NONLINEAR ANALYSIS OF SOLAR CYCLE VARIABILITY

L. Pastorek, Z. Vörös

1 - Slovak Central Observatory, 947 01 Hurbanovo, Slovak Republic, e-mail: pastorek@suhv.sk
2 – Space Research Institute, AAS, 8042 Graz, Austria, e-mail: zoltan.voeroes@oeaw.ac.at
3 - Geophysical Institute, SAS, 947 01 Hurbanovo, Slovak Republic, e-mail: geomag@geomag.sk

ABSTRACT

Modern time series analysis methods are used to study the dynamical features of high-frequency fluctuations present in daily sunspot number data. The results are discussed in terms of solar dynamo models incorporating turbulent gas motions within convective regions.

INTRODUCTION

The period of the solar cycle and its amplitude are rather variable. The most remarkable activity fluctuation is known as the Maunder minimum between 1670-1715. Besides the well known 22/11 year variability other secular periodicities of 80-90, 210 and 2400 years long were identified using C14 abundance measurements in sediments and in atmosphere (Vos et al., 1996; Hood and Jirikowic, 1990).

Not even the basic 22 year period can be fully explained, however, by solar dynamo theories. In dynamo theories the intermittent character of the MHD turbulence within convection regions plays a key role. One of the unanswered questions addressed by dynamo models is the problem of large ratio (>100) between mean cycle period and characteristic correlation time of MHD turbulence (Rüdiger and Arlt, 1996). It is clear that a nonlinear dynamo theory is needed to explain possible oscillation regimes and couplings between those processes occuring on very different time scales. Otmianowska et al. (1997) have studied nonlinear turbulence models in which non-stationary dynamo parameters were generated by turbulent gas motions within convective zone. Turbulence models reproduce the basic solar cycle variability exhibiting remarkable variations in magnetic cycle amplitude and produce also oscillating magnetic fields with periods about 8 years or with more complicated temporal behaviour characterized by rather broad power spectrum.

In this paper we analyse high – frequency oscillations present in sunspot number data. To this end we use modern nonlinear time series analysis methods proved to be useful for analysis of non-linear dynamical systems or turbulence.

RECURRENCE QUANTIFICATION ANALYSIS (RQA)

RQA represents a tool suitable for description of the time evolution of degrees of complexity. The method was introduced by Eckmann et al. (1987). Here we use its modification, developed by Webber and Zbilut (1994). It consists of an embedding technique employed for reconstruction of N-dimensional phase space from 1D time series X(tj)

\[ \mathcal{I}(Y_j) = \{X(t_j), X(t_j + \tau), ..., X(t_j + (N-1)\tau)\} \]

where \( \tau \) is an appropriate time shift, \( \mathcal{I}(Y_j) \) is a vector in N-dimensional phase space. Further, \( \mathcal{I}(Y_j) \) and \( \mathcal{I}(Y_k) \) vectors are compared in all possible j,k combinations and the corresponding differences are converted into a colour code at j,k coordinates. The resulting recurrence plots exhibit structures which characterize the dynamics of system under consideration. Webber and Zbilut (1999) have introduced several quantitative descriptors based on recurrence plots. Here we use a descriptor of determinism (DET), which quantifies the percentage of recurrence points organized into upward diagonal patterns in recurrence plots. Small values of DET correspond to dispersed recurrent points.

REGULARITY ANALYSIS (RA)

We consider the accumulated amount of signal energy \( E(t_j) \) within a window \((t_j,W,t_j)\) computed as a sum of squared amplitudes of differentiated time series \( \delta X(t_j) \) through

\[ E(t_j,W) = \sum_{j=1,2,...,N} \delta X^2(t_j) \]

where N is the total number of data points and

\[ \delta X(t_j) = X(t_j+\tau) - X(t_j) \]

Then we introduce the so-called regularity (or Hölder) exponent

\[ \alpha(t_j,W) = \log E(t_j,W) / \log W \]

taking different window lengths W. \( \alpha(t_j) \sim 1 \) for all \( t_j \) if signal energy is homogeneously distributed; \( \alpha(t_j) \sim \text{const.} \) describes self-affine fluctuations, while a time dependent \( \alpha(t_j) \) corresponds to irregular or intermittent distribution of signal energy (e.g. multifractal distributions).


