Damage detection using multivariate recurrence quantification analysis

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Abstract

Recurrence-quantification analysis (RQA) has emerged as a useful tool for detecting subtle non-stationarities and/or changes in time-series data. Here, we extend the RQA analysis methods to multivariate observations and present a method by which the “length scale” parameter $\varepsilon$ (the only parameter required for RQA) may be selected. We then apply the technique to the difficult engineering problem of damage detection. The structure considered is a finite element model of a rectangular steel plate where damage is represented as a cut in the plate, starting at one edge and extending from 0% to 25% of the plate width in 5% increments. Time series, recorded at nine separate locations on the structure, are used to reconstruct the phase space of the system’s dynamics and subsequently generate the multivariate recurrence (and cross-recurrence) plots. Multivariate RQA is then used to detect damage-induced changes to the structural dynamics. These results are then compared with shifts in the plate’s natural frequencies. Two of the RQA-based features are found to be more sensitive to damage than are the plate’s frequencies.

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1. Introduction

In many applications of signal analysis and system identification the goal is to track changes to a system’s dynamics based solely on time-series measurements of the system response. One such example is the analysis of recordings of the human heart where there exists a premium on the early

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detection of anomalies that could signify the onset of a devastating event e.g. stroke, heart attack [1]. Researchers in neuro-science are also interested in extracting signatures from EEG (brain wave) data that could provide an early warning for epileptic seizures [2]. Likewise, physiologists have attempted to detect the dynamical changes in signals associated with breathing patterns and muscle fatigue [3]. Other researchers are interested in monitoring dynamical transitions in chemical reactions [4]. Several of the above-cited works utilised recurrence plots as a tool for detecting changes to the dynamics. In structural engineering, the practitioner is interested in the related problem of tracking damage-induced changes to a structure’s dynamic response. Metrics derived from the analysis of recurrence plots are therefore well suited to the damage-detection problem and are the primary focus of this work.

The field of vibration-based structural health monitoring (SHM) represents an effort to prevent catastrophic failure, minimise maintenance costs, and reduce the costly “downtime” associated with structural degradation based on the structure’s vibrational response. Here, the term “structure” is used generically to refer to buildings and bridges, vehicles (planes, boats, etc.) and/or machinery. The general philosophy is to look for early indications (vibrational signatures) of a problem such that appropriate action may be taken before more serious problems arise. One of the standard approaches is to excite the structure with ambient or applied loading, record the dynamic response, and look for some “feature” of the response signal that can be used to classify the presence, type, location, and scope of the damage. The problem can be viewed as one of detecting damage-induced non-stationarities in one or more time-series records of a structure’s response. One of the major difficulties in SHM is that few approaches have demonstrated the sensitivity required to detect small amounts of degradation to a structure. Modal properties (e.g. frequencies, damping ratios, mode shapes) are known to be insensitive to certain types of damage yet are the most commonly utilised features (see [5–7]). These features are derived from linear time-series analysis techniques that make certain assumptions about the underlying process. As has been pointed out by Brown and Adams [8], damage evolution is often a non-linear process. We therefore consider the possibility that the more probabilistic measures from the non-linear time-series analysis literature may produce better results in tracking damage than do the more standard approaches.

Here, we present the extension of an analysis technique referred to in the literature as recurrence quantification analysis (RQA). This particular technique is based on a graphical description of a system’s dynamics dubbed the recurrence plot [9]. In short, a recurrence plot provides a global picture of the autocorrelation in a time series over all available time-scales. It is further able to reveal structure i.e. determinism in time-series data not readily available to many approaches, especially methods involving linear transformations of the data (e.g. the Fourier transform). Measures derived from recurrence plots have been shown to possess greater sensitivity to changing dynamics than do linear approaches such as frequencies [3] making them ideal candidates for damage-sensitive features. Recent applications of RQA include the detection of non-stationarities in human heart rate data [1], tracking bifurcations [10], detecting determinism in noisy signals [11] and examining coupling in non-linear systems [12].

The goal here is to extend the use of recurrence plots and RQA to include multivariate observations and to then use RQA to detect damage in a simple structure. We begin with a brief discussion of the technique as it has been used with univariate data and then present a straightforward means of including multiple observations. We also develop a quantitative
approach for selecting the key parameter used in RQA, the recurrence length scale. The technique is then tested on a two-dimensional finite element plate “structure”, where damage is taken to be various sized cuts across the width of the plate. These results are then compared to the sensitivity of the plate’s modal frequencies to damage. The indication is that for this particular system metrics derived from recurrence plots are more sensitive to damage-induced changes than are modal frequencies.

