Gear Fault Diagnosis Based on Recurrence Network

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Abstract—Gear is one of the most important components in rotary machine systems. The vibration signals generated from gearbox show strong nonlinearity or chaotic behavior. To identify the complex nonlinear behavior of gear faults, recurrence network is introduced to extract features of gear vibration under different conditions. Quantitative characteristics (such as mean degree centrality, global clustering coefficient, or assortativity of the recurrence network) related to the dynamical complexity of a time series are chosen to help classify the different faults. Experimental study on four different gear conditions has proved that the recurrence network provides a good alternative approach to characterize gear fault.

Keywords—nonlinear time series; recurrence network; rotary machine; fault diagnosis

I. INTRODUCTION

Gear is one of the popular and important components in the rotary machinery for transmission. However, if a gear fault occurs, it will influence the performance of the whole machine system. In some serious cases, the gear fault may lead to equipment failure and great economic losses. Detecting the gear fault at an early stage while the machine is still in operation can help avoid abnormal events as well as reduce productivity loss.

The gear vibration analysis has often been performed in time domain, frequency domain, and time–frequency domain for fault feature extraction [1-6]. However, as we know, the failure of a mechanical system is always accompanied with changes from linear or weak nonlinear to strong nonlinear dynamics. Various nonlinear vibrations including chaotic dynamics can be found in a gearbox system. Nonlinear time series analysis may provide a useful alternative for gear fault diagnosis.

Recently, the recurrence-based complex network provides a new approach for the study of nonlinear time series in dynamical systems. Local and global measures of recurrence network analysis method have been applied to various disciplines ranging from detection of cardiovascular (CV) disorders, understanding the dynamical properties of the brain using electroencephalography, power grids, to neuronal networks[7-10].

According to these examples, recurrence network is a powerful nonlinear tool for time series analysis of complex dynamical systems. Recurrence complex network has been used in fault diagnosis of rolling bearing and rotors[11], whose diagnosis depended on the mean value of different parameters respectively. However, few research has been done to apply this powerful tool to vibration signals of gear fault. In this paper, recurrence network is introduced to reveal the characteristics in nonlinear time series of gearbox and different parameters have been used to diagnose the gear fault. The experiment results demonstrate that recurrence network is effective to take care of nonlinearity existing in the dynamic system and can be successfully applied to detecting faults of a gearbox.

This paper is organized as follows: the recurrence network is introduced in section 2. The experimental study is then conducted in a gearbox test system and the results of nonlinear fault detection to the vibration signal of a gearbox system are presented in section 3. Finally, some conclusions are drawn in section 4.

II. RECURRENCE NETWORK

A. Transforming Time Series into Phase Space

According to the theory of dynamical systems, the most prominent feature of a nonlinear system is its phase space. Given a time series of a single observable, embedding is a traditional way to characterize properties of the phase space of unknown dimension. Therefore, it can be utilized to infer the multi-dimensional phase space from a one-dimensional time series. We use a suitable m-dimensional embedding and time delay τ [12] to transform a scalar time series \( x(t) \) \((t = t_1, \ldots, N)\) to \( x^{(m)}(t) = (x(t), x(t+\tau), \ldots, x(t+(m-1)\tau)) \). Then the binary recurrence matrix \( R \) can be obtained as

\[
R_{ij} = \Theta(\epsilon - ||x_i - x_j||) \quad (1)
\]

where \( \Theta(\cdot) \) is the Heaviside function, \( ||\cdot|| \) denotes a suitable norm in the considered phase space (maximum norm is used in this paper) and \( \epsilon \) is a threshold distance that should be reasonably small [13-14]. In this paper, we use the abbreviation \( x_i = x^{(m)}(t = t_i) \) (with \( t_i \) being the point in time associated with the \( i \)th observation recorded in the time series).

So far, network theory has been applied in various fields of research successfully, which has motivated first attempts to generalize this concept for a direct application to time series [15-17]. According to complex network analysis, important complementary features of dynamical systems can be obtained. And these features are based on spatial
dependences between individual observations instead of temporal correlations. That is to say recurrence networks (RN) are complex networks constructed from the time series of dynamical systems based on the specific property of recurrence [18].

