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Use of Fiber-optic Strain Sensors and Holder Exponents for Detecting and Localizing Damage in an Experimental Plate Structure

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ABSTRACT: The vibration-based structural health monitoring paradigm is predicated on the practitioner's ability to acquire accurate structural response data and then to use that information to infer something about the structure's health. Here the authors combine advances in both sensing and signal analysis and demonstrate the ability to detect damage in a simple experimental structure. A distributed network of nine fiber-optic strain sensors is used to acquire time series data from a rectangular steel plate where damage is considered as a cut of varying lengths. Both the sensors and the associated optical hardware are described. A new feature, Holder continuity, is then introduced as a means of identifying the presence and location of the cut length. This particular metric is derived from the field of nonlinear dynamics and is based on a phase space description of a structure's dynamic response. Specifically, the authors compute the Holder exponent which quantifies the differentiability of the functional relationship between an ‘undamaged’ and a ‘damaged’ structural response. As damage is incurred, this relationship is expected to degrade. Both univariate and multivariate applications of the method are presented. The metric shows sensitivity to damage comparable to that exhibited by the plate’s modal frequencies, a traditionally used feature in health monitoring applications.

Key Words: Holder exponents, fiber Bragg gratings, structural health monitoring.

INTRODUCTION

S T R U C T U R A L health monitoring (SHM) represents a wide body of research concerned with assessing the condition, performance, and life expectancy of civil structures, vehicles, machinery, and/or the various components that comprise them. Research in this area is driven largely by the potential cost reduction associated with condition-based monitoring as opposed to the currently utilized time-based approach. Fixed maintenance schedules are necessarily designed around the ‘worst case scenarios’ for component life and are consequently costlier to maintain than needed in the majority of cases. By allowing the condition of the structure to dictate repair schedules or removal from service the costs associated with structural ownership are reduced and the operational lifetime is increased. Research in the area of SHM has received a great deal of attention in the literature. A recent survey conducted by Sohn et al. (2004) lists a number of papers devoted to the subject on a diverse set of topics. Two primary areas of current interest are improved sensing technology for data acquisition and the signal analysis techniques used in analyzing structural response data. This work represents an effort toward combining advances in both arenas to detect damage in a simple experimental structure. Specifically, the authors employ a novel fiber-based strain sensing system for acquiring low-noise structural response data and then use a new ‘feature’, the Holder exponent, for assessing the presence, magnitude, and location of structural degradation.

Advances in the SHM field can be attributed to the development of a new generation of sensors that are lightweight, provide low-noise measurements, and can be easily multiplexed. Indeed, the SHM problem requires that the sensors interfere minimally with the structure’s performance/specifications while providing precise measurements from multiple structural locations. In essence researchers are actively seeking a ‘smart structures’ approach whereby a distributed network of sensors is placed on (possibly embedded in) the structure in question to provide for continuous monitoring/control capabilities. To this end, efforts have included the use of accelerometers (Worden and Manson, 2003),
piezoceramic sensors (Inman et al., 2001; Fukunaga et al., 2002), and fiber optic sensors (Leng and Asundi, 2003). The system employed here is a fiber-based sensor network. First described in Todd et al. (2001a), this system utilizes a Fabry–Perot filter and a Mach-Zehnder interferometer to allow for easy multiplexing, low-noise floor, and higher sample rates than have been achieved with other fiber-based systems (see Todd et al., 2004 for an overview). Furthermore, because the sensing element is ‘etched’ into a single strand of optical fiber, there is essentially no weight penalty.

In the area of signal analysis, the goal is to take the information provided by the sensors, extract the appropriate damage sensitive feature, make a confidence-based judgment regarding the presence, type, location, and possibly the scope of the damage, and ultimately offer a prognosis. The assumption underlying vibration-based SHM is that damage will manifest itself as some unique signature in the structural response data and that the feature being extracted will capture this signature. Typically the practitioner computes the feature from the undamaged data and then compares to features computed from time series collected from a degraded structure. To date, a variety of features have been proposed and have achieved varying degrees of success depending on the structure being studied, the quality of the recorded data, and the type of damage incurred. Popular choices include those derived from a modal analysis e.g., frequencies, damping ratios, mode shapes (see Doebbling et al., 1998), wavelet-based features (Okafor and Dutta, 2000), and a variety of features derived from auto-regressive (AR) modeling techniques (Fassois, 2001; Owen et al., 2001; Bodeux and Golinval, 2003). While important differences exist, each of these approaches relies on an assumption of linearity at some level and a shared hypothesis that frequency content is the appropriate metric for detecting and classifying the damage. Modal methods and wavelet-based approaches are designed to track frequency content directly. Likewise changes in the properties of an AR model are also directly related to frequency content as the coefficients of such a model define the signal’s rational transfer function (under the assumption of Gaussian excitation). As has been pointed out by Adams and Nataraju (2002) (see also Brown and Adams, 2003), damage evolution is often appropriately modeled as a nonlinear process. Furthermore, the dynamics of many structures are most appropriately represented by nonlinear models. It is therefore appropriate to consider the possibility that features adapted from the nonlinear time series analysis community may be more effective at tracking and possibly localizing damage.

