Nonlinear dynamics of thin-walled elastic structures for applications in space

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ABSTRACT

Driven by the need for multi-functionality and increasing demands for low mass and compact-stowing, unfolding, self-deploying or –morphing smart mechanical structures have become popular space engineering designs for flexible appendages. Extensive research has been conducted on the use of tape springs as hinge deployment mechanisms for space booms, solar sails, or optical membranes or directly for used as antennas. However, the vibrational behaviour of tape springs and its related dynamics have rarely been addressed in detail, even though missions are underway with similarly flexible appendages installed.

By conducting quasi-static bending tests on a tape spring antenna, we evidence hysteresis behaviours in both the opposite- and equal sense bending directions. Apart from the well-known snap-through buckling, the structure exhibits torsional buckling in the equal sense bending direction before collapsing. Micro-vibrational excitation triggers nonlinear jump phenomena and the period-doubling route to chaos. Using a computational tape spring model and simplified environmental loads similar to those encountered in near-Earth orbits, coupling between the first bending and torsional modes generates a dynamic instability which is predicted by a complex eigenvalue analysis step. The current study highlights that high perturbation sensitivity and system-inherent nonlinearities can lead to stability issues.

In the course of designing a spacecraft with thin-walled appendages, system-level trade-offs are routinely performed. Since it is unclear how severely the vibrations of flexible appendages might affect their proper functioning or the control of the spacecraft, it is of paramount importance to validate experimentally thin-walled structures thoroughly for their dynamic and stability behaviours.

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1. Introduction

Compact, light-weight nano- or pico-satellites such as cubesats have become popular alternatives to conventional satellite designs with production and launch costs often being reduced to only a few hundred thousand US$ [1–3]. However, higher performance in the form of larger antennas, optical mirrors and reflectors, or solar sails and panels, and scientific advancement, strongly push for the development of miniaturised, efficient and highly packed, area deployment devices.
[4–7]. Consequently, the thickness of thin-walled structures must be further reduced for enhanced stowage or reduced weight [4–9]. It is known, however, that high imperfection sensitivity fosters buckling instabilities [4,7,9,10]; further, that thin-walled, slender structures often show significant geometric instabilities and therefore suffer from large deflections even with miniscule loads applied. While operating, especially at altitudes up to 800 km, in low Earth orbit, a spacecraft is exposed to both, external and internal perturbations. External perturbations include changing environmental loads such as caused by solar radiation pressure, large temperature differentials, electromagnetic and gravitational fields and residual atmosphere; internal perturbations include micro-vibrations as caused by thrusters, electric motors or the attitude control system such as a reaction wheel assembly or magnetorquers [11,12]. As a consequence, large amplitude nonlinear vibrations of thin elastic shells [13] can lead to undesired dynamics and possibly a performance reduction, which may trigger unforeseen satellite behaviour, even putting a mission at risk [14–18].

Owing to their structural simplicity, self-deploying and self-locking characteristics, thin-walled structures — especially open cylinder segments such as spring tape measures — have attracted much attention in recent years [4–7,9,18–26]. Yang et al. [20] tested single layer and double layer tape spring hinge designs and highlighted the good quasi-static deployment performance with regards to structural stress, and the peak and steady moment development. Kim and Park [21] studied solar array hinges and were concerned with the overshoot behaviour during deployment. Jeong et al. [22,23] developed a new shape memory alloy dampers for solar array hinges with higher bending stiffness and shock spectrum characteristics to accommodate the launch dynamics. Jennings et al. [24] studied the kinematics during deployment and pointed out an overshoot of about 60° of the boom before the steady state dynamic behaviour is reached, which was related to the character of the tape spring hinges.

Recently, Dewalque et al. [25] studied using numerical analyses and experiment, via motion analysis and a synchronised force plate, the nonlinear behaviour of tape spring measures to be used as a deployment mechanism. Results show that the deployment can be divided into three phases, which are characterised by different types of folds, oscillation frequencies and damping behaviours [25]. Damping is essential in determining the correct deployment dynamics of tape spring booms [13]; however, with increasing deployment length tape springs show decreasingly lower damping values accompanied by an increasing modal density, rendering their vibrations difficult to control [4]. Piergentelli et al. [26], Reveles et al. [27], and Fernandez et al. [28] employed tape spring structures as lightweight antennas or space booms; however, only launch qualification is reported with no detailed dynamic vibration analysis being conducted — even though their structures are rather thin-walled and slender and susceptible to complex vibrations [13,28].

Hoffeit et al. [18], Guinot et al. [29], Soysakap [6,30] and Soysakap et al. [31], Walker [7,32], Seffen and Pellegrino [9] or Seffen et al. [33] studied the mechanical properties of tape springs including their moment-rotation, static stability, profiles or temperature dependency. Tape springs are rigid owing to their longitudinal or transverse (cross-sectional) curvatures [4,9,33]. Bending a tape spring in the ‘opposite sense’ requires more force than applying load to the ‘equal sense’ bending direction [4,5,7,9,18,29]. When the moment rotation relationship of a clamped tape spring is measured, it shows a strong hysteretic behaviour in the opposite bending sense of tape springs is not as well documented to the best of the authors’ knowledge. An experimental, ground-based validation of thin-walled slender structures remains difficult [4]; modelling and analysis require consideration of local material and global, geometric nonlinearities [8,28,29]. Further, despite their popularity as design elements nowadays [4–7,9,18,20,24,29,31–34], the dynamics of tape springs due to vibrations has not been studied in detail yet.

