applications in diverse fields [6–8] and its use in studying various biological systems [9,10] including brain [11] has been reported in literature. RQA can also deal with signals from nonlinear systems of any nature in quantifying the activity of system and is model-free. Thus, for analysis of physiological signals like EEG which are often nonstationary and depict nonlinearity, this tool is particularly suitable. Hence, RQA is a good choice to reveal changing dynamics of biomedical systems.

RQA quantifies the activity of a system independent of its dynamical nature as also deals directly with the signal without the need for system identification. Even though nonlinear dynamical technique has been applied to study epileptic EEG [12], the application of RQA to study EEG signals appears very promising for the reasons mentioned above.

Phase synchronization takes place between two signals when their respective frequencies and phases are locked. We are concerned here with the application of synchronization to seizure analysis. We would like to observe that phase synchrony does not imply seizure. However, during seizure we observe synchrony in the brain and the waveform patterns are characteristics of seizure. Also our interest in phase synchronization is due to the fact that many biological systems exhibits phase synchronization. It is well known that alpha band increases in synchrony in the occipital region of the brain when eyes are closed. It is also observed that many cognitive tasks involve synchronization. Synchronization patterns measured from EEG in many psycho physiological experiments have also been reported in literature [13] Synchronization is a universal phenomenon with global importance with its applications for chaotic oscillators being recognized [14,15] and subsequently studied by many researchers [16,17]. Literature contains several applications of this phenomenon to natural systems like magnetoencephalography [18], neuronal systems [19,20], cardiorespiratory systems [21] and ecological systems [22]. Efforts have been made in the application of nonlinear phase synchronization in seizure analysis. Lachaux et al. [23] have proposed a method for the quantification of synchronization using phase locking statistics to investigate recordings from an epileptic patient who is performing a visual discrimination task. Chavez et al. [24] have discussed about the estimation of phase synchronization from EEG signals recorded from an epileptic patient. Mormann et al. [25] have presented a technique for detection of pre-seizure state using mean phase coherence as a measure of phase synchronization. Le Van Quyen et al. [26] have discussed ways of characterizing neurodynamic changes in preictal state using nonlinear analysis of brain signals. EEG signals three groups viz. control, seizure and mania have been studied by Bhattacharya [27] to investigate the effect of these pathologies on the degree of phase synchronization between cortical areas. Studies have also been made on the multichannel seizure analysis using nonlinear measures. McSharry et al. [28] have introduced a nonlinear measure, multidimensional probability evolution, for multichannel seizure analysis. Gupta et al. [29] have shown that tracking changes in phase synchronization of narrow band activity of multichannel EEG can be useful for seizure onset detection and prediction. Bianchi et al. [30] have studied multichannel EEG synchronization by evaluating the mutual synchronization among several EEG channels using a measure called cross conditional correlated entropy. This paper discusses the application of a measure called Correlation between Probability of Recurrence (CPR) [31] which uses the concept of probability of recurrences in phase space. The definition of CPR has been extended here to study multichannel seizure EEG signals. The utility of nonlinear recurrence measure CPR has been demonstrated for both focal and global epilepsy cases using surface and contour plots of CPR matrix as well as through brain headmaps. The method employed here, being nonparametric, makes very few assumptions, making it suitable for studying brain function in a data driven way.

It is of interest to observe that many of the synchronization studies are based on linear correlation which has rather limited interpretability in the broader biological context. Specifically, it has been recognized that phase synchronization can be studied using distributed information processing in the brain occurring for many higher order brain processes [32]. While linear correlation can be used for detecting zero-lag first order phase synchronization, it cannot detect higher order and lagged synchronization [32,31]. Thus for a massively interconnected nonlinear complex system such as human brain, nonlinear methods seems better suited for synchronization studies in EEG. Hence the use of a nonlinear measure like CPR seems more appropriate for characterizing phase synchronization in the brain. This paper has validated this by showing that the nonlinear correlation measure CPR is superior in performance to the linear correlation measure. Here synchronization analysis of multichannel EEG signals has been made through a novel use of a nonlinear and non-parametric recurrence based phase synchronization measure.