2. Recurrence plots

First suggested by Eckmann et al. [9], the recurrence plot is a graphical technique designed to highlight structure (i.e. determinism) in signals. It may be thought of as a global, probabilistic autocorrelation function that considers the relative frequencies at which a system returns to a given dynamical state. Assume a dynamical system governed by $\mathbf{x} = \mathbf{F}(\mathbf{x}) \in \mathbb{R}^m$. Based on the system response recorded at $N$ discrete points in time $(\mathbf{x}(i), i = 1 \ldots N)$ the threshold recurrence plot is formally constructed by forming the matrix

$$
R_{ij} = \Theta(\varepsilon - ||\mathbf{x}(i) - \mathbf{x}(j)||),
$$

where $\varepsilon$ is a threshold parameter representing the specific length scale of focus and $|| \cdot ||$ takes the Euclidean norm of the $m$-dimensional distance vector. Making use of the Heaviside function $\Theta(\cdot)$, the values of $R_{ij}$ are 1 or 0 depending on whether the distance between points $i, j$ is less than or greater than $\varepsilon$, respectively. A plot of the recurrence matrix is referred to as the recurrence plot (RP). Appropriate choice of $\varepsilon$ will be discussed later.

Eq. (1) assumes that each of the $m$ system variables has been measured directly which is typically not possible. In the case of a single measured variable $x(n)$ (the usual case in an experiment) the familiar delay coordinate approach may be used (see [13–15] for theoretical development). Delay coordinate reconstruction is the standard first step in most non-linear time-series analysis algorithms and proceeds by forming the reconstructed dynamics

$$
\mathbf{x}(n) = (x(n), x(n + T), \ldots, x(n + (m - 1)T))
$$

with delay $T$ and embedding dimension $m$. Procedures for choosing delay and embedding are given in [16,17], respectively. Unlike many approaches in non-linear time-series analysis, however, recurrence plots do not require the underlying system’s dynamical attractor to be faithfully reconstructed for many applications. Recurrence plots and the measures derived from them (to be discussed) simply quantify patterns in the dynamics and do not require that the “true” underlying dynamics be preserved in order to be effective. Recent work by Iwanski [18] has in fact shown that the ability of measures derived from recurrence plots to detect changing dynamics is largely independent of the embedding dimension $m$ and results obtained using $m = 1$ (no embedding) are often sufficient. For simplicity, we therefore propose using $m = 1$ in order to eliminate the selection of parameters $m, T$ from consideration.

Recurrence matrices, $R_{ij}$, are shown in Fig. 1 for three different processes: Gaussian noise, a sine wave, and the output of the chaotic Lorenz oscillator. Each of the plots were based on time series consisting of $N = 1000$ points and using $m = 1$ in Eq. (2). For constant $\varepsilon$, recurrence plots are symmetric about the diagonal, i.e. if points $i, j$ are close so too will be points $j, i$. Furthermore,
the main diagonal will always be populated. Analysis of these plots focuses on the diagonal line structures where a “line” is defined as \( l \geq 2 \) adjacent points with no intervening white spaces. Vertical lines reflect the traditional notion of autocorrelation. Closely sampled points on a single trajectory are near neighbours in both time and space. Diagonal line structures are representative of deterministic dynamics. Points that are near neighbours as a result of visiting the same region of phase space remain near neighbours for some number of time steps (i.e. one trajectory “shadows” the other). Periodic signals will therefore be characterised by the banded structure seen in the sine-wave plot of Fig. 1 where the distance between bands corresponds to the period and the thickness of the lines is dictated by the autocorrelation. A purely stochastic process will have little probability of exhibiting any kind of structure as near points at time \( i \) will not likely be near at time \( i + 1 \). Likewise, points on different strands will rarely remain near as time progresses, hence the absence of diagonal line structures. An RP of a Gaussian noise process is shown in the left plot of Fig. 1. Chaotic processes will typically exhibit finite (usually short) diagonal structures of length related to the inverse of the positive Lyapunov exponent (Lyapunov exponents quantify the local exponential divergence or convergence of nearby trajectories). Trajectories will remain near for a short time before diverging due to the sensitive dependence on initial conditions. This effect can be seen in the RP of a chaotic Lorenz time series shown at the right of Fig. 1.