The approach presented here builds on previous efforts by the authors to extract damage sensitive features that are based on a phase space description of the dynamics. This class of features is popular in the physics and nonlinear dynamics communities for detecting subtle changes in time series data and is therefore appropriate for the problem posed by SHM. Specifically the authors focus on quantifying the difference between an ‘undamaged’ and a ‘damaged’ phase space object, or attractor, through the use of a Holder exponent. Holder exponents are a mathematical construct used to assess the continuity and differentiability of a function and are used here to track the breakdown in the functional relationship between attractors formed from ‘undamaged’ and ‘damaged’ response data. It should be mentioned that Holder exponents have been used in SHM to detect local discontinuities in structural response data (Robertson et al., 2003). Here, however, the function being analyzed is not the time series itself but rather the hypothesized relationship between two dynamical attractors.

VIEWING SHM IN PHASE SPACE VIA CHAOTIC EXCITATION

Phase space analysis of time series is a common approach used in the nonlinear dynamics community. Phase space methods have been used to estimate the dimensionality of a system (Nichols and Virgin, 2001), make predictions (Abarbanel et al., 1994), reduce noise (Kantz et al., 1993), and track changes in system dynamics (Gao et al., 2003). The primary advantages of using these tools are (1) freedom from many assumptions about the underlying process, (2) metrics are easily localized in time (alternatively phase space) as opposed to being globally averaged quantities (e.g., frequencies, mode shapes), and (3) have been demonstrated to possess increased sensitivity to changes in system parameters relative to certain linear-based measures (Webber-Jr. and Zbilut, 1994; Marwan et al., 2002; Zbilut et al., 2002). The improved sensitivity to change coupled with the knowledge that damage may be producing nonlinearities in the system has motivated the authors to consider how these tools might be used in the SHM community. Recent metrics used by the authors have included measures of attractor variance (Todd et al., 2001b), attractor-based prediction error (Nichols et al., 2003b), and continuity (Moniz et al., 2004). Other recent works applying phase space methods to damage detection include Chelidze et al. (2002), Trendafilova (2003), Epureanu and Yin (2004), and Bukkapatnam et al. (2000).

Univariate and Multivariate Attractor Reconstruction

The common first step to any attractor-based approach is to form the system’s attractor through either direct measurement or reconstruction. A system’s
dynamical attractor is defined by the state variables representing the system dynamics. For a system governed by a set of ordinary differential equations, \( x(n) = F(x(n)) \), the attractor is comprised of the post-transient (steady-state) solution, \( x(n) \). Here we use boldface type to denote a vector i.e., \( x(n) = (x_1(n), x_2(n), \ldots, x_N(n))^T \) for an \( N \)-D.O.F system where the \( x_i(n) \) are the state variables observed at discrete time \( n \). A phase space description of this system can be realized by simply plotting the state variables against each other. Ideally the experimenter will directly measure the attractor; however, the typical experiment allows for only a limited number of measurements. In this case, the practitioner may use the embedding theorems (see (Takens, 1981; Sauer et al., 1991) which state that a single measurement, \( x_1(n) \) for example, may be used to form the delay coordinate map

\[
x(n) = (x_1(n), x_1(n + T), x_1(n + 2T), \ldots, x_1(n + (m - 1)T))
\]

where the integer \( T \) is a measure of time delay and \( m \) is the embedding dimension. The dynamical description given by Equation (1) is proven by the embedding theorems to be topologically equivalent to the ‘true’ attractor \((x_1(n), x_2(n), \ldots, x_N(n))\) and preserves many attractor-based measures. Methods for choosing the delay and embedding dimension are well established and the reader is referred to Williams (1997) for a discussion.

If the practitioner has access to some number of state variables \( K \), a multivariate reconstruction may be performed using the same procedure, i.e., form the delay vectors

\[
x(n) = (x_1(n), x_1(n + T_1), \ldots, x_1(n + (m_1 - 1)T_1),
\]

\[
x_2(n), x_2(n + T_2), \ldots, x_2(n + (m_2 - 1)T_2),
\]

\[
\ldots,
\]

\[
x_K(n), x_K(n + T_K), \ldots, x_K(n + (m_K - 1)T_K)
\]

(2)

where each measurement has an associated delay and embedding dimension. Under the multivariate approach, delays are chosen in the same fashion as for the univariate case; however, a multivariate approach to selecting the \( m_i \) (number of delay coordinates per time series) is required. The reader is referred to Boccaletti et al. (2002) and Moniz et al. (2002) for a discussion.

