How to assess the efficiency of synchronization experiments in tokamaks

This content has been downloaded from IOPscience. Please scroll down to see the full text.
2016 Nucl. Fusion 56 076008
(http://iopscience.iop.org/0029-5515/56/7/076008)

View the table of contents for this issue, or go to the journal homepage for more

Download details:
IP Address: 141.20.40.163
This content was downloaded on 25/08/2016 at 08:19

Please note that terms and conditions apply.

You may also be interested in:

Application of transfer entropy to causality detection and synchronization experiments in tokamaks
A. Murari, E. Peluso, M. Gelfusa et al.

Extensive statistical analysis of ELMs on JET with a carbon wall
A Murari, F Pisano, J Vega et al.

ELM pacing and high-density operation using pellet injection in the ASDEX Upgrade all-metal-wall tokamak
P.T. Lang, A. Burckhart, M. Bernert et al.

Characterizing electrostatic turbulence in tokamak plasmas with high MHD activity
Z O Guimarães-Filho, G Z dos Santos Lima, I L Caldas et al.

Prediction of PAF using RP-based features of the RR-interval signal
Maryam Mohebbi and Hassan Ghassemian

ELM pacing and trigger investigations at JET with the new ITER-like wall
P.T. Lang, D. Frigione, A. Géraud et al.

ELM control strategies and tools: status and potential for ITER
P.T. Lang, A. Loarte, G. Saibene et al.
How to assess the efficiency of synchronization experiments in tokamaks

A. Murari\(^1,2\), T. Craciunescu\(^3\), E. Peluso\(^4\), M. Gelfusa\(^4\), M. Lungaroni\(^4\), L. Garzotti\(^5\), D. Frigione\(^6\) and P. Gaudio\(^4\) and JET Contributors\(^a\)

EUROfusion Consortium, JET, Culham Science Centre, Abingdon, OX14 3DB, UK
\(^1\) EUROfusion Programme Management Unit, ITER Physics Department, Culham Science Centre, Abingdon OX143DB, UK
\(^2\) Consorzio RFX (CNR, ENEA, INFN, Università di Padova, Acciaierie Venete SpA), Corso Stati Uniti 4, 35127 Padova, Italy
\(^3\) National Institute for Laser, Plasma and Radiation Physics, Magurele-Bucharest, Romania
\(^4\) University of Rome ‘Tor Vergata’, Via del Politecnico 1, 00133 Rome, Italy
\(^5\) CCFE, Culham Science Centre, Abingdon, Oxon, OX14 3DB, UK
\(^6\) Unità Tecnica Fusione—ENEA C. R. Frascati, via E. Fermi 45, 00044 Frascati (Roma), Italy

E-mail: andrea.murari@euro-fusion.org

Received 25 March 2016
Accepted for publication 12 May 2016
Published 15 June 2016

Abstract

Control of instabilities such as ELMs and sawteeth is considered an important ingredient in the development of reactor-relevant scenarios. Various forms of ELM pacing have been tried in the past to influence their behavior using external perturbations. One of the main problems with these synchronization experiments resides in the fact that ELMs are periodic or quasi-periodic in nature. Therefore, after any pulsed perturbation, if one waits long enough, an ELM is always bound to occur. To evaluate the effectiveness of ELM pacing techniques, it is crucial to determine an appropriate interval over which they can have a real influence and an effective triggering capability. In this paper, three independent statistical methods are described to address this issue: Granger causality, transfer entropy and recurrence plots. The obtained results for JET with the ITER-like wall (ILW) indicate that the proposed techniques agree very well and provide much better estimates than the traditional heuristic criteria reported in the literature. Moreover, their combined use allows for the improvement of the time resolution of the assessment and determination of the efficiency of the pellet triggering in different phases of the same discharge. Therefore, the developed methods can be used to provide a quantitative and statistically robust estimate of the triggering efficiency of ELM pacing under realistic experimental conditions.

Keywords: ELM pacing, transfer entropy, Granger causality, recurrence plots

(Some figures may appear in colour only in the online journal)

1. ELM pacing as synchronization experiments and causality

Among the instabilities affecting H-mode plasmas, ELMs are particularly problematic. They almost always cause a significant reduction of the energy confinement by compromising the edge transport barrier [1]. Due to this sudden degradation of the edge confinement, energy and matter are expelled from the plasma on a sub millisecond time scale, which can have unacceptable consequences for the divertor in the next generation of devices and in the reactor. Many statistical analyses of ELM instabilities have produced different and contrasting results regarding their dynamics. In some cases, a quasi-periodic behavior has been found; in other instances ELMs seemed to behave chaotically. More recently, evidence of quasi-chaotic ELMs has also been reported [2, 3]. Irrespective of the details...
of their dynamics, in the perspective of ITER and DEMO it is imperative to control ELMs carefully, to alleviate their detrimental effects on the plasma-facing components, particularly in the divertor. Indeed, DEMO will probably have to be operated in ELM-free scenarios. On ITER, some form of active ELM control is also considered essential. Therefore, to support the development of reactor-relevant scenarios, in many machines various ELM pacing techniques have been tested. This subject assumes a particularly relevant role in the present programme of JET with an ITER-like wall (ILW) on the route to the next full deuterium-tritium (DT) campaign [4].One of the most promising methods consists of pacing ELMs with pellets [5, 6]. The long-term objective of this approach would consist of controlling the ELM frequency by triggering them with pellets. The ELM pacing could be adjusted so that the gradients at the edge would not have time to increase excessively between subsequent ELMs. In this way, it is expected that the expulsion of energy and matter can be kept to manageable levels, not capable of damaging the plasma-facing components. These pacing experiments are typical cases of synchronization techniques.