Recurrence plots were designed to detect non-stationarity in time-series data and are therefore a candidate for detecting damage-induced non-stationarities in structural response data. This particular view of system dynamics does not involve a linear transformation of the data (e.g. FFT, AR modelling) and represents a more general, probabilistic approach to extracting structure in time-series data. Furthermore, analysis of these plots has shown superior performance to linear-based approaches in detecting non-stationarity in time-series data in previous applications [1,3].

2.1. Cross-recurrence plots

The approach described above is easily altered to allow comparisons of two different processes, \( x(n) \), \( y(n) \), by forming the cross-recurrence matrix

\[
CR_{ij} = \Theta(\varepsilon - ||x(i) - y(j)||).
\]
Two systems that tend to occupy similar states will have a large percentage of recurrence points. Furthermore, systems that “shadow” one another by following similar dynamical paths will tend to produce diagonal line structures the length of which is related to the degree of dynamical similarity between the two processes. This particular type of dynamical comparison is well suited to the problem posed in SHM, that is, to make a probabilistic assessment of the degree to which the dynamics of the undamaged system, $x(n)$, are different then those of the system under various stages of degradation, $y(n)$. The focus in the next section is therefore on the “features” that can be extracted from these plots, their meaning, and their respective abilities to detect damage in structures.

2.2. Extension to multivariate observations

In structural dynamics it is often the case that the practitioner has some number $K$ of sensors distributed on the structure being monitored. Sensors at different spatial locations will record different information. Changes in the dynamics at one location on a structure may be visible using RQA while at other locations no change is detected. Using RQA metrics computed from time series at varying locations will be the subject of future work. Here, we wish to incorporate the information from each of the sensors into the analysis. Extending RQA to multivariate data is related solely to the way in which the phase space required of Eqs. (1) and (3) is constructed. We begin with a multivariate time series of structural response data $x_i(n), n = 1 \ldots N$ which denotes the $i$th signal (time series) sampled at discrete time $n$. In general, the system’s dynamical attractor may be reconstructed by considering delayed copies of each of the signals forming the delay vectors

$$x(n) = (x_1(n), x_1(n + T_1), \ldots, x(n + (m_1 - 1)T_1),$$
$$x_2(n), x_1(n + T_2), \ldots, x(n + (m_2 - 1)T_2),$$
$$\ldots,$$
$$x_K(n), x_1(n + T_K), \ldots, x(n + (m_K - 1)T_K).$$

(4)

As in the univariate case, we simply take $m_i = 1$ $i = 1 \ldots K$ giving the phase space representation $x(n) = (x_1(n), x_2(n), \ldots, x_K(n))$. Once the multivariate reconstruction has been performed the multivariate recurrence and cross-recurrence plots (MRP, MCRP) may be formed by applying Eqs. (1) and (3).

3. RQA

RPs and CRPs are rich in information content. As previously mentioned the various line structures reflect autocorrelation and the presence of deterministic dynamics. In the case of CRPs, these lines relate to the probability that two different systems are obeying the same dynamics. Webber and Zbilut [3] termed the use of measures derived from recurrence plots RQA. Five basic metrics appear in the literature, several of which are appropriate for use in SHM.
Percent recurrence is simply the percentage of darkened points. For an \( N \) point time series this measure is simply given by

\[
\% \text{ recurrence} = \frac{\sum_{i=1}^{N} \sum_{j=1}^{N} R_{ij}}{N^2}.
\]  

(5)

This particular metric reflects the frequency that a given trajectory will visit a local region of phase space defined by \( e \). In the case of cross-recurrence plots this metric quantifies the probability that two different systems are occupying the same region in phase space. As the dynamics become more dissimilar, one would expect this probability to decrease.

Percent determinism is the percentage of darkened points occupying significant line structures where “significant” is user-defined. Due to the aforementioned rarity of consecutive recurrence points for random data \( l_{\text{min}} = 2 \) is usually used. Denoting \( N_l \) the number of lines of length \( l \), percent determinism is given by

\[
\% \text{ determinism} = \frac{\sum_{l=l_{\text{min}}}^{N} l N_l}{N_r},
\]  

(6)

where \( N_r \) are total the number of recurrence points given by the numerator of Eq. (5). Deterministic systems will re-visit the same area of phase space given a long enough time. By definition deterministic systems follow some dynamical rule (e.g. \( \dot{x}(n) = F(x(n)) \)) and therefore their nearby trajectories will evolve in similar fashion, at least over short time scales. The measure defined by Eq. (6) directly quantifies this notion of determinism. Computed from a CRP, \% determinism reflects similarity in the way the two constituent systems are evolving. Damage is expected to decrease the probability that trajectories from different systems (damaged and undamaged) will follow the same dynamical path, hence the measure is expected to decrease.