© European Space Agency • Provided by the NASA Astrophysics Data System
WAVELET ANALYSIS (WA)

Fourier transform of a function $f(t)$ gives the information about the frequency content of a signal, but it gives no information about the location of these frequencies in time. Though time localization can be achieved by windowed Fourier transform, it is not suitable for analysis of multi-scaled data, when both high- and low-frequency components are present in investigated signals. Wavelet transform (WT) also allows dilation, that is a change of scale that narrows or widens time-frequency windows. WT of a function $f(t)$ is a kind of integral transform (Chui, 1992).

$$W(\lambda, t) = \int_{-\infty}^{\infty} f(u) \lambda^{-1/2} \psi((u-t)/\lambda) \, du$$

where $\lambda > 0$ is a scale parameter, $t$ is a location parameter, $\psi_{\lambda}(u)$ is called wavelet, which has certain features (fast decay, zero mean, etc.). In this paper we use the Morlet wavelet and the algorithm proposed by Torrence and Compo (1998).

DATA ANALYSIS

In order to show that there is a connection between the ~11 year periodicity and high frequency fluctuations, combination of RQA, RA and WA will be applied to Wolf sunspot number (WSN) data.

Figure 1a shows daily WSN time series from 1850 to 2000. The corresponding wavelet time-scale representation is depicted in Figure 1b, while Figure 1c shows the global wavelet spectrum dominated by a peak around 11 years. Simple visual examination of wavelet scalogram (Figure 1b) already indicates that there is a non-negligible intermittent power within period range 0.5 – 8 years, which might be associated with fluctuations naturally present in turbulent fields. As more power is concentrated here around years when the amplitudes of WSN are large (mainly after 1950 – Figure 1a), we suppose that, a kind of energetic coupling exists which at least operates over time scales from a few years to ~11 years solar cycle. On this basis, however, we cannot say too much about the possible relation of ~8 year oscillations predicted by turbulent dynamo model (Otmanowska et al., 1997) to solar cycle variations. Using RQA and RA for the same WSN data we attempt at least to specify other characteristic features of high-frequency (0.5-8 year) fluctuations.

Figure 2 shows the 2D recurrence plot computed from WSN data using the embedding technique. Time shift was chosen to be 400 days. Grey level code is used in $(j,k)$ coordinates which correspond to number of days. The ~11 year cycle is well visible as a repeating pattern always after ~4000 days.

Figure 3 shows that WSN and DET are anticorrelated, which means that when WSN increases DET decreases and vice versa. Moreover, large WSN amplitudes seem to be associated with decreasing values of DET. We note, DET was computed within a sliding window of 100 days over the recurrence plot.

Figure 4a shows the regularity exponent computed from WSN time series. Figure 4b contains the associated wavelet scalogram, and the global wavelet spectrum is depicted in Figure 4c. $T=400$ and $W=800$ days were used.
RA shows that $\alpha$ is not constant, but time dependent, which means that WSN energy is non-homogeneously distributed. $\alpha$ is closer to 1, however during the decreasing phase of solar activity, while the increasing phases are associated with $\alpha > 1$.

Large WSN amplitudes are also associated with increased power of regularity exponents in corresponding times. Wavelet scalogram shows that, besides the $-11$ year periodicity, energy is intermittently distributed over the period range (0.5 – 8) years and global wavelet spectrum in Figure 4c also exhibit a second peak between 4-8 years and an increased power between 2-4 years. These characteristics of higher frequency fluctuations are not visible in global wavelet spectrum computed directly from WSN data (Figure 1c). It indicates that the understanding of dynamical features of high frequency fluctuations estimated within appropriate windows may be of fundamental importance.
CONCLUSIONS

In this paper modern time series analysis methods suitable for both linear and nonlinear systems were used. The results show that during the increasing (decreasing) phase of solar cycle, dynamical descriptors of high-frequency fluctuations change: determinism (DET) as well as regularity (a) decreases (increases). In other words, when the energy distribution over scales is more non-homogeneous, determinism is smaller. If these dynamical features of fluctuations may have some relation to turbulence in dynamo models, turbulence characteristics should change in concert with solar activity itself. This notion can lead to more precise prediction of solar activity.

Acknowledgement: This work was partially supported by VEGA grant 2009/22.

REFERENCES
Chui, Ch.K., An introduction to wavelets, Academic Press, 1992;
Eckmann, J.P., S. Kamphorst, and D. Ruelle, Recurrence plots of dynamical systems,

Hood, L.L., and J.L. Jirikovic, A probable approx. 2400 year solar quasi-cycle in atmospheric delta C-14, In:Climate impact of solar variability, NASA, 1990;