One of the main difficulties in developing such experimental solutions resides in the interpretation of the experimental results. This is due to the fact that ELMs are periodic or quasi-periodic in nature. As a consequence, after any sudden perturbation with such a pellet, if one waits long enough, an ELM always occurs. To properly understand the effects of pellets and to evaluate their triggering effectiveness, it is crucial to develop techniques capable of reliably assessing the time interval during which the pellets can really have a triggering capability. This means determining the time interval over which the pellets can have a causal influence on the ELM dynamics. The difficulty with this analysis can be appreciated by the inspection of figure 1, which reports the $D_\alpha$ for ELMs and pellets in a JET discharge with the ILW. It is evident that the task of determining how many pellets have triggered an ELM is quite challenging, given the limited diagnostic information and the high level of noise in the measurements. This is the typical problem of assessing causality relations between different events within a probabilistic framework.

Unfortunately, causality is a concept which has so far resisted a precise and quantitative definition. Even if in recent years there has been an explosion of theoretical work about causality by philosophers, scientists and epistemologists still do not necessarily agree on a unique definition of causality. It is, therefore, impossible to converge on a single, unique mathematical measure of cause–effect relationships. On the other hand, it is worth pointing out that originally causality was conceived to explain strong regularities in nature, but today our view of reality is probabilistic. Therefore, causal relationships are often investigated within the framework of uncertainty. As a consequence, again given the uncertainties affecting our knowledge of many phenomena, the capability of also quantifying the strength of the causal relationships has become very important. Therefore, a probabilistic and not only a deterministic account of cause-and-effect relations, is appropriate in the case of scientific problems affected by uncertainty. These considerations are particularly relevant to ELM pacing, given the complexity of the experiments, the strongly nonlinear character of these instabilities, the limited diagnostic capability of the plasma edge in many devices and the not negligible level of noise and uncertainties in the measurements. Therefore, a statistical analysis becomes indispensable to complement more traditional dynamical studies.

In this work, three statistical criteria to determine causality between external perturbations and ELMs are introduced. The first two are purely statistical approaches and implement a concept of causality as increased predictability, originally proposed by Wiener [7]. Even if the original proposal was quite vague, it was the first one which permitted mathematical precise quantification. The main idea is that, given two measurements, if the knowledge of the past of the second allows better prediction of the future of the first, then the second signal can be considered causal to the first. This specific and limited sense of causality, as increased predictability, was developed by Granger, who proposed a practical way of calculating it [8]. This Granger causality (GC) has been deployed successfully in many fields, ranging from economics to the Earth
sciences and medicine. This is the first method which, properly adapted, has been applied to the problem of assessing the efficiency of ELM pacing on JET. On the other hand, GC has been typically implemented with linear autoregressive models, which are not fully general. In the case of strongly nonlinear systems, therefore, the conclusions of the traditional GC could be questioned. The same definition of causality, as enhanced predictability, has therefore been further refined and finally implemented also using a non-parametric, nonlinear indicator, called transfer entropy (TE) [9]. TE can be thought of as the information-theoretic functional of probability distribution functions, which measures the exchange of information between signals. TE has been expressly developed to study time series and investigate their causal relationships in terms of predictability and information transfer.

The third method developed is more typical of nonlinear dynamical studies and it is based on recurrence plots (RPs). Indeed, the recurrence behavior is a fundamental characteristic of dynamical systems. RPs are very powerful tools for the descriptive study of the statistical properties of dynamical systems [10]. An RP is a plot depicting the times when a phase-space trajectory returns roughly to the same region in phase space. RP refinements, called joint recurrence plots (JRs), can be used to relate the behavior of one signal with the one of another. By quantifying the properties of JR through recurrence quantification analysis (RQA), it is possible to determine the maximum interval of information transfer between two time series [10]. As shown later, this proves to be a very good estimate of the interval over which a system exerts a causal influence on another.

It is worth emphasizing that, in this paper, the term causality is used only in the restricted sense of improved predictability and information transfer between time series, without any reference to the philosophical implications of the word. Given the nature of the application presented, this is not a real issue because it is known a priori that pellets can trigger ELMs and not vice versa; therefore philosophical discussions about root and real causes are not relevant.

As far as the structure of the paper is concerned, the next sections provide the mathematical background to the main criteria used in the paper: GC is explained in section 2, TE is introduced in section 3 and the background on RPs is provided in section 4. Some numerical tests with synthetic data are reported in section 5, to illustrate the potential of the proposed techniques to identify the interval of cause–effect relations in time series of the type encountered in practice. The proposed indicators are then applied to ELM pacing with pellets in section 6, using data of JET discharges with the ILW. In the last section of the paper, in addition to the conclusions, future developments of the proposed techniques and their applications are also discussed.

2. Granger causality: main idea and mathematical background

GC is a form of ‘predictive causality’ in the sense that causal relations are measured by determining the capability to predict the future values of a time series from the knowledge of the earlier values of another time series. A time series \( J \) is said to Granger-cause \( I \) if it can be demonstrated, through appropriate statistical tests on lagged values of \( J \), that those previous \( J \) values provide such significant information about \( I \) to allow better prediction of \( I \) future. GC is therefore based on two main principles:

1. The cause happens earlier in time than its effects.
2. The cause contains unique information about the future of its effects.