RP entropy is a measure derived from information theory and can be used to quantify the complexity of a recurrence plot. Let the probability of finding a line of length \( l \geq l_{\text{min}} \) be denoted by

\[
P(l) = N_l / \sum_{\xi=l_{\text{min}}}^{N} \xi N_\xi.
\]  

(7)

The Shannon entropy (see [21] for a detailed discussion) is then given by

\[
\text{Entropy} = - \sum_{l=l_{\text{min}}}^{N} P(l) \log_2 P(l).
\]  

(8)

Entropy reflects the information content in a given variable, in this case, line lengths. If all lines are of length \( l = 2 \), \( P(2) = 1.0 \) \( P(l > 2) = 0.0 \), the entropy is zero and the dynamics are not very complex with respect to line length. On the other hand, a wide variety of line lengths implies greater complexity and will produce a higher entropy value. Entropy is therefore an effective feature for tracking changes in dynamical complexity. It should be noted that this particular metric should not be thought of as an estimation of the entropy of the system but is instead simply the entropy of the recurrence plot (or CRP). While it may produce reasonable estimates of the “true” entropy, more accurate methods can be found in the literature.
The procedure for using these features in a health-monitoring context is as follows. Using the baseline data, form the RP (or MRP) and compute the above-described metrics in order to assess their values in the event of no damage to the system (i.e. compute baseline statistics). As damage is incurred, CRPs (or MCRPs) are then computed between the undamaged and various “damaged” response data. The idea, of course, is that as the joint phase space of the two sets of response data become more dissimilar, the structure in the CRP will be lost which will in turn be reflected in the metrics % recurrence, % determinism, and entropy. The only remaining step is to select the appropriate scale for constructing the recurrence plot.

4. Selection of $\varepsilon$

The crucial parameter in RQA analysis is the threshold length scale $\varepsilon$. Obviously, $\varepsilon$ must be greater than some measure of the noise floor, otherwise the RP or (CRP) will display no dynamical structure and will therefore be unable to resolve damage-induced changes to the dynamics. Because most of the RQA metrics are designed to focus on diagonal line structures in a recurrence plot (or cross-recurrence plot) it makes sense to employ an objective function that specifically highlights such structures. At the same time, the focus of this type of analysis is on the local (in a geometric sense) evolution of points associated with underlying system dynamics i.e. $\varepsilon$ must remain small. Most importantly, we would like to choose $\varepsilon$ such that subtle changes to the dynamics (i.e. damage) will have a large effect on the resulting RQA metrics. Here, we employ an empirical procedure that utilises the baseline data $x(n)$ to establish the appropriate $\varepsilon$.

As a first step, one can compute the number of statistically significant diagonal lines by summing along the diagonals of the baseline RP to create the histogram

$$h_i = \sum_{k-j=i} R_{jk}$$

and then counting the number of significant lines as

$$N_p = \sum_i \Theta(h_i - (\mu_h + 3\sigma_h)),$$

where $\mu_h$, $\sigma_h$ are the mean and standard deviation of the histogram, respectively. Here we take $>3\sigma_h$ to be “significant”. This step is identical to the one outlined by Matassini et al. [22] in selecting $\varepsilon$ for noise reduction purposes. To guarantee we are above the noise floor we want to choose $\varepsilon$ such that a “stable” pattern is present that is indicative of the system dynamics rather than noise. Ideally we want $\Delta N_p(\varepsilon)/N_p(\varepsilon) \leq \delta < 1$, where $\delta$ is a user-defined threshold for convergence, chosen so that the number of significant peaks is nearly constant as a function of $\varepsilon$. In practice, for $\varepsilon$ below the noise floor, $N_p$ will vary considerably as no stable pattern emerges. Beyond the noise floor $N_p$ will typically settle down to some value and then change slowly with $\varepsilon$. However, as was previously mentioned, we also want to keep $\varepsilon$ at a minimum such that small changes to the dynamics will disrupt this pattern and be clearly highlighted when computing RQA metrics from a CRP. By minimising the $\varepsilon$ used we are essentially “balancing” the RP on the edge
of the noise floor such that the slightest change to the dynamics will destroy the structure in the CRP and damage can be seen.