2. Materials and methods

2.1. Data

We give an account of the multichannel EEG data used in this paper. 16 channel EEG was recorded from 16 subjects who had
been diagnosed with epilepsy. The data was recorded at 128 Hz sampling frequency for 45 s per subject. The 10–20 international system of electrode placement was used as the recording montage. Information about the seizure focus is obtained from the patient’s clinical diagnostic data. Data was collected with patient’s informed consent and after clearance from the Ethics committee.

The patients were recorded for clinical purpose on the requisition of the neurologist for approximately of 45 min to 1 h duration. During EEG recording, direct signal digitization were carried out from the real panel of Nihon Khoden Machine (Neurofax-21) using DT2841 ADC coupled with DT2740 Array processor to an IBM i386/387 PC. The DT 7020 array processor carried out online filtration and base line corrections. We have randomly taken 8 generalized and 8 focal epilepsy cases for this study. All 16 data records were visually screened off line on a computer. Only artifact free and clearly discernible seizure components were visually identified and then segmented for analyses. The identification of artifact both on paper and on digital data is performed by the treating neurologist by visual inspection.

2.2. Recurrence plots and phase synchronization

A signal is associated with a state space trajectory specific to it. In natural systems we often do not have access to all the state variables of the system from which the phase space trajectory is constructed. According to Takens [33], the procedure for mapping of a discrete signal x into phase space is as follows. Given a signal with Nt terms, x1, x2, ..., xNt, it is mapped into phase space by constructing vectors yj of dimension D with lag (delay) defined as

\[ y_j = [x_k, x_{k+n}, x_{k+2n}, ..., x_{k+(D-1)n}] \]  

Where \( D \geq 1, d > 1, j = 1 \) to \( N_t, k = j \mod N_t - (D - 1)d \)

We call this as embedding the signal in phase space of dimension D with lag d. The variable \( y_j \) gives the trajectory of the signal in D dimensional phase space and it completely represents the original state space topologically and thus it preserves all its properties. A recurrence occurs when two points \( y_i \) and \( y_j \) in phase space, corresponding to time instants \( i \) and \( j \), are close to each other based on a certain pre-defined norm. Since recurrence is a topological property of state space, it is preserved by phase space and hence further analysis on such a trajectory can be performed.

In order to visualize the recurrence of states in a phase space, Eckmann et al. [3] have introduced a tool called recurrence plot (RP). Usually, a phase space has higher dimensions and hence it can only be visualized directly by projection into two or three dimensional sub-spaces, which will not provide a global representation. However, the D-dimensional phase space trajectory can be investigated through a two-dimensional representation of its recurrences by an \( N \times N \) matrix whose elements take values 0 or 1. Each element of the matrix is mapped to a unit black or white pixel at the corresponding location, where black and white pixels represent the presence and absence of recurrences respectively. Thus we calculate an \( N \times N \) matrix

\[ R_{ij} = \theta(|y_i - y_j|) \]  

where \(|,|\) is a norm, \( \epsilon \) is the recurrence threshold and \( \theta \) is the Heaviside step function forcing \( R_{ij} \) to be either 0 or 1. Euclidean norm has been used throughout this work.

Choice of \( D, d \) and \( \epsilon \) is important. The value of \( D \) should be selected such that it is high enough to capture all state variables but it must be kept in mind that higher value of \( D \) increases the computation time. Many issues have been discussed on the applicability of nonlinear techniques to short non-stationary data like EEG. However, nonlinear techniques and computational procedures have been improved (e.g. false nearest neighbors and mutual information lag) to take care of many of these issues [34]. Many papers discuss the choice of these parameters [35,36]; we have chosen the values of \( D \) and \( d \) accordingly.

The value of threshold, \( \epsilon \) has to be chosen properly. To do this, we perform the calculations at the smallest \( \epsilon \), say 0.1, and then increase it in steps of, say, 0.1, to get a plot of number of recurrences versus \( \epsilon \). The value where the percentage of recurring points begins to rise sharply off the noise floor is selected as the threshold. In our work, we have experimented as described above for EEG signal to choose a proper value of \( \epsilon \).

Qualitative analysis alone may not be sufficient in practical applications since RP often contains subtle patterns that are not easily established by visual inspection. To address this issue, Zbilut and Webber [5] brought in the concept of recurrence quantification analysis (RQA) to quantify RPs. Additional measures have been proposed by other researchers thereafter [37]. Many efforts have been made to apply RQA to seizure EEG. Thomasson et al. [38] have shown that RQA variables are capable of distinguishing preictal from background activity. Li et al. [39] and Rabbi et al. [40] have studied the changes in dynamical characteristics of epileptic EEG in rats using RQA.