Starting from these two assumptions about causality, the identification of a causal effect of \( J \) on \( I \) is performed in the context of linear regression models with the procedure described in the following. The analysis is typically formulated as a hypothesis-testing problem. Given two stationary time series \( I \) and \( J \), to test the null hypothesis that \( J \) does not Granger-cause \( I \), one first finds the proper lagged values of \( I \) to include in a univariate auto-regression model of \( I \):

\[
I(t) = a_0 + a_1 I_{t-1} + a_2 I_{t-2} + \cdots + a_m I_{t-m} + \eta(t)
\]

where \( \eta(t) \) indicates the residuals. In the next step of the analysis, the auto-regression model is extended by including lagged values of \( J \):

\[
I(t) = b_0 + a_1 I_t + \cdots + a_m I_{t-m} + b_I J_t + \cdots + b_m J_{t-m} + \eta_2(t)
\]

where \( \eta_2(t) \) indicates now the residuals of model 2.

In the traditional Granger–Sargent test, the coefficients in equation (2) are estimated with ordinary least-squares techniques. The model order \( m \) can be selected using model selection methods such as the Akaike of the Schwarz criteria [11].

The prediction improvement \( PI_{J \rightarrow I} \) can be defined as a function of the minimized residuals:

\[
PI_{J \rightarrow I} = S_{\eta_1}(t) - S_{\eta_2}(t)
\]

where \( S_{\eta_1} \) and \( S_{\eta_2} \) indicate the variances of the residuals of the respective models. The more information signal \( J \) contains about signal \( I \), the better the model of equation (2) and, therefore, the lower its residuals. Therefore, the higher the value of the \( PI_{J \rightarrow I} \) indicator the better is the prediction of \( I \) when the signal \( J \) is taken into account. To summarize, the higher the value of the \( PI_{J \rightarrow I} \) the more information \( J \) carries about \( I \) and, therefore, the more causal \( J \) can be considered to be to \( I \). In practice, of course, random fluctuations can induce the estimator to assume positive values even if there is no causal relation between the two time series. Therefore, a criterion is needed to assess whether an influence of \( J \) on \( I \) is really present at a certain confidence level. It can be demonstrated that the quantity

\[
F_{J \rightarrow I} = (N - 3m - 1)[S_{\eta_1} - S_{\eta_2}]mS_{\eta_2}
\]

is distributed according to the Fisher’s \( F \)-law with \( (m, N - 3m - 1) \) degrees of freedom (\( N \) is the total number of available time slices) [8]. Therefore, it can be stated that causal influence of \( J \) on \( I \) exists at the significance level \( p \) if the value of \( F_{J \rightarrow I} \) exceeds the \( (1 - p) \) quantile of the respective \( F \) distribution. This means that if \( F_{J \rightarrow I} \) exceeds the \( (1 - p) \) quantile of the respective \( F \) distribution, the probability of
random error in detecting causality is less than \( p \). To obtain the results shown in this paper, the GC has been calculated using the consolidated spectral method reported in [8].

GC can be deployed to achieve two different objectives. If the relation between the time series to be investigated is unknown, GC can help in assessing their causal dependencies. In this case, it has already been proved that there is a cause–effect relation between two signals, and GC can contribute to determining the length of time over which this relation is really effective. The most direct approach consists of determining the longest interval over which \( F_{ij,t} \) exceeds the \((1 - p)\) quantile of the \( F \) distribution, i.e. the longest interval over which the null hypothesis is not valid, at the selected significance level. This is the use of GC relevant to the application described in this paper.

### 3. Transfer entropy: concept and mathematical background

The subject of this section is the introduction of the reader to the mathematical properties of TE which generalize GC. Since TE is an information-theoretic criterion, it is advantageous to start the discussion with the Kullback entropy (KE), defined as

\[
K_I = \sum_i p(i) \log(\frac{p(i)}{q(i)}).
\]

In equation (5) \( p \) and \( q \) are two probability density functions (PDFs). Even if it does not satisfy all the necessary axioms, the KE can be loosely considered as a distance between PDFs, since it always assumes positive values and it is zero only when the two PDFs, \( p \) and \( q \), are exactly the same. The PDF \( p(i) \) is typically considered the reference one and, therefore, the closer the PDF \( q(i) \) is to the reference \( p(i) \), the smaller the KE. A complementary interpretation of the KE consists of viewing it as the error committed by approximating PDF \( p \) with PDF \( q \). In terms of information-theoretical concepts, the KE can be seen as the excess number of bits that have to be used if PDF \( p(i) \) is approximated with PDF \( q(i) \).

The KE can also be expressed in terms of conditional probabilities \( p(i|j) \):

\[
K_J = \sum_i p(i|j) \log(\frac{p(i|j)}{q(i|j)}).
\]

It is also possible to express KE in terms of joint probabilities, which allows the introduction of the mutual information \( M_{IJ} \) between the two processes \( I \) and \( J \). The \( M_{IJ} \) can be defined as

\[
M_{IJ} = \sum_i p(i,j) \log(\frac{p(i,j)}{p(i)p(j)}).
\]

In the formulation of the mutual information, the reference is the case of independent events, since the two PDFs are factorized at the denominator of the logarithm. Alternatively, the mutual information can be seen as the excess of the code required if one makes the erroneous assumption that the two systems \( I \) and \( J \) are independent. Again, complementary to the interpretation in terms of the additional amount of code required, the \( M_{IJ} \) can also be considered the error committed by considering independent PDFs, which in reality it is not.