The phase space vectors are considered as nodes of a network and recurrences are identified with links. The binary adjacency matrix \( A \) can represent an undirected and unweighted network, where a connection between nodes \( i \) and \( j \) is marked as \( A_{i,j} = 1 \). \( A \) is obtained from the recurrence matrix by removing the identity matrix.

\[
A_{i,j}(\epsilon) = R_{i,j}(\epsilon) - \delta_{i,j}
\]  

where \( \delta_{i,j} \) is the Kronecker delta introduced here in order to avoid artificial self-loops.

**B. Quantitative Characteristics of Recurrence Networks**

Relationships between recurrence network entities and corresponding geometrical objects and their properties in phase space are described in Table I.

| TABLE I. RELATIONSHIPS BETWEEN RECURRENCE NETWORK AND PHASE SPACE |
|-----------------------------|-----------------------------|
| Recurrence Network | Phase Space |
| Vertex | State \( x_i \) |
| Edge | Recurrence of states |
| Path | Overlapping sequence of \( \epsilon \)-balls |

Due to the natural interpretation of vertices, edges and paths, the topological characteristics of a recurrence network catch the essential phase space properties of the dynamical system.

Quantitative characteristics of the topological features of recurrence networks can be considered as novel and complementary measures. Local network properties and global network properties will be defined in the following part.

Quantifying the importance of a vertex in a recurrence network, the degree centrality (local recurrence rate) of a vertex \( v \), \( k_v \), is defined as the number of neighbors directly connected with \( v \):

\[
k_v = \sum_{i=1}^{N} A_{i,v}
\]

According to the definition, usually the sum is taken over all \( i \neq v \). However in some situations, it is more meaningful to pay attention to the mean degree of all vertices rather than the full distribution of degree centralities in a network.

\[
\langle k \rangle = \frac{1}{N} \sum_{i=1}^{N} k_v = \frac{2L}{N}
\]

The mean degree centrality \( \langle k \rangle \) is a simple characteristic quantity for this distribution. And the total number of edges in the recurrence network \( L \) is defined as follows.

\[
L = \sum_{i,j} A_{i,j} = N(N-1)\rho
\]

According to equation (5), the mean degree centrality \( \langle k \rangle \) is directly proportional to the edge density \( \rho \) of the network.

Through normalizing degree centrality by the maximum number of possible connections, \( N-1 \), the local connectivity is obtained.

\[
\rho_v = \frac{1}{N-1} \sum_{i=1}^{N} A_{i,v} = RR_v
\]

From the recurrence plot point of view, the local connectivity corresponds to the local recurrence rate \( RR_v \) of the state \( v \). Global recurrence rate (edge density) can be obtained as

\[
\rho = \frac{1}{N} \sum_{i=1}^{N} \rho_v(\epsilon) = \frac{2}{N(N-1)} \sum_{i=1}^{N} A_{i,v}(\epsilon)
\]

\[
= \frac{2}{N(N-1)} \sum_{i=1}^{N} \Theta(\epsilon - ||x_i - x_j||)
\]

\[
= RR
\]

The clustering coefficient of a vertex \( v \), \( C_v \), is considered to characterize the density of connections in the direct neighborhood of this vertex in terms of the density of connections between all vertices that are connected to \( v \). In this paper, the definition of clustering coefficient is proposed by Watts and Strogatz [19].

\[
C_v = P(A_{i,j} = 1 | A_{v,i} = 1, A_{j,v} = 1) = \frac{P(A_{i,j} = 1, A_{v,i} = 1, A_{j,v} = 1)}{P(A_{i,j} = 1, A_{v,i} = 1, A_{j,v} = 1)}
\]

(8)

using Bayes’ theorem with

\[
P(A_{i,j} = 1, A_{v,i} = 1) = \frac{1}{(N-1)(N-2)} \sum_{j=1}^{N} \sum_{i=1, i \neq j}^{N} A_{i,j} A_{v,i} A_{j,v}
\]

(9)

and \( P(A_{i,j} = 1, A_{v,i} = 1, A_{j,v} = 1) \) is obtained as a similar expression.