Considerable literature exists on RQA and its applications to seizure EEG. However our concern here is not on RQA but more on the probability that each of the samples of the trajectory returns to its own neighborhood after some delay. Further, phase synchronization (PS) between any two signals can be detected by using this probability of recurrence. In fact, a measure called Correlation coefficient between Probabilities of Recurrence (CPR) [31] can be derived from that. It is a Cross Recurrence Quantification Analysis measure. Such a cross recurrence measure is suitable for the analysis of multichannel EEG signals. The value of CPR in our studies lies in the fact that it gives the degree of phase synchronization between two signals. The details of calculation of CPR are given below.

The case of \( R(i,j) = 1 \) with \( i = j + \tau \) corresponds to the trajectory that returns to the neighborhood of \( i \) after a delay \( \tau \). Considering such occurrences for all \( (i, i+\tau) \), one can get an estimate of the probability \( P(\tau) \) of the system returning to a pre-defined state after a delay \( \tau \). Then one can estimate the probability \( P(\tau) \) that each of the samples of the trajectory returns to its own neighborhood after \( \tau \) samples delay as

\[ P(\tau) = \frac{1}{N_t - \tau} \sum_{i=1}^{N_t-\tau} R_{i,i+\tau} = \frac{1}{N_t - \tau} \sum_{i=1}^{N_t-\tau} \Theta(\epsilon - ||y_i - y_{i+\tau}||) \]  

\( P(\tau) \) has been used to estimate the dynamical invariants of the system by means of recurrences in phase space without embedding [41] and has found applications to geophysical data [6].

2.3. Correlation between probabilities of recurrence for two signals

Phase synchronization (PS) between any two signals can be detected by using the probability of recurrence, \( P(\tau) \) of two signals [31]. The coincidence of the positions of the maxima of \( P(\tau) \) for both signals is of value since we can quantify the degree of phase synchronization by looking at it. In fact, a measure called Correlation coefficient between Probabilities of Recurrence (CPR) [31] can be derived from that. Evaluation of phase synchronization may be made by first computing the probabilities of recurrence \( P_1(\tau) \) and \( P_2(\tau) \) for the signals 1 and 2 respectively and then computing the correlation coefficient between probabilities of recurrence

\[ \text{CPR} = \frac{\sum_{\tau = \tau_0}^{\tau_m} \left( P_1(\tau) - m_1 \right) \left( P_2(\tau) - m_2 \right)}{\sigma_1 \sigma_2} \]  

where \( m_1 \) and \( m_2 \) are the mean, and, \( \sigma_1 \) and \( \sigma_2 \) are the standard deviations of \( P_1(\tau) \) and \( P_2(\tau) \) respectively. \( \tau \) ranges from \( \tau_0 \) to \( \tau_m \)}
where $\tau_2$ is the value of $\tau$ for which $P(\tau)=1/e$. We usually choose the value of $\tau_m$ to be half the length of the longer signal.

The degree of phase synchronization (PS) between two signals is given by CPR. If two signals are in perfect PS, the probability of recurrences has peaks for some delay, $\tau_1$, and CPR $= 1$. In contrast, if the signals are not in PS, the peaks of the probability of recurrences do not coincide and CPR will then have a low value. The nonlinear recurrence measure CPR does not seem to have been applied for the analysis of seizure EEG signals.

In this paper, the definition of CPR has been extended to study the overall synchrony of multichannel signals. EEG is recorded simultaneously using electrodes placed at various locations on scalp to obtain multichannel seizure EEG signals (16 channels in this paper). CPR is calculated between all pairs of 16 channels. We thus obtain a $16 \times 16$ matrix, $\text{CPR}_{16 \times 16}$,

$$\begin{align*}
\text{CPR}_{16 \times 16} &= \begin{bmatrix}
\text{CPR}_{1,1} & \cdots & \text{CPR}_{1,N} \\
\vdots & \ddots & \vdots \\
\text{CPR}_{N,1} & \cdots & \text{CPR}_{N,N}
\end{bmatrix} ; \quad \text{CPR}_{i,j} \\
&= \text{CPR between signals of channels } i \text{ and } j
\end{align*}$$

(6)

The overall synchronicity of the brain is represented by mean value of all the 240 elements of the matrix (excluding principal diagonal elements). Application of CPR for the case of multichannel seizure EEG signals has been discussed in this paper for the cases of both global epilepsy and focal epilepsy.