Unfortunately, the mutual information is a time-symmetric quantity. Therefore, \( M_{IJ} \) does not provide any indication about directionality, which is an essential aspect of causality. On the other hand, \( M_{IJ} \) can be given a direction by introducing a time lag \( \tau \) between the processes

\[
M_{IJ}(\tau) = \sum_n p(i_n,j_{n-\tau}) \log(\frac{p(i_n,j_{n-\tau})}{p(i_n)p(j_{n-\tau})}) \quad (8)
\]

This last step suggests that, in order to obtain information about the dynamical structure of the processes to be investigated, one can reformulate the previous quantities in terms of transition probabilities instead of static probabilities. In this perspective, the formalism of the Markov processes comes in handy. In the case of words of length \( k \), using the synthetic notation \( i_n^k = (i_n, \ldots, i_{n-k+1}) \), the following relation is satisfied by a Markov process of order \( k \):

\[
p(i_{n+1}|i_n, \ldots, i_{n-k+1}) = p(i_{n+1}|i_n, \ldots, i_{n-k}). \quad (9)
\]

At this point, one important quantity to remember is the entropy rate \( h_I \), which quantifies the number of bits required to encode one additional state of the system if all previous ones are known. The definition of the entropy rate is

\[
h_I = -\sum p(i_{n+1}|i_n^k) \log p(i_{n+1}|i_n^k). \quad (10)
\]

The entropy rate is the most fruitful quantity to extend to two processes for the study of causal relationships. This can be achieved by measuring the deviation from the generalized Markov property

\[
p(i_{n+1}|i_n^k) \neq p(i_{n+1}|i_n^k, j_n^l). \quad (11)
\]

The main idea behind relation (11) is that, if there is no flow of information from \( J \) to \( I \), the state of \( J \) should have no influence on the transition probabilities of \( I \). A generalization of the KE, called the TE, can be adopted to quantify the inadequacy of this assumption. The TE is defined as

\[
T_{I\to J} = \sum p(i_{n+1}|i_n^k, j_n^l) \log \left( \frac{p(i_{n+1}|i_n^k, j_n^l)}{p(i_{n+1}|i_n^k)} \right). \quad (12)
\]

In this case, the cause–effect relation between the two signals under investigation is already known, as in the application described in this paper, TE can contribute to determining the time span over which this causal influence is effective. This can be achieved by inspecting the trend of equation (12) when scanning parameters \( l \) and \( k \). A full mathematical derivation of the TE is provided in [9] and the references therein.

### 4. Recurrence plots

As mentioned already, the recurrence behavior is a fundamental characteristic of dynamical systems. RPs are very powerful tools for the descriptive study of the statistical properties of complex and chaotic systems. An RP is a plot showing the times at which a phase-space trajectory returns to more or less
the same region in phase space. In more detail, the RPs depict the collection of pairs of times at which the trajectory returns sufficiently close to the same place. RPs are based on the following matrix representation

\[ R_{ij} = \Theta(\varepsilon - \|\vec{x}_i - \vec{x}_j\|), \quad i, j = 1, \ldots, N, \quad (13) \]

where \( \vec{x}_i \) indicates the point in phase space where the system is located at time \( i \), and \( \varepsilon \) is a predefined threshold. \( \Theta(x) \) is the Heaviside function and, therefore, the matrix consists only of the values 1 and 0. The graphical representation of the RPs is an \( N \times N \) grid of points, which are depicted as black for 1 and white for 0.

Therefore, if a certain point in the RP is black, this means that the system returns to an \( \varepsilon \)-neighborhood of this point in phase space. This aspect of showing graphically the recurrence properties of systems gives the name to the method. A number of examples, for different types of dynamical systems, are reported in figure 2.

Multivariate extensions of RPs have been developed. The most relevant for the applications described in this paper are the JRPs. JRPs can be defined as the Hadamard product of the RPs of their sub-systems [10], e.g. for two systems \( x \) and \( y \) the JRP is

\[ \text{JR}(i, j) = \Theta(\varepsilon - ||\vec{x}(i) - \vec{x}(j)||)\Theta(\varepsilon - ||\vec{y}(i) - \vec{y}(j)||)\vec{x}(i) \in \mathbb{R}^m, \vec{y}(j) \in \mathbb{R}^n, \quad i, j = 1, \ldots, N_{x,y}. \quad (14) \]

JRPs have been explicitly devised to study the relation between the recurrence properties of two or more systems; for example, they can be deployed to investigate phase synchronization.

In addition to allowing an easy visualization of the periodicities of dynamical systems, RPs and JRPs permit us to quantify very important properties of a system phase space: this is the so-called RQA. RQA condenses in a series of useful indicators the information about the distributions of the diagonal, horizontal and vertical lines in the RP. The combination of RPs and RQA constitutes a powerful tool to investigate the properties of dynamical systems and indeed both methods have been widely applied in many fields such as physics, economics, health and atmospheric sciences.

Of particular relevance for the subject of this paper is the fact that RQA provides several measures, which can be related to the causal relation between signals:

- The recurrence rate:

\[ \text{RR} = \frac{1}{N^2} \sum_{i,j} R_{ij}(\varepsilon) \quad (15) \]

\( \text{RR} \) is the density of recurrence points in an RP. The phase space of deterministic systems consists of intervals during which the trajectories tend to run parallel for some time. This behavior is reflected in the formation of diagonal line structures in the RP. \( \text{RR} \) quantifies this aspect and can be considered the probability that any state will recur.