Hence, starting with static bending tests, we extract hysteresis curves for both the equal and opposite sense bending direction of clamped-free, single layer, tape spring configurations and the buckling instability margins (BIM) are determined. To study the effects of internal vibrations as caused e.g. by the periodic excitation of a reaction wheel assembly, the vibration response dynamics of a periodically excited thin-walled structure is studied. To evaluate the risk of self-excitation, a complex eigenvalue analysis is finally conducted using a numerical model of an experimentally updated tape spring [4].

2. Buckling instability margins

2.1. Experimental setup

The geometric parameters of the tape spring (Stanley 30–497, total length 5000 ± 0.2 mm, width 19.1 ± 0.2 mm) are depicted schematically in Fig. 1(a). We obtained the characteristic curvature, the elastica, by bending the tape spring in the opposite sense direction and holding it together at both ends [7,19]. A secondary curvature along its longitudinal axis in the opposite bending direction (Fig. 1(a)), similar to curved tape springs, led to a slight deflection of 8 mm at 1300 mm deployment length which is assumed to follow a parabola [4,5,9,33]. We used a clamped-free configuration to
experimentally validate the static stability of the tape springs if bent in different directions and plotted the average buckling instability margins (BIM) as well as hysteresis curves \( n = 4 \) samples. The tape springs were clamped into an adaptor which had the same curvature as the elastica and which was held by a burette, which was attached via a rod to a heavy, stamped steel base (Plated 5/16\(^{\circ}\) steel rod), Fig. 1(b). To load the tip of the beam, we attached slotted weights of 10 g, 20 g and 50 g (plus weight holder). For smaller loads, attachable weights of 25 mm\(^2\) area were cut from magnetic foil (0.1372 ± 0.0130 g AEA 250 g, Adam Equipment Co Ltd, Milton Keynes, UK). Using the slotted weights and strips of magnetic foil we obtained both the critical loads at which buckling occurred as well their associated deflections in \( y \)-direction (Kingcrome\textsuperscript® vernier caliper, ±0.01 mm uncertainty) \[4,5\]. The uncertainty corridors were calculated using the standard deviation of the experimentally measured critical loads and adding an estimated maximum measurement error of 1.34 mN, which corresponded to the minimum average weight increment possible. The contributing moments \( mgl \) as well as the angle of rotation \( \beta \) were calculated about the clamping point \( P \) using the relation \( mgl = |i| + |j| \) with \( |i| \) and \( |j| \) representing the absolute components tangential and perpendicular to the bent tape spring (Fig. 1(b)). To illustrate the tape springs’ hysteresis behaviour (buckling points, restoration lengths), angles and moments were calculated by deducting the buckling point distance \( d_f \) relative to \( P \) from their hypothenuse \( l \), then normalised with their absolute maximum value. These are plotted against each other in Fig. 1(b), cf. \[6,7,9\].

A numerical model of a tape spring was generated, by integrating FEMTools\textsuperscript® 3.81 with ABAQUS\textsuperscript® and vibration testing (Polytec\textsuperscript® laser vibrometry), as detailed in \[4\]. The experimentally obtained critical load were compared to results of a finite element model setup in ABAQUS\textsuperscript® using linear and nonlinear buckling (Riks) analysis \[4,5,35\].

2.2. Results

Fig. 2 depicts the experimentally and the numerically calculated BIMs. Buckling modes as related to torsion or snap-through buckling, with either two- or three-dimensional folds (combinations of torsional and bending modes), were identified \[4,7,33\].

The experimental results of the oppositely bent tape spring showed a widening BIM until the tape spring eventually buckled under its dead weight, having been deployed beyond a length of 1730 mm. By extrapolating the median curve depicted in Fig. 3 to the median failure lengths estimated on 35 arbitrarily chosen tape springs of the same type (distribution on \( x \)-axis), the remaining load capacity was estimated to be about 5.6 ± 2.7 mN. As for lengths greater than 1 m in the opposite sense direction, the linear buckling method increasingly failed to converge, so the buckling loads were calculated using a nonlinear buckling step in ABAQUS\textsuperscript® by applying the Riks method. For this, up to 1000 increments were used, with an initial and minimum increment of 0.05 and 1E–35 and an adaptive stabilisation ratio of 0.15. The even sense bent tape spring was calculated using a linear buckling analysis step (30 vectors, 30 iterations, 100 vectors per iteration) \[4,5,35\].