**Relationships between Attractors in the Context of SHM**

Although many phase space methods (including the one described here) are applicable to stochastically excited structures (see for example (Nichols, 2003; Trickey et al., 2004) it is generally considered desirable to minimize the volume of the phase space (control the dimension). A stochastic signal is, in theory, infinite dimensional and the number of points required to adequately ‘fill’ a systems phase space i.e., populate the attractor, scales as a power of dimension. Large embedding spaces will, for a fixed amount of data, decrease the density of points on an attractor thus decreasing the quality of attractor-based metrics. One way to force a structure’s response to be low dimensional yet still involve the desired number of structural modes is to use chaotic forcing. Chaotic excitation signals are deterministic, that is to say exact knowledge of the state of a chaotic system at discrete time \( n \) combined with the governing equations is entirely sufficient to predict the exact state of the system at any future time \( n + T \). As a direct result, chaotic excitation can be shown theoretically and experimentally (Nichols et al., 2003a) to produce low-dimensional structural response data. By contrast, signals typically used for excitation purposes are stochastic (e.g., Gaussian noise). Stochastic signals are by definition randomly chosen values with probability of occurrence dictated by their probability density function. Stochastic signals are considered ideal for health monitoring applications because they tend to be broadband in the frequency domain and can excite a number of structural modes. However, many chaotic signals are also broadband in the frequency domain allowing for a large number of structural modes to be excited while at the same time minimizing the phase space dimension of the response.

Let the dynamics of the entire system, structure + excitation, be governed by the following set of ordinary differential equations

\[
\frac{dz}{dt} = G(z)
\]

\[
\frac{dx}{dt} = Ax + Bz
\]

(3)

where the dynamical variables \( z \) are the solution to a chaotic system described by the nonlinear function \( G \) and the variables \( x \) are the structure’s response. Here we have assumed a linear structure (see discussion at the end of this section) governed by the constant coefficient matrix \( A \) which will be expected to change as damage to the structure is incurred. The matrix \( B \) simply acts to couple the output of the chaotic system to the structure. It has been shown (Nichols et al., 2003a) that if the ‘speed’ parameter \( \eta \) associated with the chaotic system is adjusted properly (changing \( \eta \) simply decreases or increases the bandwidth of the chaos), the structure’s response will be (a) low dimensional and (b) reflect damage as a change to its phase space properties specifically the dimension. It is noted that for a linear structure, dimension changes in the response can only be guaranteed using a chaotic driving. Sinusoidal excitation will, for example, produce a low-dimensional structural
response (1-D). However, regardless of the amount of damage the dimension of the response will remain unchanged.

The paradigm used here is therefore as follows. The undamaged structure is excited with a chaotic signal, designed using the approach described in Nichols et al. (2003b). The time series is then recorded at one or more locations on the structure and the corresponding ‘undamaged’ attractor is reconstructed in order to characterize the healthy phase space dynamics. Data are also recorded at a later time when the structure is in a (presumably) damaged condition; these data are used to reconstruct the ‘damaged’ attractor. It is explicitly stated that the goal here is to discern the difference between a ‘damaged’ and an ‘undamaged’ attractor by analyzing properties of the functional relationship between the two. The idea is that as damage is incurred these differences will be highlighted by a degradation of this relationship. A schematic of this approach to the problem is illustrated in Figure 1. The drive attractor \( Z \) excites the structure giving rise to undamaged \( X \) and damaged \( Y \) response attractors.

The assumption of a linear structure in Equation (3) is not strictly necessary for the proposed approach to work. In fact much of the motivation for using phase space methods is their applicability to linear and nonlinear systems alike. A linear structure is, however, much easier to deal with theoretically, and chaotically driven linear structures are no different in this regard. The theorems that justify our approach relate specifically to the system described by Equation (3), a so-called ‘skew-product’ system studied extensively by Davies and Campbell (1996). In this work the authors adhere to this specific forcing/structure model but note that the described approach and associated algorithm(s) can work for linear or nonlinear structures subject to deterministic or stochastic driving.

**HOLDER CONTINUITY**

The central question of interest is what happens to the functional relationship between system outputs \( (X, Y) \) in Figure 1 as damage is incurred? Assume that two time series have been recorded, one from the structure in an undamaged configuration denoted \( x(n) \), and the other from the structure with some unknown amount of degradation denoted \( y(n) \). Each of these time series may be used to reconstruct the associated attractors \( x(n), y(n) \). For the case of no damage (and neglecting other sources of changes to the dynamics), the relationship between two different system outputs should be the identity \( x(n) = y(n) \). As the structure becomes damaged it is hypothesized that this relationship will change such that \( y(n) = \psi(x(n)) \).

Although a linear structure has been assumed, the functional relationship \( \psi \) may vary nonlinearly with damage. There has been quite a bit of work done on the nature of the function from drive to response in the case of a linearly filtered nonlinear signal but little work has been done on the nature of the function from one response to another. Various theorems (Davies and Campbell, 1996; Hunt et al., 1997; Afraimovich et al., 2001; Hadjiloucas et al., 2002) indicate the differentiability and/or Holder continuity of the function from drive to response is dependent on the relation of the characteristic exponents (eigenvalues) of the linear filter to the driving function’s parameters. Thus it is conjectured that as damage is incurred there is a loss of differentiability of the function \( \psi \) between the two responses directly as a result of the change in stiffness of the material and the corresponding changes in the characteristic exponents of the filters. If this conjecture is correct, an optimal metric for detecting damage would be one that quantifies the differentiability of the function \( \psi \). This may be accomplished by computing an estimate for a local Holder Exponent at various attractor locales.