- The determinism (predictability) is a measure based on diagonal lines:

\[ \text{DET} = \frac{\sum_{l=t_{\text{min}}}^{t_{\text{max}}} \text{IP}(l)}{\sum_{l=1}^{t_{\text{max}}} \text{P}(l)} \quad (16) \]
DET is based on the distribution of the lengths of the diagonal lines $P(l)$ of length $l$ that appear in the plot. DET can be understood by considering that processes with uncorrelated, stochastic or chaotic behavior present few and very short diagonals. In contrast, deterministic processes exhibit longer diagonals and less isolated recurrence points. DET quantifies this aspect and, therefore, it is a measure of the determinism (predictability) of the system. The threshold $l_{\text{min}}$ is introduced to exclude the diagonal lines which are due to the tangential motion of the phase-space trajectories. The great interest in the diagonal lines, and implicitly in DET, derives from the fact that they are linked to the largest Lyapunov exponent, of course, under the assumption that the plots describe the behavior of an underlying dynamical system. The length of the lines is related to the inverse of the largest positive Lyapunov exponent [10].

- **Average diagonal length**: The averaged diagonal line length can be calculated as

$$L = \frac{\sum_{l=l_{\text{min}}}^{l_{\text{max}}} lP(l)}{\sum_{l=l_{\text{min}}}^{l_{\text{max}}} P(l)}$$

where $P(l)$ is the frequency distribution of the lengths $l$ of the diagonal lines. This indicator is related to the predictability horizon of a dynamical system.

- The **entropy of the diagonal length** is defined as the Shannon entropy of the frequency distribution of the diagonal lengths

$$\text{ENTR} = \sum_{l=l_{\text{min}}}^{l_{\text{max}}} p(l) \ln p(l)$$

where

$$p(l) = \frac{P(l)}{\sum_{l=l_{\text{min}}}^{l_{\text{max}}} P(l)}$$

In other words, ENTR is the Shannon entropy of the probability $p(l) = P(l)/N$ to find a diagonal line of exactly length $l$ in the RP. This entropy, therefore, provides a quantitative measure of the amount of information required to recover the system. ENTR is also an indicator of the complexity of the RP with respect to the diagonal lines. For uncorrelated noise, the value of ENTR is typically quite small.

**Figure 3.** Top, synthetic signals; middle, TE exhibiting a clear maximum at $\tau = 6d\tau$; bottom, determinism shows also a maximum at $\tau = 6d\tau$. 
JRPs, and in particular the parameters which can be derived using RQA, can be used to determine the causality horizon, the longest time interval over which it is reasonable to assume that a system influences another. Indeed, various parameters obtainable with RQA show a clear maximum for the appropriate lag time corresponding to the causality horizon. This is the criterion adopted to derive the results reported in the rest of the paper.

5. Results of numerical test with synthetic data

The three proposed methods have been extensively tested with synthetic data to prove their capability to identify causal relationships. In particular, it has been verified that they can properly determine the right interval over which signal \( J \) exerts a causal influence on signal \( I \). In this section, some examples of the systematic numerical tests performed to qualify the performance of the various criteria are reported.

In general, the analysis of synthetic data has proved how the proposed methods can be applied to investigate the behavior of time series affected by noise. The first numerical example involves more traditional and smooth signals. The second example is meant to prove the capability of the three techniques to also handle spiky signals, of the type characteristic of ELMs.

The first test reported consists of twenty-one time series of functions with a trapezoidal shape. Each synthetic signal has then been introduced as the argument of a sinusoidal function with a delay of \( \tau = 6d \) (see figure 3). Finally, Gaussian noise, of standard deviation equal to 5% of the plateau of the trapezoidal curve, has been added to each time sequence and the TE computed. GC identifies the right lag time of 6d at 5% significance level. Figure 3 depicts the functions and the TE results, where the brighter the color the higher the TE. The figure shows very clearly that the delay has been correctly detected. The JRPs also show a clear distinctive peak at the right time lag, as reported in figure 3 for the determinism.

As mentioned earlier, a second specific example is reported to verify the applicability of the methods to spiky signals. The main reason for this type of test is the fact that, in real experiments, the three methods are to be applied to signals measuring various aspects of instabilities, which can present quite abrupt variations. It has, therefore, been considered appropriate to double-check that the three tools can properly handle even this typology of data. An example of the signals used for the systematic tests performed with synthetic data is reported in figure 4. A series of triangular-shaped pulses, each with slightly different slope and height, has been generated first, simulating the \( I \) signal (red triangles in figure 4). Then, a lower number of triangular-shaped pulses has been generated, to simulate the signal \( J \) (blue triangle in figure 4). Then, for each spike in the \( J \) signal, a corresponding spike on \( I \) has been generated after a temporal lag of \( \tau = 15d \). As usual, in the end, Gaussian noise has been added to all the time series. In order to improve the clarity of figure 4, the amplitude of each peak is one thousand times the standard deviation of the noise, but absolutely similar results have also been obtained for a signal-to-noise ratio of ten. The test shown in the paper has been performed using twenty couples of time sequences,
each presenting a different time length, but all with the same lag time.

The GC identifies the right mean time lag at 5% significance level, adopting the usual criterion of choosing as lag time the longest interval for which the null hypothesis is falsified. With the TE, the estimate has been derived calculating the mean of the TE for each lag unit between all the couples. The lag time is then assumed to be the time at which the TE assumes the maximum value. The results indicate clearly that the ratio $\frac{TE}{JI}$ \approx 2.5 and that the TE has a maximum at $\tau = 15\delta t$. The described test, therefore, confirms that the TE can identify both the real causal relationship and also the right lag time for spiky signals. The derived lag time versus the number of samples is shown in figure 5.

The RPs also manage to properly identify the correct lag time. Moreover, various indicators obtained with RQA provide the same estimate. In figures 6 and 7 the recurrence rate and determinism are plotted versus the lag time. Both indicators exhibit a clear peak for the correct delay of 15 time units, for both the case with and without noise.