The numerical computations very closely match the experiments and cross the uncertainty bounds only for lengths greater than 1500 mm. For lengths \( l > 1500 \) mm, the uncertainty is dominated by the finite resolution of the incremental loads. The failure length of the equal sense bent tape spring is significantly lower compared with the opposite sense bent
spring (>1730 mm), the maximum extension of the doubly-curved structure of 760 mm approximates better the experimental maximum buckling length (751 mm) than the singly-curved tape spring (730 mm). Those numerically simulated tape spring segments with lengths greater than 1600 mm bent in opposite sense direction provide an overestimated static stability limit and are hence found outside the uncertainty corridor. Contrary to that, the tape spring bent in equal sense direction under-predicts the experimentally obtained stability limit.

Fig. 2. Buckling instability margins for opposite and equal sense bent tape springs; experimental results are indicated by a mean and standard deviation corridor (dashed line, \( n = 4 \) different tape springs) and failure lengths on x-axis (\( n = 35 \) arbitrarily chosen tape springs); the numerical values correspond to a singly curved and doubly curved tape spring model (i.e. with secondary curvature).

Fig. 3. (a) Static bending tests on for opposite (1st quadrant) and equal sense bending (3rd quadrant) of different tape spring lengths; (b) 400 mm long tape spring test with mean and its standard deviation corridor (\( n = 9 \)) highlighting the spring’s hysteresis behaviour in equal sense bending direction; the six markers/numbers indicate the different behaviours 1–2: straight bending; 2–3 bending with torsion; 3–4 buckling; 4–5 minimum bending owing to almost vertical position; and 4–6 relaxation.
Fig. 3(a) depicts the static bending test of the tape spring in opposite and equal sense bending directions using the setup shown in Fig. 1(b). In both bending directions a hysteresis loop develops which also shows how the stiffness reduces with length, especially in the opposite sense bending direction. Fig. 3(b) magnifies the third quadrant and plots the average hysteresis of the moment-rotation curve in equal sense bending direction ($l = 400$ mm). Dashed lines indicate an uncertainty corridor (average ± standard deviation); numbers '1' to '6' highlight different solutions the tape spring experiences during hysteresis due to changing the load. After a steep increase (1–2) the tape spring exhibits torsional buckling which is accompanied by a partial loss of stability as indicated by a smaller absolute slope (2–3). This torsional buckling mode in point 2 is unique for the even sense bent tape spring which is similar to a ship’s rump shortly before capsizing, i.e. a sudden tilting motion after point '3', and is here, to the best of the authors’ knowledge, described for the first time for tape spring structures. After point '3', the tape spring loses stability completely, with the trajectory coming to a halt at point '4', where only little rotation can be observed if the load is further increased: the tape simply dangles from its clamping point in a fashion similar to a string. By reducing the load again, the tape spring relaxes (4–6) until it eventually snaps back after reaching point '6'.

Fig. 4 depicts the change of stiffness with deployment length for (1) the opposite sense bent tape spring prior to snap through buckling and (2) the equal sense bent tape spring in point '2' prior to torsional buckling (Fig. 3(b)) and in point '3' prior to collapsing. The stiffness of the equal sense bent tape spring is almost as high as for the opposite sense bent spring tape (on average over all lengths tested it has 65% of its stiffness), until point '2', then the stiffness is reduced to on average about 4% of the opposite sense bent structure. While the stiffness in both bending directions should be identical in theory, the deadweight of the structure and gravity induce qualitatively a nonlinear preload when bent in equal sense direction even in the '0–1' portion. This difference in deflection is the root cause for the increasing stiffness deviation between the equal- and opposite-bending directions at larger deployment lengths.

3. Forced vibrations

3.1. Experimental setup

As shown in the schematic of Fig. 5(a), the tape spring was clamped to a frame, excited close to its clamping point and a laser scanning vibrometer, setup at a distance of 3.6 m, was used for contactless measurement of its vibrations (Fig. 5(b)). By extending the tape springs, similarly to a deployment of an antenna in space, starting with a length of 400 mm and ending with a length of 1700 mm, the forced responses, modal damping and their coherences were compared.

A two-piece adaptor fixed the structure to a frame, using a three-finger metal clamp. Since the tape spring deflection became greater with distance from the clamping point (CP), an excitation source close to the CP was preferred. To establish contact, the tape spring was clamped vertically to limit the influence of gravity on the bending and torsional vibrations. Then, the armature of the electro-dynamic shaker (Bruel & Kjaer (B&K) 4809) was zeroed and then the shaker was carefully moved into position, adjusting the tension in the shock cords attached to the test structure to establish contact without inducing a preload.

The shaker was suspended on shock cords to simulate the free-field condition of the excitation source. The test article was connected horizontally using bee's wax (B&K YJ-0216) with the force transducer (B&K 8200). The connection maintained the
zero position of the armature to minimise pre-stressing of the tape spring and to avoid its mass loading through the shaker. A stud connected the force transducer with the shaker, which was connected to a power amplifier (B&K 2706, set to 20% load) and a signal generator (RIGOL DG 1022), which produced a random burst excitation signal (rectangle waveform, 1 cycle at 2.5 kHz excitation frequency, Fig. 5 (c)).