Fig. 1a. (a) EEG signals (channels 1 and 2) and their $P(\tau)$ curves before seizure (CPR = 0.25).
3. Results and discussions

RP is obtained using the following parameters: $D = 8$, $d = 10$, $\varepsilon = 0.5$. These values are selected based on the procedure already discussed. Analysis of PS in multichannel EEG signals is done for seizure dataset of neurological patients with general epilepsy as also for seizure data of a patient with focal epilepsy.

We first obtain the $16 \times 16$ CPR matrix. CPR is calculated between all pairs of combinations of the 16 channels of EEG. Thus, a $16 \times 16$ matrix of CPR values is obtained. Surface plot is obtained
as the 3-D plot of colored parametric surface defined by the CPR matrix, where the 2 dimensions are matrix indices and the third dimension being its magnitude. A contour plot is the level curves of the surface plot. It is a 2-D representation which shows the sliced contours obtained from surface plot. In this work, contour plot is obtained by taking ten slices of the surface plot.

3.1. Phase synchronization in multichannel EEG signals with global epilepsy

We first compute CPR between channels 1 and 2 for pre-seizure and seizure conditions. Figs. 1a and 1b show the probability of recurrence curves in pre-seizure (low synchronization) and seizure cases (high synchronization) respectively. During pre-seizure case, there are no significant peaks in P(r), and also the curves appear to be uncorrelated with each other, and hence CPR is low (0.25). However during seizure case, there are significant overlapping peaks resulting in high CPR (0.83). 16 channels (8 on left and 8 on right) of EEG data of a patient affected by global epilepsy are considered. Fig. 2 shows the running CPR (rcPR) curve with frames of 320 samples (2.5 s). The curve is shown between channels 1 and 2: before (samples 1–14), during (15–23) and after (24–39) seizure. It is clearly observed that CPR value increases considerably during seizure which is due to increased synchronization between channels during seizure. It should be noted that phase synchronization provides additional insights into the functionality of the brain, which is not possible otherwise.

3.1.1. A seizure classifier based on CPR

Segments of EEG before and after seizure are considered for CPR computation. The manually selected segments and their respective durations for each of the 16 subjects are shown in Table 1. CPR is calculated between all pairs of combinations of the 16 channels of EEG for each subject. Thus, a 16 × 16 matrix of CPR values is obtained for each of the 16 subjects. Fig. 3 shows the surface and contour plots of CPR matrix before and during seizure for a chosen subject. Contour plot is obtained by taking ten slices of the surface plot. By visual inspection, it is clear that at almost all points the CPR values before seizure are less than that during seizure.

Diagonal entries of the matrix are always equal to 1, since it is the PS of the signal with itself. Mean value of the entries of the 16 × 16 matrix, excluding the diagonal entries, is obtained for both before and during seizure. Table 1 summarizes these values for all the 16 subjects in the dataset. These mean values for the cases before and during seizure are denoted by CPRb and CPRt respectively.

The statistically significant measure p-value is shown in Table 1. In the evaluation of 16 × 16 CPR matrix, we get one p-value for each (i,j) of CPR(i,j), which would finally give 256 p-values for each matrix. For each subject, the maximum p-value among all the 512 p-values is given in Table 1. If the p-value is small, say less than 0.05,

