Recurrence quantification analysis of experimental turbulent plasma data series

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1 Introduction

The understanding of plasma turbulence and the associated cross-field transport in magnetic confinement fusion devices has been a matter of continuous investigation for many years. A vast amount of evidence points to turbulence as the main phenomenon responsible for generating most of radial transport and the corresponding confinement degradation. On the other hand, it is also apparent that a deeper understanding about the dynamics of a system may sometimes be obtained by looking at experimental data from a non-standard point of view. The current work must be understood from this perspective. We test a new approach to characterize turbulence where the concept of recurrence takes a principal role. The formal concept of recurrence (in conservative systems) was introduced first by Poincaré in 1890 [1] when he was studying the three body problem and the chaotic behaviour of orbits. In his work, he mentioned that in volume-preserving flows with bounded orbits only, the system recurs infinitely many times as close as one wishes to its initial state. Although our system does not fulfill entirely these requirements, we can still follow the evolution of the system state and search for recurrences in experimental data. Then, we try to draw associations between the observed typical features of these recurrences and the underlying dynamics of turbulence in these plasmas.

2 Recurrence plots

Recurrence plots (RPs) were introduced in 1987 by Eckmann [2] to visualize in an efficient manner the recurrences of phase space trajectories of dynamical systems. These techniques can also be applied to time series of experimental data, which must reflect the underlying dynamics. As an illustration, Fig. 1(a) shows the time evolution of the fluctuating potential measured with Langmuir probes over a temporal window encompassing 2 milliseconds of a discharge in the TJ-II stellarator. In Fig. 1(b) the corresponding recurrence plot is depicted. This plot has been constructed by considering the scalar time series, \( v_i = v(i\Delta t) \), where \( i = 1, \cdots, N \), being \( N \) the number of points and \( \Delta t \) the inverse sampling frequency. We create an “artificial” or reconstructed phase space by embedding
Figure 1: (a) Sort time interval (500 samples after filtering) of fluctuating potential time series and (b) its corresponding recurrence plot.

our original series in a higher dimension space. Our choice is the time delay method:

\[ \bar{y}_i = \sum_{j=1}^{m} v_i + (j-1)\tau \hat{e}_j, \]

where \( m \) is the embedding dimension, \( \tau \) is the time delay and \( \hat{e}_i \) are unit vectors perpendicular to each other (\( \hat{e}_i \cdot \hat{e}_j = \delta_{i,j} \)). The RP is constructed by forming the recurrence matrix:

\[ R_{i,j}(\varepsilon) = H(\varepsilon - \| \bar{y}_i - \bar{y}_j \|) \quad i, j = 1, \ldots, N, \]

where \( H(\cdot) \) is the Heaviside function and \( \| \cdot \| \) is a norm. In this way, a RP is constructed by assigning different colors for the two different values its binary entries can take (e.g. black dots for \( R_{i,j} = 1 \) and white ones for \( R_{i,j} = 0 \)), the former ones corresponding to recurrences. The lines and patterns in an RP give us idea of the recurrences in the system [2]. Indeed, there is always a main diagonal, because a system is always identical to itself at the same time. Non-principal diagonal lines represent moments in which the system state passes close to its initial value after a certain time. Vertical (and horizontal) lines represent, on the other hand, cases in which the system basically has not changed much. Quantifying the amount of time (or the probability, given by the average lengths and statistics of these lines) that the system spends in these regimes can be a useful (nonlinear) tool to analyze and characterize the dynamics of the system.

In order to get a credible RP, it is crucial the correct election of the different parameters that enter in the calculus. One of them is \( \varepsilon \). If it is chosen too small, there may be almost no recurrences in our RP. On the other hand, if it is chosen too large, the opposite effect is going to happen, bringing a very dense (lots of black dots) RP and therefore entangling the subsequent results. Our election for \( \varepsilon \) has been made in such a way that the recurrence rate (RR) is about 5% in all calculations (i.e. about 5% of points in \( R_{i,j} \) are recurrent points), and this value for the RR is achieved by assigning a numerical value to \( \varepsilon \) equal to the variance of the scalar time series over the corresponding temporal window, so \( \varepsilon \) is
not constant. Values for the time delay $\tau = 20$ and the embedding dimension $m = 4$ have been chosen (see Refs. [3,4] for the criteria used).

3 Diagnostics

As previously mentioned, patterns as shown in Fig. 1(b) are linked to a specific behaviour of the system. Regarding diagonal lines, and based on the histogram of diagonal lines $P(\varepsilon, l)$, we can define the determinism ($DET$) of the system:

$$DET = \frac{\sum_{l=1}^{N} lP(l)}{\sum_{l=1}^{N} lP(l)},$$

(3)

It is close to unity if the behaviour is deterministic (or predictable) and approaches to zero as random behaviour dominates. In (3), $l_{\text{min}}$ is the minimum length for a line to be considered as a diagonal line. Based on the probability $p(l) = P(l)/N(l)$ to find a diagonal line of length $l$, it is defined the Shannon entropy ($ENTR$) of the system:

$$ENTR = - \sum_{l=l_{\text{min}}}^{N} p(l) \ln p(l).$$

(4)

Regarding vertical lines, and based on the histogram of vertical lines $P(\varepsilon, v)$, we can define the laminarity ($LAM$) of the system:

$$LAM = \frac{\sum_{v=v_{\text{min}}}^{N} vP(v)}{\sum_{v=1}^{N} vP(v)},$$

(5)

which in an analogous way to the determinism, quantifies the occurrence of laminar states or states which do not change much in time.

4 Features of recurrences in fluctuating potential data

The experimental data used in this work were obtained from the edge of the TJ-II stellarator (in Ref. [5] there is a general description of the edge fluctuating general properties in TJ-II). A rack with twelve Langmuir probes placed at varying radial locations at the outer region of the plasma is used. Since the data series is long, we split in windows of constant size, and compute the diagnostics described on each window. Fig. 2 shows the averages of the diagnostics over the windows as a function of the radial position of the probe. Clearly, the temporal average of $DET$, $ENTR$ and $LAM$ increase as we approach the last close surface (LCS), and remain roughly constant moving further out. This implies that the system dynamics become increasingly less predictable/deterministic as we move into the plasma. In order to check whether the reason why this happens is attributable to the presence of an increasing shear of the poloidal rotation, we have also computed the mean value of the shear of the poloidal rotation and its absolute value over each of the windows. Fig. 3 plots the values of $DET$ against both shear and shear magnitude for different locations of the same discharge. Although this is a preliminary
Figure 2: Radial profiles of (a) determinism, (b) laminarity and (c) entropy throughout the whole discharge. The values are obtained by time averaging the different temporal windows’ results and the error bars stand for the standard deviation of the averages.

analysis, there is a trend that seems to indicate that higher values of the shear magnitude indeed correspond with lower values of DET, and similar behaviors are observed from LAM and ENTR. Further analysis of this kind of data for this and other discharges is currently underway, but our initial data already show that RQA can be a very useful tool in this context.

Figure 3: (a) Dependency of DET on shear. Each dot represents a temporal window and each color represents different radial positions, all of them for the same discharge. In (b), the average of the magnitude of the shear, instead of the signed shear has been calculated.

References