For the full-field measurements of the tape spring, a Polytec PSV-400 laser scanning vibrometer (PSV-I-400 scanning head) was setup at a distance of 3.6 m and centred at about 100 mm from the excitation source near the clamping point (Fig. 5 (b)). The laser vibrometer was connected to an OFV-5000 controller, a PSV-E-401 junction box and a personal computer with Polytec PSV 8.7 analysis software installed. The measurement grid density of the tape spring was 5.7 mm across the width and every 7.5 mm along the length (i.e. at 400 mm length ca. 160 points) and we took 15 measurements at each point. The measured response signal had a frequency resolution of 45 mHz with a Hamming window length of 1024 samples and 50% overlap [36].

To measure the effects of micro-vibrational internal perturbations on the dynamic response of a tape spring segment (500 mm length), a study of forced response to periodic excitation was conducted. A single point response due to low amplitude vibrational forcing (from about 0.3 mN to about 1 mN) was measured, at a point centred about 100 mm from the excitation source near the clamping point. The frequency of the sinusoidal forcing was varied from 22 to 28 Hz in steps of 250 mHz and the micro-vibrational forcing amplitudes were altered from 0.302 mN to 0.507 mN, 0.725 mN and 0.926 mN. The maximum steady-state amplitudes were recorded as output values and plotted over excitation frequency.

Then, we studied the nonlinear behaviour of the tape spring more systematically by up- and down-swinging the excitation forces. The voltage signal was ramped up from 1 V to 12 V in 0.5 V steps, then ramped down to obtain both a steady state response of sufficient length and a continuous signal. The excitation frequency was set to 15 Hz, which was above the lower bound of 12 Hz of shaker, reasonably far away from the first bending mode at about 8 Hz, but close enough to a torsional mode at about 17 Hz. The excitation frequency was then successively increased to 15.5 Hz and to 15.75 Hz. Welch’s method was employed to estimate the power spectral density using 6000 FFT lines and a Hamming window of 1024 samples with 50% overlap [36]. The power spectral density can provide insights into the dynamics to distinguish various periods, sub-harmonic or broadband spectra. The dynamics’ phase space and its attractor are re-constructed via Kennel’s delay embedding by using the average auto-mutual information (MI) and the global false nearest neighbour algorithm [37]. The MI determines the optimal delay to de-correlate the data points and to span the attractor in a large enough space.
(embedding dimension) [37]. Recurrent plots qualify dynamic states of the attractor in phase space (2000 continuous samples) and indicate over diagonal lines, whether periodic, multi-harmonic or chaotic regimes are present. As environment, a fixed number of neighbours of 10% of the maximum phase space diameter is selected [38]. To study the effect of damping on the tape spring, we laminated a larger (hence more sensitive) tape spring segment of 1000 mm with Kapton\textregistered tape on its convex side along the centre line to 50%, then fully to 100%.

3.2. Results

In the full-field vibration measurements, dynamics of tape spring lengths up to 700 mm could be well measured as indicated by coherence values close to unity; by extending the tape spring further to 1000 mm length, the modal density increased significantly and the average coherence of the measurements deteriorated (Fig. 6, cf [4]). Reasons for a lower average coherence are thought to be the increased modal density and the lower signal to noise ratio found in some anti-resonances. Another reason is the tape spring’s increasing degree of nonlinearity due to larger deflections. Larger deflections were further accompanied by an increasingly lower stiffness, higher sensitivity and by more prominent, as well as more difficult to measure torsional vibration modes [4,5]. Fig. 6 exemplifies this development for tape spring lengths of 500 mm, 1000 mm, and 1700 mm.

To study the nonlinear response of a 500 mm long test article, the maximum steady-state vibration amplitudes due to continuous upward and downward forcing were recorded and plotted against frequency variations (Fig. 7). By setting the forcing amplitude to one volt (302 \( \mu \text{N} \)), only the resonance at 26 Hz showed up. However, by decreasing the voltage from about 2 V (equivalent to 507 \( \mu \text{N} \)), a sudden down-jump in the response at about 25.5 Hz showed up; for higher voltages this jump became more pronounced and occurred at lower excitation frequencies. The softening behaviour showed maximum response amplitudes of 20 mm s\(^{-1}\); the larger the forcing amplitude, the longer the tape spring oscillated in a large amplitude regime. Finally, at 4 V (0.926 mN), the amplitude jump occurred at 23.5 Hz.

By applying nonlinear time series analysis tools, possible routes to instability were studied in the following. The excitation level of the sinusoidal signal was varied from 6.5 V (\( ~1.532 \text{ mN} \)) to 12 V (\( ~2.680 \text{ mN} \)), while the frequency was kept constant at either (I) 15 Hz, (II) 15.5 Hz or (III) 15.75 Hz, respectively which is between the first bending mode at 7.8 Hz and the first torsional mode at 17 Hz (cf. Fig. 6(a)). The colours plotted were normalised to unity and ranged from blue (low response level) over green to yellow for the maximum response level (\( ~2.5 \text{ m s}^{-1} \)).