<table>
<thead>
<tr>
<th>Subject</th>
<th>Before seizure</th>
<th>During seizure</th>
<th>p-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subject-1</td>
<td>0.6851</td>
<td>0.9461</td>
<td>&lt;e-7</td>
</tr>
<tr>
<td>Subject-2</td>
<td>0.4928</td>
<td>0.8212</td>
<td>&lt;e-3</td>
</tr>
<tr>
<td>Subject-3</td>
<td>0.6891</td>
<td>0.9283</td>
<td>&lt;e-5</td>
</tr>
<tr>
<td>Subject-4</td>
<td>0.6755</td>
<td>0.8703</td>
<td>&lt;e-7</td>
</tr>
<tr>
<td>Subject-5</td>
<td>0.6968</td>
<td>0.9437</td>
<td>&lt;e-17</td>
</tr>
<tr>
<td>Subject-6</td>
<td>0.4903</td>
<td>0.7969</td>
<td>&lt;e-2</td>
</tr>
<tr>
<td>Subject-7</td>
<td>0.6709</td>
<td>0.9347</td>
<td>&lt;e-5</td>
</tr>
<tr>
<td>Subject-8</td>
<td>0.6843</td>
<td>0.9371</td>
<td>&lt;e-8</td>
</tr>
<tr>
<td>Subject-9</td>
<td>0.5161</td>
<td>0.8209</td>
<td>&lt;e-3</td>
</tr>
<tr>
<td>Subject-10</td>
<td>0.6192</td>
<td>0.8557</td>
<td>&lt;e-2</td>
</tr>
<tr>
<td>Subject-11</td>
<td>0.6697</td>
<td>0.8951</td>
<td>&lt;e-4</td>
</tr>
<tr>
<td>Subject-12</td>
<td>0.6563</td>
<td>0.8614</td>
<td>&lt;e-6</td>
</tr>
<tr>
<td>Subject-13</td>
<td>0.6587</td>
<td>0.8968</td>
<td>&lt;e-7</td>
</tr>
<tr>
<td>Subject-14</td>
<td>0.4693</td>
<td>0.7934</td>
<td>&lt;e-2</td>
</tr>
<tr>
<td>Subject-15</td>
<td>0.6452</td>
<td>0.8157</td>
<td>&lt;e-4</td>
</tr>
<tr>
<td>Subject-16</td>
<td>0.6916</td>
<td>0.9203</td>
<td>&lt;e-9</td>
</tr>
<tr>
<td>Mean</td>
<td>0.6257</td>
<td>0.8774</td>
<td></td>
</tr>
<tr>
<td>STD</td>
<td>0.0824</td>
<td>0.0551</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 3. Surface plot (left) and contour plot (right) of 16 × 16 CPR matrix before (above) and during (below) seizure.
then the correlation is said to be significant. It is observed from the Table 1 that the results are statistically significant. The overall mean and standard deviation for the two cases is also shown in Table 1. We can observe that the standard deviations are small compared to their respective overall mean values. Mean values of the individual subjects before and during seizure are shown in Fig. 4. It is evident that there is a clear distinction between the two classes. There is higher synchronization between brain regions during seizure. This is not an indicator of higher coordination between regions; rather this represents the synchronous firing of the neurons, which is a characteristic of epileptic seizures.

Massively synchronous discharge in adults may be an indicator of brain dysfunction such as seizure. Analogy can be drawn to an electronic circuit in which currents of equal magnitude travel in the same direction in all the wires irrespective of the semiconductor elements present there. It would malfunction. The circuit needs every element and connection to carry only the assigned volume of current during the time it is supposed to.

To test the accuracy with which one can separate the two classes, two thresholds are defined: minmax and mean. The minmax puts the threshold, \( T \), at the midpoint between the minimum of CPR\(_a\) and maximum of CPR\(_b\).

\[
    n_d = \min(CPR_a), \quad x_0 = \max(CPR_b), \quad T = \frac{(n_d + x_0)}{2} \tag{7}
\]

The second method of evaluation involves the calculation of the mean of the two curves, and then obtaining the threshold as the midpoint between them.

\[
    m_d = \text{mean}(CPR_a), \quad m_b = \text{mean}(CPR_b), \quad T = \frac{(m_d + m_b)}{2} \tag{8}
\]

From Table 1, the thresholds obtained are as follows: Minmax threshold = 0.7451, Mean threshold = 0.7515. For both the thresholds, the classification accuracy is obtained as 100%. Fig. 4 shows CPR\(_a\) and CPR\(_b\) along with the mean threshold. These results show that CPR is a good quantifier of phase synchronization and results on seizure EEG signals suggest that there is a clear distinction in synchronization in the brain between seizure and non-seizure conditions.