There is still another consideration when choosing the value of $\varepsilon$. Each of the metrics described in Section 3 will saturate when the dynamical structure in a CRP is lost. For example, if we have optimised $\varepsilon$ for detecting subtle changes, both % determinism and entropy will tend toward zero for large changes in the dynamics. In other words, once the structure of an RP or CRP collapses, these metrics will be unable to resolve differences between large amounts of damage. In many cases, where early detection of damage is the goal, this does not pose a problem. However, if one wishes to use this approach to also classify the magnitude of the damage, one must use a slightly larger $\varepsilon$. This effect will be explored in the results.

As an example, consider again the output of the chaotic Lorenz oscillator, depicted in Fig. 1 as a RP. Fig. 2 shows the function $\Delta N_{p}/N_{p}$ for 0%, 10%, 20%, 25% additive Gaussian noise. The threshold for convergence is indicated by the dotted lines and was taken to be $\delta = \pm 0.15$. This value for $\delta$ may seem large, however it will often represent a change of only $\Delta N_{p} = 1$ or 2 peaks as $N_{p}$ will typically be in the range $15 \leq N_{p} \leq 30$. From the figure it is clear that as the noise floor increases, so too will the value of $\varepsilon$ required to produce a meaningful RP. For the no noise case a value of $\varepsilon < 0.01$ would be appropriate while for the case of 25% additive noise $\varepsilon \geq 0.03$ would be required. The 10% and 15% noise cases indicate $\varepsilon = 0.018$ and $\varepsilon = 0.022$, respectively.

For structural health monitoring purposes we therefore prescribe (1) compute the RP using the baseline data, (2) forming $h_{1}$ (Eq. (9)) and computing $N_{p}(\varepsilon)$, (3) numerically differentiating this function and searching for the minimum $\varepsilon$ such that $|\Delta N_{p}(\varepsilon)/N_{p}(\varepsilon)| \leq \delta$. We have found that generally $0.1 \leq \delta \leq 0.2$ coincides with $\Delta N_{p} = 1$ or 2 and tends to produce good results. The

![Figure 2](image-url)
resulting value for $\varepsilon$ is taken as optimal in the sense that it will produce RQA metrics that are stable under the condition of no damage to the system, yet exhibit large changes as damage is incurred. If the magnitude of the damage is also desired, the practitioner may want to increase $\varepsilon$ slightly in order to expand the meaningful range of RQA metric values. We now turn our attention to the performance of the RQA metrics with regard to their respective abilities to quantify damage in a structure.

5. System

5.1. Finite element plate structure

All results presented in this work were generated from a Reissner–Mindlin finite element model of a thin plate. A direct Newmark integration scheme was used to simulate the dynamic response of a 624 element model of a plate with dimensions $0.660 \times 0.408 \times 3.175 \times 3$ m and material properties of 1018 hot-rolled steel ($E = 205$ GPa, $v = 0.3$, $\rho = 7859$ kg/m$^3$). A clamped-free boundary condition was imposed on the model. The damage mechanism in the model was a “cut” in the plate, obtained by removing the connectivity between nodes as shown in Fig. 3. Damage was imposed on the refined region of the mesh in increments of 5% of the plate width, starting with 0% and extending to 25%. A Rayleigh proportional damping model was implemented by constructing a damping matrix, $C$, which was composed of the mass and stiffness matrices scaled by coefficients $\alpha$ and $\beta$

$$C = \alpha M + \beta K.$$  \hspace{1cm} (11)

Assuming a damping ratio of $\zeta_1 = 0.009$ and $\zeta_3 = 0.004$ for modes one and three of the system with undamped natural frequencies $\omega_1 = 2\pi \cdot 39.715$ rad/s and $\omega_3 = 2\pi \cdot 101.207$ rad/s leads to coefficients equal to $\alpha = 4.418 + 0$ and $\beta = 1.184e - 6$.

Strain was observed at nine sensor locations in a $3 \times 3$ grid pattern on the plate (see circles in Fig. 3). Nodes are labelled bottom-to-top and left-to-right such that sensor 1 is located at the

![Fig. 3. Schematic of plate showing sensor locations and excitation point (left) along with a sample mesh shown with a 10% cut along the width (right).](image-url)
bottom left corner, sensor 3 is located at the top left corner, sensor 7 the bottom right corner, etc. The forcing input was applied at a node located 0.154 m from the lower left corner (in the horizontal direction) and 0.122 m from the bottom of the plate (see Fig. 3). The presence of the cut is expected to change the temporal correlations in the response time series. These differences will translate directly into a loss of both recurrence and determinism in CRP’s for the reasons mentioned in Section 3.