It is worth mentioning that, since the pulses have a finite width, the results cannot be provided with a resolution higher than an approximate lag time. A numerical investigation has also been carried out to assess the effects of the finite size and of the noise in the experimental measurements, as reported in the next section.

6. Application to ELM pacing with pellets

As mentioned earlier, probably the main issue with the interpretation of pellet pacing is the determination of its effectiveness. It is indeed a very delicate task to discriminate between the ELMs that have been really triggered by pellets from simple coincidences. On JET, on the basis of simple statistical tests, it has always been assumed that a pellet can trigger an ELM only if the time between the two events is less than or equal to 2 ms. In order to test this hypothesis, the three techniques introduced in the previous sections have been applied to a set of eight JET pulses, devoted explicitly to the investigation of various settings of the pellet-pacing system: 82885, 82886, 82887, 84688, 84690, 84693, 84696. The analysis relies on the $D_\alpha$ emission to determine both the occurrence of the ELMs and the arrival time of the pellets in the plasma [5, 6]. The analysis of $D_\alpha$ emission to determine both the occurrence of the ELMs and the arrival time of the pellets in the plasma [5, 6]. For these discharges, sufficient statistics and signals of adequate quality are available to allow robust estimates. In particular, these plasmas are sufficiently stationary, an implicit assumption for the application of the GC and TE criteria, in the form implemented in this work. The issue of
increasing the time resolution will be discussed in the next section. On the other hand, this series of shots is not to be considered representative of the average performance of the JET pellet injector, since various upgrades in the system have been recently implemented.

The main characteristics of the pellets injected in these discharges are reported in table 1. As can be seen from the table, the parameter which has been varied over the widest range is the pellet speed.

To assess the time interval over which the pellets can be considered to have a triggering effect on the ELMs, the GC has been applied to the two $D_\alpha$ time traces; the one of the ELM and the one of the pellets entering the plasma. By changing the lag time, the last instant, in which the null hypothesis is not verified at the 5% significance level, has been selected to identify the interval of causal relationship between the two phenomena.

The TE in its turn presents a very similar evolution in time for all discharges investigated. The TE shows a slight increase for a short interval, levels off reaching a constant level and then decreases quite sharply for longer lag times (see figure 8). This is a behavior quite typical of many dynamical systems. The triggering event has a significant influence for a specific window of opportunity, outside which it becomes rapidly ineffective. Pellets seem to behave in a similar way. In a short

<table>
<thead>
<tr>
<th>Pulse</th>
<th>#ELMs</th>
<th>#Pellets</th>
<th>Nominal frequency (Hz)</th>
<th>Real frequency (Hz)</th>
<th>Mass ($N_\alpha$)</th>
<th>Speed (m s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>84688</td>
<td>47</td>
<td>33</td>
<td>25</td>
<td>16.5</td>
<td>0.0299</td>
<td>80</td>
</tr>
<tr>
<td>84690</td>
<td>57</td>
<td>57</td>
<td>25</td>
<td>16.3</td>
<td>0.0299</td>
<td>80</td>
</tr>
<tr>
<td>84693</td>
<td>44</td>
<td>36</td>
<td>25</td>
<td>18.0</td>
<td>0.0332</td>
<td>80</td>
</tr>
<tr>
<td>82885</td>
<td>87</td>
<td>116</td>
<td>50</td>
<td>29.0</td>
<td>0.0349</td>
<td>176</td>
</tr>
<tr>
<td>82886</td>
<td>77</td>
<td>94</td>
<td>50</td>
<td>31.3</td>
<td>0.0349</td>
<td>170</td>
</tr>
<tr>
<td>82887</td>
<td>63</td>
<td>83</td>
<td>50</td>
<td>27.7</td>
<td>0.0349</td>
<td>200</td>
</tr>
<tr>
<td>82889</td>
<td>65</td>
<td>77</td>
<td>50</td>
<td>19.3</td>
<td>0.0349</td>
<td>174</td>
</tr>
<tr>
<td>84696</td>
<td>38</td>
<td>43</td>
<td>50</td>
<td>14.3</td>
<td>0.0332</td>
<td>80</td>
</tr>
</tbody>
</table>

Figure 7. Determinism. Left without noise, right with 5% additive noise.

Table 1. Characteristics of the pellets for the investigated discharges.

Figure 8. The evolution of TE between pellets and ELMs for shot number 84693.

Figure 9. Discharge 82855. A peak is very evident in the entropy of diagonal length. The entropy of diagonal length has been fitted with a spline to identify the maximum (red curve).
time interval, they exhibit the largest probability of triggering an ELM collapse. This probability then decays rapidly, which can be interpreted by considering that if the pellets reach the plasma too far away from the completion of the natural cycle of the ELMs they have less chance to be effective. On the other hand, if the pellets arrive too close to the end of the natural ELM cycle, they are less capable of influencing their dynamics. The triggering efficiency, defined as the interval for successful triggering of the ELMs, can be calculated as the time point when the TE decays to 95% of its peak value. This 95% interval has been chosen as a statistical ‘confidence interval’, on the basis of many numerical tests, with pulses with shape and level of noise similar to the experimental values. Indeed, these numerical tests have shown that the TE curve broadens proportionally to the width of the peaks. This translates into the fact that 95% of the TE maximum value typically provides the best estimate of the right time lag for signals of the shape typical of JET ELMs. This interval almost always provides a much more accurate value for the interval, over which causality is effective, than the simple maximum of TE. Moreover, the value of 95% is sufficiently conservative to ensure that the effectiveness of the pellet triggering is not overestimated.