The mathematical definition of a Holder exponent is as follows. There exist global quantities \( C \) and \( 0 < \alpha < 1.0 \) such that for all \( x, x_0 \in X, \) one has

\[
\| \psi(x) - \psi(x_0) \| \leq C \| x - x_0 \|^\alpha
\] (4)

The quantity \( \alpha \) is the Holder exponent, and \( \psi \) is then said to be Holder continuous. It can be noted that \( \alpha = 1.0 \) is a necessary condition for differentiability of \( \psi \). Thus, the Holder exponent quantifies the degree of nondifferentiability of the function \( \psi \). To define local Holder continuity, one does not require the constants \( C \) and \( \alpha \) to be defined globally. Instead, a small neighborhood is taken around each \( x \in X \) and it is required that \( \psi(x) \) be bounded in the same way, i.e., for a
small deviation $\delta$ around $x(f)$, there exist $C$ and $0 < \alpha < 1.0$ such that:

$$\| \psi(x(f) + \delta) - \psi(x(f)) \| \leq C \| \delta \|^\alpha$$ (5)

More simply, for a small deviation $\delta$ around $x(f)$ the local Holder exponent $\alpha$ quantifies the expansion of $\delta$ under the action of $\psi$. Changes in the distribution of local Holder exponents capture the gradual transformation of $\psi$ from an identity function to a non-differentiable function as the linear filter (structure) changes.

Making this mathematical definition realizable in an algorithm involves first reconstructing the system’s dynamical attractor via the delay coordinate approach using either univariate, or multivariate observations. A randomly selected fiducial point with time index $f$ is then selected on the attractor $x(f) \equiv (x(f), x(f + T), \ldots, x(f + (m - 1)T))$. The nearest $M$ neighbors to this point are then selected using the Euclidean distance in phase space. These neighbors become candidate points for computing the Holder exponent as they are all a small distance $\delta$ away from $x(f)$. For each of the $M$ points

$$\delta(j) = \|x(j) - x(f)\| \quad j = 1 \ldots M$$

is computed and the corresponding distance on the second (presumably ‘damaged’) attractor

$$\hat{\delta}(j) = \|\psi(x(f) + \delta(j)) - \psi(x(f))\|$$

$$= \|y(j) - y(f)\| \quad j = 1 \ldots M.$$ 

The operator $\| \cdot \|$ takes the Euclidean norm. The Holder exponent is then extracted as

$$\alpha = \max \left[ \frac{\ln \hat{\delta}(j) - \ln C}{\ln \delta(j)} \right] \quad j = 1 \ldots M. \quad (6)$$

In this work the constant $C$ is assumed to be unity so that Equation (6) simply reduces to the ratio of the logs of the two distances. This constant acts as a scaling parameter and may be increased for data with large variances where the distances $\delta$ are greater than unity. If the situation arises where $\delta < 1$ and $\hat{\delta} > 1$ a negative value for $\alpha$ will result (in theory $\alpha$ should reside in the range $[0, 1]$). All data in this study are normalized to zero mean and unit standard deviation prior to analysis so that $C=1$ is deemed sufficient. Similar ‘scaling’ issues can occur due to the embedding process. Higher dimensional embeddings will, for a finite amount of data, necessarily increase the average interpoint spacing. The simple solution is to scale the distance vectors by the dimension $m$ being used in the computation. For example, when computing the Euclidean distance between points in $m$-dimensional space, one can scale by the dimension so that $\|x(j) - x(f)\| = 1/m \sqrt{\sum_{i=1}^{m}(x(j+i-1)T) - x(f+i-1)T)^2}$. This same approach was applied in Finn et al. (2003) for scaling the distances involved in the estimation of information dimension and is employed here.

The final value of $\alpha$ is taken to be the maximum value of the $M$ possible candidates as dictated by the inequality (5). Some of the points $j$ will be near to $f$ due to noise rather than the actual system dynamics. Such points will likely produce spuriously low values of $\alpha$ that are not reflective of the ‘true’ properties (e.g., continuity, differentiability) of $\psi$. Tolerance to noise can be adjusted by simply increasing the ‘candidate’ size of nearby points, $M$. Values of $\alpha$ near unity suggest (although do not prove) the function $\psi$ connecting the two attractors is differentiable while values near 0 indicate a lack of differentiability. It is expected that as damage is incurred, the Holder exponent measured between an undamaged and a damaged attractor will drop. The above-described process can be repeated for some number of randomly chosen fiducial neighborhoods resulting in a collection of Holder exponents which will be denoted as $\hat{\alpha}$. This vector of Holder exponents can be recorded at each damage level and used to assess the health of the structure.

One final numerical issue concerns the rare case where $\delta < \hat{\delta}$ resulting in $\alpha > 1$. The practitioner can either set $\alpha = 1$ or amend Equation (6) such that the maximum $\alpha$ is taken such that $\alpha < 1$. Here the latter approach is taken although it is noted that both produce nearly identical results. Despite the apparent complexity, this and other phase space algorithms are quite simple to implement. The key step in attractor-based algorithms is the near neighbor searching. If done in the naive $O(N^2)$ fashion (for an $N$ point time series), phase space methods are not practical. However, there exist two different approaches that reduce the computational burden to $O(N \log N)$ for near neighbor searches; these are the box-assisted approach outlined in Grassberger (1990) (source code for this algorithm is given in Kantz and Schreiber, 1997), and K–D trees, described in Bentley (1979) and more recently in Hjaltsi and Samet (1995). Once the near neighbors have been found, the rest of the Holder algorithm simply involves computing Euclidean distances and taking the ratio of their logs.