5.2. Excitation

The vibration-based SHM paradigm is predicated on being able to excite the structure in question through either ambient or applied loading. Much of the work reported in the literature utilises broad band or impact excitation sources for inducing a vibrational response from the structure in question. Broadband sources are considered ideal if the features of interest are the structure’s modal properties (e.g. natural frequencies, damping ratio, mode shapes, etc.). However, we are interested not in modal content, but in metrics reflecting the probabilistic recurrence structure of the vibrational response. Previous work by the authors [23,24] has demonstrated that using chaos to interrogate a structure can be used effectively in applying attractor-based or phase space methods to SHM. Because many of these methods are based on the idea of spatial recurrence (such as the approach used here) it is convenient to try and minimise the dimension of the response signal. Chaotic excitation signals can be shown theoretically and experimentally [25] to produce low-dimensional structural response data. From a practical standpoint generating and applying a chaotic signal is no different then using Gaussian noise. Essentially, one can take advantage of the broadband nature of chaos in order to excite the desired number of structural modes while at the same time inducing a low-dimensional, deterministic structural response.

To this end we impart a chaotic vibration on the plate. Specifically, the excitation signal was generated as output of the Lorenz oscillator which is described by the three first-order differential equations

\[
\begin{align*}
\eta \dot{z}_1 &= \sigma (z_2 - z_1), \\
\eta \dot{z}_2 &= rz_1 - z_2 - z_1z_3, \\
\eta \dot{z}_3 &= -bz_3 + z_1z_2,
\end{align*}
\]

(12)

where \(\sigma = 10\), \(r = 28\) and \(b = \frac{8}{3}\). The parameter \(\eta\) is used to adjust the bandwidth of the oscillator; smaller \(\eta\) increases the bandwidth, larger \(\eta\) decreases the bandwidth. It should be mentioned that RQA is not dependant on deterministic structural response data. To the contrary, the technique was designed as a way of analysing short, noisy time series possessing non-stationarity. Many of the applications of RQA to date have in fact been in physiology, where biologically generated signals are being analysed. This analysis technique is therefore not restricted to systems where the form of the excitation can be controlled.

Fig. 4 shows the time series and power spectral density (PSD) of the input signal and the plate response. The excitation exhibits a linear decay in PSD amplitude with frequency, a characteristic of the Lorenz oscillator. The first several structural modes can be seen in the PSD of the plate’s response beginning at 39 Hz. Excitation was applied and the response recorded for each of the six
plate conditions (undamaged + five damage levels). Each of the strain response time histories consisted of \( N = 45\,000 \) points and were recorded at a sampling rate of 2 kHz. Prior to analysis the time series were normalised to zero mean and unit standard deviation. It should be mentioned at this time that the magnitude of the damage was unknown a priori. The analysis began with five “damaged” time series (labelled A, B, C, D, E) and attempted to rank them according to damage magnitude. The only “known” condition was the undamaged or pristine plate response. In order to generate a ranking it was assumed that the progression of RQA metric value with damage would be monotonic.

6. Results

A phase space description for the undamaged process was constructed by incorporating all nine time-series records to give \( \mathbf{x}(n) = (x_1(n), x_2(n), \ldots, x_9(n)) \). The recurrence plot was then formed, and the ratio \( (\Delta N_p/N_p) \) computed for a variety of \( \varepsilon \) values. Results of this computation are shown.

![Fig. 4. Time series and power spectral density of the input (top) and plate response (bottom) recorded at location (A) on the plate.](image-url)
in the left plot of Fig. 5 and indicate an $\varepsilon \geq 0.01$ will generate a stable, structured RP. Here we have used a “settling” threshold of $\delta = 0.2$, indicated by the dotted lines. Smaller $\varepsilon$ values show the large fluctuations indicating a lack of a stable dynamical pattern. Based on this result a value $\varepsilon = 0.15$ was deemed appropriate for performing the RQA analysis. The right plot of Fig. 5 shows the histogram $h_i$ corresponding to $\varepsilon = 0.15$. The dotted horizontal line in this plot corresponds to the $\mu_h + 3\sigma_h$ “cut-off” for what constitutes a significant diagonal line.