Fig. 8(a) and (b) depict the excitation and the response levels for a variation of the voltage from 6.5 V to 12 V and from 12 V to 6.5 V in 0.5 V steps. After initial transients, the response settles into a steady state and its behaviour is rather proportional to the changed excitation signal. Characteristic dynamic regimes are indicated by arrows and named A and B as studied in detail within subplots (c) to (f). Fig. 8(c1) shows a periodic relaxation motion, characterised by its curved increasing and linearly decreasing flanks, similar in character to a saw-tooth function [41]. Increasing the excitation level to about 2.6 mN only lets the amplitude grow – Fig. (c2). The power spectral density (PSD) estimate for regime A (Fig. 8(d1)) only indicates the excitation frequency \( f_e \) of 15 Hz and its harmonics, whereas that of regime B shows a small extra peak at about half the excitation frequency at 7.6 Hz (in Fig. 8(d2)). Other than that, the power in the low frequency region is merely raised.
by about 10 dB. The attractors shown in Fig. 8(e1) and (e2) are that of period-1 and period-2 motion, respectively. The limit cycles are that of a nonlinear system, as can be seen from the shape of the waveform and the higher harmonics in the spectrum [37–40]. The recurrence plots depicted in Fig. 8(f1) and (f2) clearly show the driving frequency of 15 Hz, corresponding to 160 samples between subsequent diagonal lines. The sub-harmonic component of regime B is yet too weak and would only be detectable on a different time scale in the recurrence plot and is, hence, not visible in the recurrence plot [37,38].

Exciting the structure with a driving frequency of 15.5 Hz using identical force amplitudes as before, produces a very different dynamic response (Fig. 9(a)). The regimes for 6.5 V to 11 V are that of period-1 or period-2 dynamics (hence identical to regimes A and B of Fig. 8) but once a voltage of 11.5 V is reached, the response amplitude swings up from 0.3 m s\(^{-1}\) to 1.8 m s\(^{-1}\), showing large amplitude, complex oscillations as indicated in Fig. 9(b) and its insert. These large amplitude vibrations are persistent and only diminish slowly by reducing the strength of the excitation to below 8.5 V after ca. 300 s. This is a common behaviour in structures with hysteresis. The time series and spectrum of regime C at 11.5 V indicates period-7 motion (Fig. 9(c1), (d1)), with its spectrum being lifted up by about 23 dB (Fig. 9(d1)) compared to that of regime B (Fig. 8(b1)). Also visible is a sub-harmonic frequency component at about 7.2 Hz, as well as higher harmonics. The recurrence plot seems to confirm that the dynamics are already at least slightly chaotic, since many of the diagonal lines are a broken, each of them representing a recurrent but non-periodic state (Fig. 8(e1)) [4]. The remaining undisrupted diagonal line corresponds to the frequency of 7.27 Hz (330 samples distance), which is about half the driving frequency. The dynamic regime D at 12 V is similar to that of regime C comparing its time series and recurrence plot.

However, the peaks in the PSD show broader sidebands, which cause the attractor to be more diffuse with the trajectory, resembling more a band than a line. Both attractors describe a weakly chaotic state as indicated by the non-periodic trajectories, driven with 7.27 Hz and an unstable periodic orbit acting as the attractor’s skeleton [39,42,43]. Increasing the frequency further to 15.75 Hz complicates the dynamics visibly (Fig. 10). For low voltages, periodic vibration similar to that found for regimes A and B is again present (Fig. 8). However, for stronger excitations beyond 8.5 V the dynamics become complex, displaying dynamics of C and D (Fig. 9); yet only after 11.5 V (case E) the motion becomes strongly irregular with high amplitude vibrations (Fig. 10(c1)). For case F (12 V) turbulent dynamics show up and the amplitudes exceed that of 2.2 m s\(^{-1}\) (Fig. 10(c2)).

The PSDs for E and F are broadband in nature with an approximately 10 dB higher response level at 12 V. While the attractor of case E has discernible features that of case F resembles a cloud of points. The recurrence plots show the forcing frequency of about 7.5 Hz as a continuous diagonal line; apart from that all other diagonals are disrupted, which represents a fingerprint of chaotic dynamics [37,40,41]. Once in the chaotic regime, the tape spring keeps vibrating even though the voltage has been reduced significantly.