**STATISTICAL CONSIDERATIONS**

As a final step the practitioner must consider whether or not the collection of features is showing a statistically significant change as damage progresses. This step is complicated by the fact that for many features one does not know the underlying feature distribution a priori. Rather the distributions must be formed empirically and the practitioner must make a judgment at a prescribed level of confidence as to whether or not two different
distributions are the same. More formally, one seeks to test the null hypothesis

$$H_0 : \tilde{a}_0 = \tilde{a}_k$$

(7)

for a given damage level 'k'. The null hypothesis states that there is no difference between the feature distributions (in this case holder exponents) of the undamaged $\tilde{a}_k$ and damaged $\tilde{a}_k$ feature sets. It is tested against the alternative

$$H_1 : \tilde{a}_0 \neq \tilde{a}_k$$

(8)

that there is a statistically significant difference and that damage does exist. In the absence of an underlying distribution one may utilize the empirical Kolmogorov–Smirnov (K–S) test (Press et al., 1992). This test compares the cumulative distribution functions of two different data sets and looks for the maximum difference between the two, $D$, as a measure of dissimilarity. This difference is referred to as the K–S statistic or K–S distance and will be denoted $D$. The distribution for $D$ is known (to approximation) and therefore $D$ values can be compared against those expected in the case that $H_0$ holds, i.e., no damage. Rejection of the null indicates damage at the prescribed level of confidence. The K–S statistic is a particularly useful metric for tracking damage evolution in a system and has been used recently to detect damage in ball bearings (Chinmaya and Mohanty, 2004) and in detecting ‘rubbing’ phenomena in rotating machinery (Hall and Mba, 2004).

EXPERIMENT

Structure and Excitation

The structure of interest considered here is a thin steel plate structure measuring 764×408×3 mm³. The plate is clamped along the two shorter edges in a ‘fixed–free’ configuration as shown in Figure 2. Damage in the plate was induced by first cutting the plate into two at approximately two-third the length and then reconnecting the two pieces with a series of 20 2×2 in.² brackets made from the same material (Figure 2). A 3 mm gap was left between the two halves to ensure that the sides did not make contact during vibration testing. Damage to the structure is controlled by removing various combinations of the brackets allowing both the location and the magnitude of the degradation to be varied.

In order to perform vibration testing, an MB Dynamics shaker was coupled to the plate by means of a threaded rod at a location 123 mm from the free edge of the plate and 210 mm from the nearest fixed edge (Figure 2). Both the shaker and the stinger were oriented in the vertical direction, perpendicular to the plate surface. In-line with the stinger is a 25 lb load cell used for recording the input signal. The excitation signal was generated as output of the Lorenz oscillator which is described by the three first-order differential equations

$$\eta \ddot{z}_1 = 10(z_2 - z_1)$$

$$\eta \ddot{z}_2 = 60z_1 - z_2 - z_1z_3$$

$$\eta \ddot{z}_3 = -(8/3)z_3 + z_1z_2.$$  

(9)

These three differential equations collectively represent the nonlinear function $G(.)$ alluded to in Equation (3). The parameter $\eta$ is used to adjust the bandwidth of the oscillator; smaller $\eta$ increases the bandwidth, larger $\eta$ decreases the bandwidth. Here $\eta = 15.00$ is used. This value is large enough to excite the first two structural modes yet still produce a relatively low-dimensional response. Equation(s) (9) were integrated numerically using a standard fifth order Runge–Kutta algorithm and then sent to the shaker controller at an update rate of 10 kHz.

![Figure 2. Experimental setup (left) and schematic of the plate.](image-url)
Fiber-optic Strain Measurement System

The plate’s vibrational strain response was recorded using nine fiber Bragg grating strain sensors located in a $3 \times 3$ grid spanning the surface of the plate. A complete description of the sensing system, summarized here, can be found in Todd et al. (2001a). The practitioner interrogates the sensors by illuminating them with a broadband light source, in this case a super luminescent light emitting diode (SLED) (Figure 3). Each fiber Bragg grating acts as an optical filter that reflects a specific wavelength, $\lambda_j$, $j = 1 \cdots K$ within the source bandwidth for each of the $K$ gratings. The reflected wavelengths are passed to a tuneable scanning Fabry–Perot (SFP) optical filter. The filter is driven (‘tuned’) with a periodic triangle waveform so that it scans through the reflected wavelengths sequentially in wavelength order (and then back down in reverse order). In this fashion, the signals from many gratings can be multiplexed on a single optical fiber. At this stage, the practitioner may insert a photodetector converting light intensity to a voltage. This voltage signal is monitored for a peak, indicating that the filter has swept past a particular grating wavelength. A pre-defined voltage-to-wavelength conversion can then be made mapping a point (voltage) on the driving triangle waveform to a specific wavelength. The primary disadvantage of this approach is that the resolution of the measurement is limited by the level to which the driving waveform can be discretized (each point on this waveform must map to a unique wavelength). This technique is also limited in bandwidth as a result of an inability to drive the SFP filter faster while maintaining the integrity of the filter response.