An example of JRP, for the pellet and ELM time series, is shown in the right hand plot of figure 2. For the JPRs, the evolution of several analysis measures with the lag time has been calculated: average of diagonal length, entropy of diagonal length, recurrence time, determinism. Events (=peaks) have been sought in the trends of these parameters with the lag time. Once an event is detected in the evolution of multiple parameters, the peak position is determined using the most distinctive peak. This peak defines the period of maximum correlation between the dynamics of the two systems and, therefore, it is assumed to identify an appropriate range over which the causal relation is effective. An example is shown in figure 9.

In table 2, the lag times calculated with the three proposed techniques have been reported. The estimates are very similar and basically agree with each other within the error and uncertainties in the experimental data. The efficiency of the ELM triggering is also reported in table 2. This efficiency is calculated as the number of pellets triggering an ELM divided by the total number of pellets. A pellet is considered to have triggered the subsequent ELM if it has reached the plasma within the time lag identified by the corresponding criterion. For completeness the efficiency of the pellet triggering is reported also for the case of 2 ms lag time, traditionally used in JET.

The first interesting observation emerging from the previous analysis is that the three indicators provide very similar results. The discrepancy is typically of the order of a fraction of a millisecond between the estimates of the TE and JRP, which is certainly the limit of the accuracy which can be achieved given the measurements available. The maximum discrepancy is 0.6 ms. The weakest estimate is typically the GC, which is known to be quite sensitive to noise and pulse shaping. Indeed, it should not be overlooked that the experimental signals are not of a very high quality, considering the complexity of the phenomenon. Indeed, time resolution of the measurements is quite low and the level of added noise not negligible. In any case, since the three approaches are numerically fully independent, the found agreement increases significantly the confidence in the obtained estimates. Moreover, the maximum discrepancy between the indicators can be treated as a confidence interval in the estimates. This is another important added value of the proposed approach, since using a fixed lag time of 2 ms does not allow us to provide any confidence interval in the results.

The other aspect to note is that the discharges can exhibit a very different behavior. Therefore, the assumption of a single time interval of 2 ms, for calculating the number of ELMs triggered by pellets, is not supported by the present analysis.

<table>
<thead>
<tr>
<th>Pulse</th>
<th>∆t GC (ms)</th>
<th>GC % triggering</th>
<th>∆t TE (ms)</th>
<th>TE % triggering</th>
<th>∆t JRP (ms)</th>
<th>JRP % triggering</th>
<th>∆t = 2 (ms)</th>
<th>2ms % triggering</th>
</tr>
</thead>
<tbody>
<tr>
<td>82885</td>
<td>1.5</td>
<td>6</td>
<td>1.5</td>
<td>6</td>
<td>1.5</td>
<td>6</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>82886</td>
<td>3.5</td>
<td>15</td>
<td>3.8</td>
<td>17</td>
<td>3.3</td>
<td>15</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>82887</td>
<td>3.6</td>
<td>19</td>
<td>4.1</td>
<td>23</td>
<td>4.2</td>
<td>24</td>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>82889</td>
<td>4.2</td>
<td>21</td>
<td>4.5</td>
<td>21</td>
<td>4.5</td>
<td>21</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>84688</td>
<td>1.8</td>
<td>9</td>
<td>1.3</td>
<td>3</td>
<td>1.8</td>
<td>9</td>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>84690</td>
<td>3.8</td>
<td>30</td>
<td>3.2</td>
<td>21</td>
<td>3.4</td>
<td>25</td>
<td>2</td>
<td>14</td>
</tr>
<tr>
<td>84693</td>
<td>3.1</td>
<td>25</td>
<td>3.5</td>
<td>28</td>
<td>3.2</td>
<td>25</td>
<td>2</td>
<td>17</td>
</tr>
<tr>
<td>84696</td>
<td>3.4</td>
<td>9</td>
<td>3.5</td>
<td>9</td>
<td>3.5</td>
<td>9</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 2. Percentage of triggering for the lag times calculated with GC, TE, JRP and with the usually assumed interval of 2 ms.

Figure 10. Average diagonal length for discharge 82854. The presence of two lag times is quite evident. The second peak corresponds to a specific interval of the discharge between 51–52 s (see figure 11).
since the lag times found with the three proposed methods range between 1.5–4.5 ms. The 2 ms interval previously assumed is probably an acceptable estimate on average, but should be particularized for each discharge. Indeed, assuming a fixed 2 ms interval, the actual triggering efficiency can be either overestimated or underestimated, depending on the shot. Such a choice can, therefore, be misleading and compromise the interpretation of the actions taken in the experiments, which can be wrongly attributed the opposite effect on the triggering efficiency rather than the one they really have.

It is also worth mentioning that the present results confirm and strengthen the conclusions already reached in [12] with only TE. In any case, on average the choice of 2 ms lag time leads to an unnecessary underestimate of the triggering efficiency of the pellets, at least for the discharges considered in this paper.

7. Discussion and conclusions

In this paper, three different statistical approaches to the assessment of the triggering efficiency of ELMs have been introduced. Two of them, GC and TE, implement a causality concept based on increased predictability. TE does not rely on any assumption about linearity of the underlying dynamics and, therefore, is more general than the implemented version of GC. On the other hand, the two criteria are numerically completely independent. The third method exploits the properties of JRPs and is based on the analyses of the recurrence properties of dynamical systems. A series of numerical tests has shown the potential of the three techniques to determine the time interval during which a causal–effect relationship is really taking place. All three criteria have proved to possess excellent qualities of prediction and also produce the expected results for spiky signals of the type typical of ELMs.