**Fig. 7.** Maximum velocity versus frequency for different voltage settings for the 500 mm long tape spring (Fig. 6(a)) close to resonance at 26 Hz. After a voltage setting of 1 V hysteretic, softening behaviour increase the amplitude of the response with a down jump phenomenon with trend to lower frequencies.
Fig. 11 shows the effect of laminating a 1000 mm long tape spring using Kapton\textsuperscript{®} tape. The purpose of the lamination is to increase the damping properties and to reduce the tape spring’s antenna vibration response amplitudes. The colour of the vibrations are normalised to the maximum response amplitude of the non-laminated tape spring segment (Fig. 11(d)); blue indicates low vibration levels and orange higher vibration responses. If the tape spring’s surface is laminated, the coupling of modes should be promoted, since the foot of the resonance curves become broader, increasing the likelihood of energy to be exchanged. Indeed, we can observe larger vibration amplitudes, which swing up sooner (less force) than in the non-laminated state (Fig. 11(d)); the swung-up response amplitudes persist, even for rather small forces. Only for the tape spring with a fully laminated surface, did the absolute amplitude slightly more than halve.

Fig. 12 depicts reconstructed delay-embedded attractors’ dynamics for all voltages applied in Fig. 11. It can be observed that the higher the damping, the sooner a quasi-periodic motion develops. While for the non-laminated case 11 periodic regimes are discernible as indicated in framed subplots of Fig. 12(a), the periodic regimes are in (b) reduced to seven for the 50% laminated case and in (c) to two for the 100% laminated case. We observed that a more damped structure starts to exhibit complex oscillations sooner. At the same time we only observed significantly decreased vibration amplitudes for the 100% laminated test article. Considering that only micro-vibrations are applied, this trade-off between amplitude and complexity could be important when dealing with the control of issues such as antenna boom vibrations when using tape springs in space applications.
4. Self-excitation

4.1. Methods

When operating in low earth orbit, thin elastic structures, such as clamped-free, single layer open cylinders employed as instrument booms or antennas, are exposed to aerodynamic drag, gravitational gradients, solar radiation pressure and magnetic forces. These forces generate loads, which may be cyclic, difficult to predict and may cause structural (binary flutter-type) instabilities [35,40,42], which are similar to and can interact with aero-elastic flutter [43–45].

An experimental investigation was conducted, employing an updated tape spring model of 1000 mm length with a Young’s modulus of $E = 202.34$ GPa, a Poisson’s ratio of $\nu = 0.254$, density of $\rho = 7815.42$ kg m$^{-3}$, and Rayleigh damping with parameters $\alpha = 0.01473$ s$^{-1}$ and $\beta = 2.12 \times 10^{-5}$ s, as validated in [4]. The equations of the autonomous structural system were solved in matrix form in a frequency range of 0.25 Hz to 1325 Hz as

$$M\ddot{y} + C\dot{y} + Ky = F_{ae}$$

(1)

where $F_{ae}$ were distributed element forces converted into equivalent nodal loadings [35,40,42]. The tape spring was pre-loaded either in opposite or equal sense bending directions with a uniformly distributed, constant load of 4.5 mN m$^{-2}$, which was orthogonal to residual gravitation of 2.5 E$-6$ ms$^{-2}$ as encountered in about 400 km height [5]. This constant pressure load was representative of the aerodynamic drag arising from the residual atmospheric pressure at a typical low Earth orbit.
altitude (atmospheric density \( \rho = 5 \times 10^{-6} \text{ kg m}^{-3} \)), the orbital speed of about 7.5 km/s and an assumed drag coefficient of \( C_d = 2.85 \) [5]). Contributions from solar radiation pressure, as well as the body forces which occur due to gravity gradients and variations in the Earth’s magnetic field around the orbit were neglected [5].

A nonlinear static prestress analysis was followed by a symmetric eigenvalue problem \((\epsilon \mathbf{M} + \mathbf{K})\mathbf{z} = 0\) using a subspace eigensolver. Then, the original matrices were projected in the subspace of real eigenvectors \( \mathbf{z} \) and a generalised unsymmetrical eigenvalue problem was solved using

\[(\lambda^2 \mathbf{M}^* + \lambda \mathbf{C} + \mathbf{K}^*) \mathbf{y} = 0, \]

where \( \mathbf{M}^*, \mathbf{C}, \) and \( \mathbf{K} \) constitute the structural mass, damping and stiffness matrices, respectively, in their transformed form using \( \mathbf{A} = (z_1, z_2, \ldots, z_N)^T \mathbf{A}(z_1, z_2, \ldots, z_N) \) [41]. This analysis step took into account Rayleigh damping as evaluated in [4] and the modified equilibrium position due to the distributed pressure acting on the deployed tape spring structure. Complex eigenvalues \( \mu \) were then extracted. Positive real parts \( \text{Re}(\mu) \) indicate a negative damping ratio \( \zeta \),

\[ \zeta = -2 \frac{\text{Re}(\mu)}{|\text{Im}(\mu)|} \]

with \( |\text{Im}(\mu)| \) being the imaginary part’s magnitude. Negative damping, however, represents the generation of additional energy and structural flutter type instabilities, and the emergence of limit cycle vibrations [42,46]. A bifurcation parameter, i.e. the control quantity which induces unstable behaviour, the slenderness ratio
\[
\lambda = \frac{L}{r_G}
\] 

was varied from 10 (~5.47 mm tape spring length) to 3150 (~1725.49 mm tape spring length). Different slenderness ratios also allowed a classification according to different static stability ranges: the strength limit, the inelastic and the elastic...
stability limit. The strength limit depends only on the failure stress of the material (no buckling or yielding possible); and the inelastic stability limit depends on the proportional limit, which is the system’s capacity to resist additional loads. For the elastic stability limit, in contrast to the strength limit and the inelastic stability limit, buckling occurs before the structure collapses or yielding and plastic deformation sets in; elastic static instabilities are able to trigger elastic waves and dynamic instabilities.