In structural dynamics it is often necessary to acquire data at higher frequencies. The problem of damage detection furthermore requires high resolution data capable of highlighting subtle differences in time series data. Both of these objectives can be met by adding a Mach–Zehnder interferometer to the optical configuration as shown in Figure 3. After passage through the SFP filter, each wavelength is then converted to a phase by means of a path-imbalanced interferometer. Interferometric devices can produce significant gains in both sensitivity and bandwidth. The particular configuration used in this experiment, for example, has a dynamic strain sensitivity of $\pm 1 \mu$e and a bandwidth of 0–2 kHz. The primary difficulty with interferometric measurements is long-term stability. Interferometers can drift as a result of changing path-imbalance due to temperature and/or other environmental effects. Compensation strategies have been proposed.

![Figure 3. Fiber-based strain measurement system.](image-url)
(see Todd et al., 2004) but are not utilized here. In this case the authors are concerned with vibrational measurements on time scales which are much faster than those at which the drift becomes observable. On exit from the interferometer, the signal enters a 3×3 coupler and is split into three separate components with 120° phase offsets between them. The signals arrive at each of the three photodetectors and a voltage, proportional to the intensity of light incident on the photodetectors, is digitized and processed. It has been shown (Todd et al., 2003) that one can recover the phase, and hence the strain, using the voltages at the three photodetectors. This particular demodulation scheme is passive and thus does not restrict the operational bandwidth. Once the phase has been extracted, the strain for a given grating location is given by

\[ \varepsilon_j = \frac{\lambda_j}{2\pi n \Delta L \beta} \Delta \phi_j \quad j = 1 \cdots K \quad (10) \]

where \( \Delta \phi_j \) is the change in phase for the \( j \)th grating, and the constants \( n, \Delta L, \beta \) are the refractive index of the fiber core, the path imbalance of the interferometer, and the photoelastic constant, respectively. In this study the gratings are uniformly spaced in the 1535–1570 nm range (\( \beta \approx 1.15 \text{ pm/} \mu \text{e} \)), the path imbalance is \( \Delta L \approx 2.75 \text{ mm} \), and \( n = 1.46 \).

The entire system (excitation, sensing, data acquisition) were controlled by LabVIEW data acquisition software. Data were collected from the plate in the undamaged configuration and for each of the five different damage levels, 5%, 10%, . . . . A sampling rate of 800 Hz was used in each case for recording both the excitation (via the load cell) and the strain response.

**RESULTS**

Figure 4 shows a sample excitation and response time series, recorded from Sensor 5, and the corresponding power spectrum. Note that the strain amplitude for this signal is ±22 με. The noise floor recorded for this sensor was ±1 με giving a signal-to-noise ratio of \( \approx 20:1 \) for this location on the plate. Other sensors produced varying results depending on their location. Sensors 8 and 9 exhibited the lowest response amplitudes and therefore had signal-to-noise ratios of \( \approx 10:1 \). All other sensors had signal-to-noise ratios of \( \approx 15:1 \). Excitation for this system was restricted to ±25 lbf (the full range of the load cell) thus preventing a larger strain response (hence better signal-to-noise characteristics). The first two modes of the plate occur roughly at 34 and 50 Hz. Using the chaotic excitation, for the most part, only the first two modes of the structure were excited. From a modal analysis point of view this would seem a poor choice of excitation, however, frequencies are not the feature of interest here.

The first damage scenario considered was a cut in the plate at the lower edge, created by removing tabs 1–5 (in order) giving damage levels of 0.5, . . . , 25%. Each experimental run consisted of exciting the plate and recording 50,000 point strain time series at each sensor location (≈1 min of data). For the undamaged plate, 10 different sets of plate response were recorded in order to make a thorough undamaged to damaged comparison i.e., generate 10 separate \( \alpha_0 \). Plate responses were also recorded at each of the damage levels giving \( \alpha_{1,5} \). Both multivariate and univariate applications of the Holder test were implemented. In both implementations, a candidate size of \( M = 20 \) points were used in the algorithm. For the multivariate case the delays
were found via the autocorrelation function to be $T_i = 9, 8, 7, 7, 6, 7, 9, 7, 7$ for sensors 1–9, respectively. It was determined through the multivariate false neighbors approach that an embedding of $m_1 = 1, 1, 1, 2, 2, 2, 1, 1, 1$ was appropriate for reconstructing the underlying dynamics (see Equation (2)).

Figure 5 shows the progression of K–S distance ($D$) with damage for the 10 undamaged runs (spread highlighted in grey) and for each of the damage scenarios. There is little change seen in the dynamics until the plate exhibits a 20% level of degradation. The undamaged spread of values is large enough such that damage below this level would not be considered to show significant change. Indeed, only $D$ values computed at the 20 and 25% damage levels violated the null hypothesis at the 95% confidence level.