Once $\varepsilon$ was selected, the baseline MRP was formed along with the MCRPs based on the “undamaged” and each of the five “damaged” sets of data. Fig. 6 shows the MRP for the undamaged vs. undamaged comparison along with the MCRP formed between the undamaged and damage scenario (D) data. As damage is incurred the dissimilarity between the baseline and damaged response data is clearly evident. The number of recurrence points is diminished as is the presence of diagonal lines.
In order to quantify these differences, the 3 RQA analysis metrics described above in Section 3 were computed from the plate response time series. To this end each time series was divided into overlapping segments (windows) consisting of \( N = 10000 \) points each. For each segment the RQA metrics were computed, the window time shifted by 1400 points (each window overlaps the next by 8600 points) and the process repeated giving a total of 25 feature values per damage level for each metric (% recurrence, % determinism, and entropy). In order to generate confidence intervals for the mean feature values we use the bootstrapping procedure described in [26]. Essentially one creates surrogate feature sets by considering randomly re-ordered groupings of the original 25 feature values. The resulting distribution of surrogate set means will have a normal distribution (according to the central limit theorem) and confidence intervals may be assigned as \( \mu_f \pm Z_{\alpha/2} \sigma_f \) where \( \mu_f, \sigma_f \) are the mean and standard deviation, respectively, of the surrogate means. The \( Z \) values are tabulated in most introductory statistics texts for a prescribed level of confidence, \( \alpha \). Here we take \( \alpha = 0.05 \) giving 95% confidence intervals. Assessing confidence in the feature values is an essential part of the SHM process, especially if the practitioner is to account for the effects of experimental variability and/or environmental fluctuations.

Fig. 7 shows the progression of mean feature value with damage for each of the five damage levels along with the corresponding (normal) probability density functions (PDFs). Confidence limits for the undamaged vs. undamaged comparison are highlighted in grey and represent the null hypothesis of no damage to the system.

Each of the damage scenarios were ranked based on an assumed monotonic progression of feature value with damage. For example, because scenario (D) had the greatest difference in percent determinism from the undamaged case it was assumed to come from the time series recorded at 25% damage. Conversely, scenarios (A and E) showed the least change from undamaged with regard to per cent determinism and were therefore assumed to correspond to the 5% and 10% damage case, respectively. Clearly, as damage is incurred the various metrics are able to discern changes to the system dynamics with notable differences in performance. Per cent recurrence is the best performer in that it clearly identified the correct ordering of the damage scenarios (A,E,C,B,D) and was also able to identify damage cases C,B,D (15%,20%,25% damage) as coming from statistically different distributions. Only the first two damage scenarios

![Fig. 7. Progression of recurrence-based features with damage (95% confidence). Shown are % recurrence, and % determinism for \( \varepsilon = 0.015 \) and entropy for \( \varepsilon = 0.025 \).](image-url)
(A and E) were unable to be resolved at the 95% confidence level. Per cent determinism showed similar results and was able to recognise the 15–25% scenarios as coming from different dynamical processes as well. Because many of the PDFs are clearly separable (the confidence limits do not overlap) the presence and magnitude of the damage can also be inferred. The entropy metric was not able to show any clear indication of damage at the \( \varepsilon = 0.015 \) length scale. Meaningful results could only be obtained by increasing the search radius to \( \varepsilon = 0.025 \) implying that the method for selecting \( \varepsilon \) described in Section 4 may be inappropriate for if entropy is to be used as a damage sensitive feature. Even with the larger value for \( \varepsilon \) the entropy metric was only able to clearly resolve the most extreme damage scenario.

Next, we compared the RQA metrics to the plate’s modal frequencies with regard to sensitivity to damage. In this numerical example the plate frequencies are known exactly (to numerical precision). Modal frequencies are an often used feature in SHM (see [5,7,27]) and provide a convenient benchmark for comparison to the proposed technique. We recognise, however, that other features derived from modal analysis may possibly perform better for this particular damage scenario. For example, mode shapes or their derivatives (curvatures) may produce better results for this example. This is almost certainly true if the mode shapes were obtained directly from the FEM model which consisted of > 600 elements. Any meaningful comparison to mode shapes as a feature would necessarily involve a discussion of the estimation procedure from time-series data with regard to the number of sensors required to achieve the desired sensitivity.