It is noted that conducting a stability analysis as presented here only serves as a first approximation, similar to the constant force caused by an aerodynamic static mean flow, as it estimates the dynamics of the structure alone [42,44]. No dynamic forces of the fluid or their feedback to the structure are incorporated, which would require a flow analysis and the calculation of fluid-structure interaction [44–46]. However, given the very low mean free path in the residual atmosphere at these altitudes, such effects are likely to be of secondary importance.

4.2. Results

In Fig. 13, the real parts of structural complex eigenvalues are plotted against the mode’s number. For slenderness ratios ranging from 10 to 40 (the ‘strength limit’), only the first 5 modes above 1325 Hz were analysed; for all other slenderness ratios all modes below 1325 Hz were extracted. As a result of increasing the tape spring’s length its modal density increases significantly so that for an extended tape spring as found in space boom or antenna applications, the elastic stability limit mostly applies. Further, for larger \( \lambda \) increasingly more modes obtain a near zero real part, which indicates that the tape spring dynamics become marginally stable. A weakly damped structure, which is marginally stable, exhibits more easily self-excited vibrations, since it requires less energy to push the damping into a negative regime.

Only for lengths falling below the inelastic and the elastic stability limit do the eigenfrequencies of the tape spring fall below 1325 Hz, and very soon the tape spring is moved into the elastic stability zone if extended. In Fig. 13(b) the damping ratio \( \zeta \) is plotted against the mode number to indicate that for a \( \lambda \) larger than 1350 the first 15 vibration modes can be classified as marginally unstable; the larger the slenderness ratio, the higher the mode number and the more modes are classified as marginally unstable [40,47,48].

Preloading the tape spring in the equal sense bending direction, however, indicates a different scenario. While the modal density increases in a similar manner as in the opposite sense bent structure (insert Fig. 13(c)), at about \( \lambda = 1950 \) a positive real part renders the damping negative, indicating flutter type instability of coupled modes. A closer look at the mode shapes reveals that energy from the first bending mode is exchanged to the first torsional mode. The torsional buckling, with static divergence and single mode instability, as evident in the hysteresis regime ‘2–3’ in Fig. 3(b), seems to trigger mode coupling. In other words, energy is exchanged between the 1st torsional and the 1st bending vibration modes – see Fig. 13. For confirmation of this phenomenon, an in-orbit experimental test with a tape spring antenna mounted on a satellite is required, so that the loading environment is truly representative.

Fig. 13. (a) Real parts of complex eigenvalue extraction steps indicating stability for various slenderness ratio; and damping ratio against mode number calculated via complex eigenvalue extraction step for different slenderness ratios by preloading the tape spring in (b) opposite sense; and (c) equal sense direction. In (b) and (c) the number of ‘marginally’ unstable modes increases significantly, but only for the equal sense bent tape spring, self-excited vibrations develop beyond a length of about 1000 mm.
Fig. 14 shows the damping ratio of the first bending and the first torsional mode for slenderness ratios ranging from $\lambda = 1830$ (2.54 and 9.24 Hz, unloaded) to $\lambda = 2180$ (1.35 Hz). A deterministic case without thickness variations (mean thickness $t = 0.112$ mm) and an uncertain case with thickness variations (uniformly distributed $\bar{t} \pm 2\sigma$, $n = 6$) are plotted; the distribution of the thickness variation is taken from thickness measurements along the tape spring length. Both the tape spring with uniform thickness and that with varying thickness become unstable. However, the mean curve of the case with thickness variations becomes unstable for shorter lengths so that the onset of instability in a real structure is expected to lie somewhere between the slenderness ratios of $\lambda = 1880$ and $\lambda = 1930$. Results of the ‘deterministic’ complex eigenvalue analysis (CEA) do not lie exactly on top of the mean curve, but within the corridor of $\bar{\zeta} \pm 2\sigma$ of the ‘uncertain’ case, indicating a 95% overlap with the ‘empirical distribution’. The fluctuations in the damping ratio highlight varying stiffness contributions due to thickness variations. At about $\lambda = 2200$ (1205.1 mm), the nonlinear prestress analysis no longer converged, indicating that the static instability might be too severe and that the system is past a static bifurcation point. Fig. 14(b) and (c) depict the unstable vibration mode pair coupled at 1.35 Hz, the stable torsional mode, which vibrates at a frequency of 9.24 Hz if unloaded and the unstable first bending mode which is found at 2.54 Hz if unloaded.