Using the same data a univariate attractor reconstruction was also employed at each sensor location, and then the behavior of K–S distance was observed. The delays used for the individual sensors are the same as those used in the multivariate approach, however, the false neighbors algorithm of Kennel et al. (1992) determined that at least $m = 4$ delay coordinates were required for each individual attractor reconstruction. Figure 6 shows the progression of K–S distance with damage for each of the nine sensors. Also shown are the cumulative distribution functions associated with the undamaged construction before proceeding with the computation of the Holder values $\alpha$. Results for this computation are

![Figure 5. Progression of K-S distance with damage for multivariate embedding (tabs 1-5 removed).](image-url)
presented in Figure 8. Again the multivariate approach exhibits a limited ability to detect the lower levels of damage. Only damage levels of 25, 30, and 35% can be clearly seen (violate the null hypothesis at 95% confidence). One possible reason for this concerns the issue of dimensionality. Even though each sensor only contributes 1 or 2 coordinates to the global attractor reconstruction, the total attractor dimension in the multivariate case is \( \sum_{i=1}^{9} m_i = 12 \). In addition, certain sensors had higher signal-to-noise ratios than others. Sensor 8, for example, had the lowest signal-to-noise ratio and was therefore contributing less information to the global picture of the dynamics while still increasing the dimensionality. The subject of multivariate attractor reconstructions is relatively new and there remain a number of unanswered questions regarding attractor size, possible weighting schemes for the various sensor data, etc.

As in the previous damage case, the univariate approach proved far more satisfactory. Results for the progression of \( D \) with damage along with a contour plot showing damage location are given in Figures 9 and 10.
The CDFs for \( \tilde{\alpha}^2 \) have been omitted as they show similar trends as those in the previous experiment. Again, there is a largely monotonic progression of \( D \) with damage for each of the sensors. Also shown in Table 2 are results of the K–S test applied to each sensor for the 95% confidence level. The sensors close to the damage again show an increased ability to detect the presence of the damage. Particularly Sensor 3 is able to detect even the 5% cut length. The performance of Sensors 3 and 5 is again suggestive that the Holder metric may possess information about the damage location. Indeed, the contour plot illustrates a clear bias in the direction of the damage indicating that the Holder metric reflects local dynamical information as opposed to global system properties. There are two apparent false positive results, obtained from Sensor 8 at the 5 and 10% damage levels (this is also evident from Figure 9). The progression of feature value with damage is, for the most part, monotonic with the exception of this particular location on the plate. The effect of noise in computing Holder exponents is to ‘skew’ the distribution of \( \tilde{\alpha} \) in the direction of no continuity (toward 0). It is most likely that the effect that is being observed in the spuriously high K–S values given by Sensor 8 are at the low damage levels.

As a comparison to a more standard approach, the first experiment was repeated where damage is considered by a removal of the tabs 1–5. The structure was excited with a broadband (0–275 Hz) Gaussian noise signal and a set of plate data was recorded using the same sampling parameters and data length as for the chaotic excitation (800 Hz, 50,000 points). Using the input and the nine sensor outputs the associated frequency response functions were computed and then the Eigensystem realization algorithm (ERA) (Juang and Pappa, 1985) was made use of for extracting the plate’s frequencies. For each set of plate response (the ERA algorithm utilizes data from all nine sensors) a moving window of 4096 points was passed through the data and the ERA algorithm was used to extract the first eight plate frequencies for each window. This process resulted in 22 separate estimates for each of the eight frequencies. Only the first six frequencies were identified reliably; these are shown in Figure 11. The first two frequencies, located at roughly 35 and 49 Hz show very little variation with damage. Only the two most extreme cases show a significant drop in these frequencies. The best performing frequency was that associated with the fourth mode. This frequency exhibits a monotonic downward trend showing damage clearly at the 10% level. The fifth frequency was essentially unchanged by damage while the sixth frequency exhibited some change.

Frequencies were also computed for the second damage progression (tabs 20–14 removed) and the results are shown in Figure 12. Again the fourth

Table 1. Results of the K–S test for the first damage scenario at 95% confidence.

<table>
<thead>
<tr>
<th>Sensor/damage level</th>
<th>5%</th>
<th>10%</th>
<th>15%</th>
<th>20%</th>
<th>25%</th>
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Figure 7. Contour plot of D statistic showing damage location.
frequency ($\approx 120\text{Hz}$) was by far the best performer in terms of sensitivity to damage. This particular frequency was consistently able to identify the damage as well as the Holder metric.

A direct comparison between Holder exponents and natural frequencies as features is difficult due to the different excitation bandwidths and in the ways these features are computed. Some conclusions may be drawn however. With regard to sensitivity, both the Holder metric and the fourth modal frequency give comparable results. For this particular structure the onset of damage would be predicted the same for both sets of features. It is noted however, that it is not often known a priori which frequency will show the most significant change with damage. Rather one has to apply a broadband excitation and excite as many frequencies as possible with the hope of capturing at least one that shows significant change. Here the same sensitivity is achieved.