We have to note that 240 CPR values (16 diagonal entries excluded) are averaged to obtain each of the 16 mean CPR values given in Table 1. Hence a total of 3840 CPR values are involved in each of the classes. A classification accuracy of 100% in our experiments suggests that CPR can be effectively used for seizure detection and classification. More generally, CPR can be tested and used for differentiating synchronized and asynchronized states in biomedical systems.
3.2. Phase synchronization in multichannel EEG signals with focal epilepsy

So far we have analyzed synchrony in global epilepsy subjects. An extension of these concepts can also be made to subjects having focal epilepsy. Patients with global epilepsy exhibit synchronous firing of neurons throughout the brain whereas patients with focal epilepsy exhibit synchronous firing of neurons only in certain smaller affected regions in the brain. The elements of CPR matrix to be selected for study of synchrony would be restricted in case of focal seizure. Hence, global parameters like mean of CPR of the entire matrix will not be meaningful.

As an illustration we have considered a 10 channel EEG signal recorded during seizure in a patient with focal epilepsy. The same methodology used previously is applied here to obtain a 10 × 10 CPR matrix. Fig. 5a shows the contour plot of CPR matrix. Similar analysis was carried using linear correlation in place of CPR and Fig. 5b shows the contour plot of linear correlation matrix.

It can be observed that focal epilepsy is clearly evident in the case of CPR with higher synchronization observed between channel pairs (3,8) and (4,7). (Note the brown colored triangle at these coordinates.) These coordinates correspond to location pairs (B3,p4) and (F4,p3) respectively in the brain according to the international 10–20 system of electrode placement. That would put the focus of epilepsy in the parietal lobe near the sensor motor area with the resolution of 10 channels available to us, which corroborates with the diagnosis done for the subject. The result for linear correlation presented in Fig. 5b is not encouraging since we are unable to find any focus. This illustrates the efficacy of CPR in analysing focal epilepsy, and also shows that CPR provides meaningful insights while linear correlation fails to do.

4. Conclusion

We have considered here the study of phase synchronization in multichannel EEG signals using recurrence plot based technique. It appears that not much study has been made on the application of RP techniques to phase synchronization in the brain and we demonstrate a novel application of phase synchronization technique to multichannel EEG data here. Applications to multichannel EEG seizure data, for the cases of both global epilepsy and focal epilepsy, have been studied. We have considered the application of synchronization analysis to two channels of seizure data for the case of global epilepsy. The contrast of running CPR value during seizure has motivated us to build a simple classifier based on simple threshold logic for detecting seizures. Sixteen channels of EEG data of sixteen subjects with records containing segments of global seizure are considered. Contour plots of CPR matrix showed clear contrast between seizure and pre-seizure segments. Threshold value is arrived at using statistical analysis of CPR matrices and a classification accuracy of 100% has been achieved. For a focal epilepsy signal, contour plot of CPR matrix gave a clear indication of the region of focus. The identification of focus is much better compared to contour plot obtained using linear correlation matrix. The main findings in this paper are that CPR could distinguish between pre-seizure and seizure states with 100% accuracy in global epilepsy case and that CPR could identify the focus of epilepsy in focal epilepsy case.

Quantifying PS based on recurrence in phase space, as employed in this paper, is relevant for attractors with a rather broadband power spectrum. As such they are very relevant for application to EEG signals which are known to have broadband spectrum. This technique, being nonlinear, is particularly suitable for a biomedi- cal system like brain which is inherently nonlinear. Since it is also nonparametric, very few assumptions are made in this method, making it suitable for studying phase synchronization in brain in a data driven way. The advantage of this nonlinear technique has been established by the results of the paper that it outperforms the conventional linear correlation technique. It is hoped that the novel application proposed in this paper will trigger interest in using recurrence based techniques for many other applications. In fact, the proposed technique can be applied to many other types of EEG signals recorded under several experimental conditions. It is also hoped that the paper will lead to many other studies on nonlinear based synchronization which in turn will help in understanding nonlinear brain function in a better way. While most of the previous studies have either employed linear models or nonlinear parametric models, this paper uses a parametric nonlinear model and shows its utility in studying synchronization in the brain using EEG signals.

Conflict of interest

None declared.

Acknowledgments

The authors would like to thank Prof. D. Narayana Dutt, Prof. T. Srinivas, Dr. Chandra. R. Murthy and Dr. T.V. Antanapadmanabha for many useful discussions.

References