The application of the three criteria to ELM pellet-pacing experiments on JET with the ILW gives very coherent and similar outputs, which can also help in quantifying the uncertainty in the estimates. The obtained results indicate that the traditional criterion used to estimate the efficiency of the pellets underestimates on average the capability of this system to trigger ELMs. However, every discharge is a different case and must be studied independently. Indeed, the appropriate lag time to calculate the number of ELMs triggered by pellets ranges between 1.5–4.5 ms. The proposed criteria, therefore, allow for the assessment of the properties of the pellets on a shot-to-shot basis, paving the way for a much better understanding and optimization of this important tool in the perspective of ITER. It is also worth mentioning that the proposed techniques, being based on the statistical relations between the pellets and ELMs, are not influenced by changes in the base plasma parameters. Indeed, the injection of pellets can induce increases in the plasma density at the edge, rendering even more difficult the interpretation of the results if a fixed lag time is used, since the change in the ELM frequency can be due to those variations of the average density and not to the pellet-pacing capability. The two effects are difficult to disentangle if a fixed lag time is used as the criterion to evaluate the pellet-pacing efficiency [5].

The proposed techniques seem to have a higher time resolution, also rendering them capable of detecting changes during a single shot. Indeed, there is no particular physical reason for the lag time to remain exactly constant over the entire discharge. JRPs have a particularly good time resolution as can be seen from the example reported in figure 10 for shot number 82854. For these discharges, the JRP criterion identifies two different lag times: the first one around 2.9 ms and a second one at about 3.8 ms. The GC also provides two estimates: at 2.7 and 3.8 ms. These two different estimates correspond to two different phases of the discharge, as can be seen in the trend of the $D_\alpha$ reported in figure 11. If the interval between 51–52 ms is removed from the analysis, only the lag time around 2.7 ms remains. In contrast, if only the interval between 51–52 ms is considered, the lag time remaining is 3.8 ms. Therefore, the increase in the ELM frequency in the period between 51–52 ms really seems linked to an increase in the efficiency of the pellet pacing. On the other hand, the statistics in these discharges is not very high so this result must be considered as preliminary. In any case, it can be stated that, although the three criteria are statistical in nature and require enough data at steady state, if
different but sufficiently long stationary intervals are present during a single discharge, they can provide specific information on these intervals. Such a capability provides the opportunity to particularize the analysis and even to perform different experiments in the same discharge.

With regard to future developments, first it should be mentioned that, to fully understand the physics behind pellet pacing, the experimental results of other devices should also be analyzed. Indeed, in Europe, multimachine databases are being built, also to investigate ELMs, and more cooperation in this field would be extremely important. Since the techniques proposed in this paper are in no way machine specific, their deployment for the investigation of the data of other devices would be both immediate and very beneficial. Also, a comparison with other methods would be appropriate, given the difficulty of really proving causal relationships between the pellets and ELMs.

The application of the developed tools to other Tokamak physics aspects also seems quite promising. Indeed, traditionally, in many cases the causal relationship between time series is determined by simple visual inspection of the signals and the determination of the time proximity of events. First of all, these visual inspection exercises are quite prone to errors and misinterpretation. Moreover, in some applications, they have been complemented with not well-founded and simplistic assumptions. Therefore, the proposed techniques, based on sound statistical and dynamical tools, are expected to have significant potential. They could indeed be used in a variety of contexts, ranging from the consolidation of preliminary investigations to the critical assessment of working hypotheses. Three very promising potential fields are the investigation of instabilities, the study of plasma dynamics and the control of impurities. Among various instabilities, the assessment of the disruption causes seems particularly relevant. The present investigations of this very important aspect have been limited so far by some weaknesses that the proposed methods could help to alleviate. First, the choice of the most significant signals for disruption prediction on JET has been based on empirical considerations without solid statistical basis. In this perspective, the three methods introduced in this paper could be usefully deployed to identify the most appropriate signals to be given as inputs to disruption predictors for the optimization of their performance [13–19]. They could also play a role in the assessment of mitigation strategies. With regard to disruption avoidance, the contribution of the proposed techniques could have an even more decisive impact, since the investigation of the best strategies to reduce the percentage of disruptions is at a more primitive stage than prediction for mitigation. With regard to plasma dynamics, an interesting subject is certainly the L-H transition, for which various models exist but no accepted dynamical theory is available. The tools presented in this paper could be deployed to provide a statistical sound analysis of the most likely quantities to be the control parameters for this transition [17, 20]. Another aspect of Tokamak control, which presents serious challenges, is impurity control [21]. This issue has become particularly relevant since the installation of the new ILW. Given the difficulties in the impurity dynamics, their link with sawteeth and ELMs and the quality of the measurements, sound statistical criteria to assess the efficiency of the various control strategies based on radio frequencies are expected to be quite beneficial. Another important potential application is the qualification of the magnetic equilibria, which are essential to control the configurations and to maximize performance. On the other hand, so far statistical methods have not been applied much to detect problems in the reconstruction of the magnetic topology and to determine the relations between possible causes of errors, including the internal measurements [22–24]. It is also worth mentioning that the techniques described in this paper are also being considered for application in other fields of applied physics, such as remote sensing and monitoring of the environment [25, 26].

Acknowledgments

This work has been carried out within the framework of the EUROfusion Consortium and has received funding from the Euratom research and training programme 2014–2018 under grant agreement No. 633053. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

References

[27] F. Romanelli and on behalf of JET Contributors 2015 Nucl. Fusion 55 104001