**Figure 8.** Progression of $D$ with damage for multivariate embedding (tabs 20–14 removed).

**Figure 9.** Progression of $D$ with damage for each sensor.
as the fourth mode even though the excitation bandwidth of the chaotic signal only covers the first two modes (see again Figure 4). Furthermore, frequencies are not able to provide information regarding the location of the damage. Variation in natural frequency with damage varies in the same fashion for all sensor locations. Computation of the plate’s mode shapes would have proven more useful in this regard, however reliable mode shape estimates are often more difficult to obtain. Finally, algorithms that extract modal properties can sometimes be quite sensitive to differences in algorithm parameters. The ERA algorithm used here, for example, requires the choice of several parameters regarding the size of the Hankel matrix and the number of modes to search for (size of the identified model). In order to obtain the results presented here the authors had to iterate both quantities over a range of values. The identification of spurious modes and the occasional inability of the algorithm to ‘find’ a certain frequency (dropouts) complicate the algorithm. For example, the sixth frequency was missed entirely by the algorithm at the 20% damage level. The Holder algorithm described here does not suffer this problem. In fact it has to be mentioned that two of the authors employed subtly different versions of the Holder algorithm using different parameters for embedding and number of near neighbors and achieved nearly identical results. Ease of implementation, good sensitivity to damage, and the ability to localize damage make the Holder metric an attractive candidate for vibration-based SHM.

It is noted that each of the above experiments were repeated several different times. The results were, for the most part, the same and the differences were almost entirely in how the K–S test interpreted results from the lowest damage scenarios (5,10%). For example, Sensor 3 of the second damage progression

![Figure 10. Contour plot of K–S distance showing damage location.](image)

Table 2. Results of the K–S test for the second damage scenario at 95% confidence.

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<tr>
<th>Sensor/damage level</th>
<th>5%</th>
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saw the damage (a ‘1’ in the K–S test result) at the 5% level for the results presented here, but not for every experiment conducted. This, of course, is not an unexpected problem since the lowest damage levels are naturally the most susceptible to ‘false positive’ readings. While an effort has been made to reduce these occurrences (10 undamaged runs were collected for every experiment) they could not be eliminated. All experiments conducted were qualitatively the same (sensors closest the damage were the most...
sensitive) and gave nearly identical results in the K–S test for 15–35% damage showing good consistency for all but the two lowest damage cases.

CONCLUSIONS

In this work a new feature has been introduced for damage detection reflecting differentiability between the system responses at various damage levels. This general approach is fundamentally different from those used previously in vibration-based structural health monitoring (SHM). Rather than tracking a feature in a time series as damage is incurred this approach is based on the analysis of properties of the functional relationship between a system’s dynamical response data. As was previously mentioned it is conjectured that a loss of differentiability occurs when the structure is damaged, hence, measuring the differentiability via the Holder exponent is an appropriate choice of feature. This feature has been demonstrated to be effective at discerning the presence and location of a cut in an experimental steel plate using both uni- and multivariate descriptions of the dynamics. This approach was also shown to compare favorably to the standard practice of examining the plate’s modal frequencies. In fact, the Holder exponent was able to clearly detect damage at the same level as the plate’s most sensitive frequency, particularly when computed from data recorded near the damage site. As implemented here the approach uses a chaotic signal as opposed to the more traditional random excitation. Such a signal is no more difficult to impart in practice than a stochastic signal and furthermore requires much less bandwidth to implement possibly giving this type of excitation a practical advantage. It should be re-iterated that the algorithm can be applied to structural response data resulting from any excitation source, stochastic or otherwise. The theorems that justify the Holder metric are no longer applicable in the case of stochastic excitation; however, in practical applications this does not seem to pose a problem. The Holder algorithm is also easy to implement requiring only the choice of delay $T$ and embedding dimension $m$ (for attractor reconstruction) along with the number of near neighbors to consider $M$. There exist well-established, automated methods for choosing both $T$ and $m$ leaving $M$ as the only ‘free’ parameter.

A ‘state-of-the-art’ distributed sensing system has also been implemented for acquiring structural response data. This system uses fiber Bragg gratings for the strain sensing elements and a novel optical configuration consisting of a scanning Fabry–Perot (SFP) filter and a 3×3 Mach–Zehnder interferometer. The primary advantages of this particular system are high accuracy and higher sampling rates than typical fiber-based systems. These properties, combined with ease of multiplexing and almost no weight penalty make this system ideal for SHM applications.

Finally, an effort has been made to place the analysis of the Holder exponents in a hypothesis testing framework by making use of the Kolmogorov–Smirnov (K–S) test. This test assesses the difference between two empirically constructed cumulative distribution functions and is ideal for the SHM problem. It is noted that occasionally the K–S test gives inconsistent results in that a lower damage level may be ‘seen’ while a higher one is found to be consistent with the null hypothesis. Such inconsistencies are likely to be a result of the lower damage levels being misclassified as damage (i.e., a false positive diagnosis) when in fact the differences in the data are due to some other factor which has not been accounted for. Interferometer drift, resulting in small amplitude changes for the recorded time series, is the likely cause of these false positives. Admittedly, this is a problem that must be overcome if this approach is to be used in an ‘online’ fashion. Future efforts will be directed toward understanding the nature of these misclassifications and stabilizing the hardware to minimize their occurrence.

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REFERENCES