4.3. Summary

We have studied the stability behaviour of tape spring structures that are increasingly employed in space engineering designs as hinges, as self-deployable, self-unfolding light-weight appendages, as space booms to carry payloads, as frames in solar sails, as extended supports for optical reflectors, de-orbiting devices or as antennas. Despite already being standard design elements, the analysis presented here is the first of its kind, studying the stability behaviour of these thin-walled, open cylinders in detail, including their vibrational behaviour.

Clamped-free, single layer tape springs have been statically tested for their stability by measuring their moment-rotation relationship in both opposite and equal sense bending directions. Hysteresis was not only found in the opposite sense but also in the equal sense bent tape spring. The hysteresis loop of the equal sense bent tape spring is, however, different in its buckling behaviour and does not simply snap-through: its moment-rotation curve steeply increased, showing stiffness values similar to those of the opposite sense bent structure, before losing only some of its stability via torsional buckling. The torsional buckling preserved some of the tape spring’s bending stiffness and the moment-rotation showed a second, less stiff regime similar to the bi-stable state reported in the deployment of a reflector shield [33].

We then forced a 500 mm long tape spring segment periodically to cause micro-vibrational perturbations similar to those occurring during the corrective action of a reaction wheel assembly [12]. By increasing, then subsequently decreasing the forcing frequency, we measured a jump in the tape spring’s vibration response, a common behaviour observable in nonlinear systems [39,48]. This jump phenomenon strengthened for higher forcing amplitudes. The stronger the forcing, the larger the response and the longer the structure stayed tuned in its regime of large amplitude vibrations. By increasing the forcing amplitude from 1.5 mN to 2.65 mN and placing the excitation frequency between the 1st bending mode and the 1st torsional...
mode, the tape spring response first showed regular limit cycle motion. For increasing excitation amplitudes and frequencies, the response, however, degenerated into irregular oscillations, following the period-doubling route to chaos.

The period doubling route to chaos is well-known and can be found for viscous fluids, biological, chemical and thermal reactions (e.g. Rayleigh-Bénard convection); it is also known for thin, elastic structures (for example see the numerical study on doubly curved shells [28]) and in classical, analytical, nonlinear systems such as the Duffing oscillator with nonlinear stiffness [39,48,49]. Thin-walled elastic structures with bi-stabilities might exhibit behaviours similar to those observed in bistable biochemical reactions in which mixed mode states result from phase-locking onto the attractor dynamics [39]. This development of large amplitude hysteretic chaos is related to the nonlinearity of the tape spring, its sensitivity to imperfections and its low damping as a spring [4]. The tape spring continued robustly oscillating in its chaotic regime, even for significantly reduced excitation amplitudes. In tests of longer tape spring segments, i.e. greater than 1 m, obtaining a simply periodic response was difficult: generally more than one resonance peaked up, quasi-periodic motion dominated and irregular motion followed soon after. Laminating the spring with Kapton© tape reduced the amplitude significantly only for 100% laminated case; however, it also increased the complexity of the dynamics at lower peak-to-peak excitation amplitudes. Once 100% of the outer area of the tape spring was laminated with Kapton© tape, the vibration amplitude was reduced by about 50%, but the system’s vibrations were irregular. Damping increases the width of a resonance foot, which promotes energy transfer (e.g. internal resonances) and coupling of neighbouring modes.

We then tested whether self-excitation is another mode of generating instabilities in tape springs. A complex eigenvalue analysis indicated that with increasing deployment length the structure’s modal density increased, with the emergence of many ‘marginally stable’ modes [47,48,50]. If pressurised in opposite sense direction, the tape springs remains stable; if pressurised in equal sense direction, springs longer than 1 m developed a flutter instability coupling the 1st torsional mode to the 1st bending mode. However, the load profiles used in the computer simulation are constant and therefore in their simplest form; only a mission in space with experimentally measured forces on thin-walled elastic structures could validate some of the results presented. Regardless, the number of marginally stable modes for a large tape spring should be considered with care: it is anticipated that the slightest perturbation from within the spacecraft or from the external environment could trigger instabilities. The low structural damping of the tape spring and the additional effect of lamination combined with increased modal density and sensitivity to imperfections render the control of thin-walled, flexible structures for space applications problematic. Thin-walled structures in small spacecraft, such as nano-satellites, may be especially prone to these effects, given the combination of very low mass and operation in orbits with cyclic magnetic and gravitational loads, and relatively large aerodynamic drag. Damping increases the likelihood of mode coupling as it promotes the exchange of energy. It is, therefore, anticipated that treating the coupling and controlling the dynamics of tape springs in space applications will be challenging and that proper vibration testing of thin-elastic structures needs to be addressed in future antenna designs [4,5,8,51,52].

Competing interests

The authors declare not to have competing interests.

Data accessibility statement

The data supporting the results will be either found within the manuscript, the supporting material or will be archived in an appropriate public repository.

Author contributions

SO designed the experiments, setup the computer models and conducted the experiments, retrieved, analysed and interpreted the data, wrote and revised the manuscript; SLT helped interpreting the results and revised the manuscript.

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