Fig. 8 shows the per cent change in mean feature value as a function of damage for the RQA metrics, and the three most sensitive plate frequencies. Clearly both % recurrence and % determinism exhibit larger shifts in value as damage is incurred than do the modal frequencies. By tuning the parameter \( \varepsilon \) to an appropriate value, the practitioner can achieve gains in sensitivity over modal analysis. One is effectively “balancing” the feature values such that tiny variations from the pristine condition will cause large changes in the cross-recurrence plots. We also point out that only the first three frequencies are being excited using the chaotic interrogation approach (see Fig. 4). The frequencies associated with these modes exhibit only a few per cent (<5%) change with damage. Comparing the RQA features to the first three modal frequencies (rather

![Fig. 8. Percent change in recurrence-based features with damage along with three most sensitive modal frequencies (left), comparison with the first three modal frequencies (centre) and % change in value for the first 20 modal frequencies (right).](image-url)
than the most sensitive) therefore provides an even greater contrast in feature progression; the results of this comparison are shown in the centre plot of Fig. 8. Furthermore, it is unlikely that the practitioner would be able to obtain accurate estimates of modes 14 and 19 (the two most sensitive plate modes shown in Fig. 8). The right plot of Fig. 8 displays the progression of the first 19 plate frequencies with damage. One of the difficulties in using natural frequencies for tracking changes in structural dynamics is that it is unknown a priori which frequency will exhibit the most change with damage. Clearly some of the plate’s frequencies show little variation for even large damage levels.

As a final test, we repeated the RQA analysis on the plate response using a variety of $\varepsilon$ values spanning the range $0.005 \leq \varepsilon \leq 0.05$. Fig. 9 shows the progression of mean RQA feature value with damage for each of the three metrics for several of these $\varepsilon$ values. The most sensitive plate frequencies are again shown (modes 6, 14 and 19). Small values of $\varepsilon$ clearly produce the largest % change in feature value at the low damage level, particularly for the recurrence measure. Larger $\varepsilon$ values are less apt to indicate small damage levels but show a clear monotonic progression of feature mean with damage. These trends are consistent for both % recurrence and % determinism. The determinism metric does not, in general, behave as well as recurrence as is evident from the figure. This particular metric is not able to identify the 15% damage level and can behave “erratically” at low $\varepsilon$ values as evidenced by the jump in feature value at the 5% damage level. Entropy was not an effective metric for detecting damage. For almost all $\varepsilon \leq 0.05$ this measure showed no clear progression with damage and could only resolve the final damage scenario for large $\varepsilon$. Because this measure is designed to reflect variability in the line lengths it can only be effective if the distribution of the $P(l)$ is changing. However, we have ensured through our choice of $\varepsilon$ that few recurrence points will be present and that any lines will most likely be of minimal length (2 or 3). We believe that the only way for the entropy measure to be effective from a health-monitoring perspective is to choose large values of $\varepsilon$.

The final decision regarding $\varepsilon$ will clearly depend on the application. Per cent recurrence is the best performer in terms of both early detection of the damage and in producing a clear monotonic progression with damage magnitude. Entropy is the least capable of the three metrics. For proper

![Fig. 9. Progression of % recurrence (left), % determinism (centre), and entropy (right) with damage as a function of length scale $\varepsilon$. Also shown is the progression of the most sensitive modal frequencies for comparison.](image-url)
7. Conclusion

This work introduces the concept of recurrence plots and recurrence quantification analysis as a viable tool for detecting damage in structures. We have shown that the approach may be easily extended to include multivariate observations such as those coming from a spatially distributed network of sensors. We have also provided a straightforward empirical approach for selecting the length scale parameter $\varepsilon$, the only parameter required of this method. Based on a multivariate phase space construction, three different RQA metrics were used as damage-sensitive features in a blind test of their ability to discern the magnitude of damage in a finite element plate structure. Two of the metrics, % recurrence and % determinism, were able to correctly order most of the damage scenarios by magnitude. Furthermore, both metrics showed a greater sensitivity to damage than did the plate’s most sensitive modal frequencies. In particular the percentage of recurrence points, obtained from a cross-recurrence plot, appears to be an extremely effective tool for diagnosing subtle changes to system dynamics. Conversely, the entropy metric performed poorly and was only able to detect the most extreme damage scenario. The sensitivity of this approach combined with the fact that only a single parameter is required for the algorithm implementation makes it attractive for structural health-monitoring applications. In addition, because the approach is probabilistic in nature it requires no assumptions of the underlying structure (e.g. linearity).

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