The average value of the clustering coefficient of all vertices of a network is considered for the degree centrality. As a global characteristic parameter of the topology of a network, the global clustering coefficient is defined as

\[
C = \frac{1}{N} \sum_{v=1}^{N} C_v
\]

(10)

A network is called assortative if vertices tend to connect to vertices of a similar degree \( k \). On the opposite side, it is called disassortative if vertices of high degree prefer to
connect to vertices of low degree, and vice versa. Hence, the Pearson correlation coefficient of the vertex degrees on both ends of all edges can be used to quantify assortativity [19-20],

\[
A = \frac{1}{L} \sum_{j>i} k_i k_j A_{i,j} - \left[ \frac{1}{L} \sum_{j>i} \frac{1}{2} (k_i + k_j) A_{i,j} \right]^2
\]

If the density of states in phase space doesn’t change within an \( \varepsilon \)-ball, which means the degrees of nodes of an edge will tend to be similar, the assortativity coefficient \( A \) will be positive. Within the framework of recurrence networks, \( A \) can be interpreted as a measure to evaluate the continuity of the density of states. Assortativity coefficient is a new characteristic, which has not yet been described by other nonlinear measures especially within the RQA (Recurrence Quantification Analysis) framework.

In a scale-free network, vertices with small degree may work primarily, which can lead to an underevaluation of the actual fraction of triangles in the recurrence network. In order to eliminate such effects, network transitivity is proposed by Barrat and Weigt [21], which is defined as [22]

\[
T(\varepsilon) = \frac{\sum_{\nu,\mu,\nu} A_{\nu,\mu} A_{\mu,\nu}}{\sum_{\nu,\mu} A_{\nu,\mu} A_{\mu,\nu}}
\]

\( C \) is used as a represent for the average local dimensionality, while \( T(\varepsilon) \) characterizes the effective global dimensionality of the system, which denotes the probability that two nodes connected to a third node are also directly connected.

III. EXPERIMENTAL EVALUATION

To utilize the quantitative measures resulted from the recurrence network for characterizing the gear states, an experimental study was conducted on a four-speed motorcycle gearbox test system [23]. As shown in Fig. 1, the electrical motor rotates at a constant nominal speed 1420 RPM. In order to eliminate the vibration, four shock absorbers are installed under the base of test system. Four different fault conditions were tested in this study, which include slight-worn gear, medium-worn gear, broken teeth of gear and one normal condition respectively.

The signals were sampled at 16384 Hz, The raw vibration signals of four conditions are shown in Fig.2.

![Figure 1. The Experimental Platform.](image)

<table>
<thead>
<tr>
<th>Faults</th>
<th>( &lt;k&gt; )</th>
<th>( \rho )</th>
<th>( C )</th>
<th>( A )</th>
<th>( T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>11.7417</td>
<td>0.0119</td>
<td>0.2886</td>
<td>0.5731</td>
<td>0.3447</td>
</tr>
<tr>
<td>Slight worn</td>
<td>1.5641</td>
<td>0.0016</td>
<td>0.0999</td>
<td>0.2891</td>
<td>0.1732</td>
</tr>
<tr>
<td>Medium worn</td>
<td>2.2785</td>
<td>0.0023</td>
<td>0.1219</td>
<td>0.4929</td>
<td>0.2029</td>
</tr>
<tr>
<td>Tooth broken</td>
<td>27.447</td>
<td>0.0277</td>
<td>0.357</td>
<td>0.5207</td>
<td>0.4221</td>
</tr>
</tbody>
</table>

To classify the conditions of the gear, support vector machine is used as the classifier. In order to prove the superiority of SVM, k-nearest neighbor (KNN) classifier is used as a contrast. 100 data samples are used to train the classifiers, and for each condition 25 data sample are used respectively. 100 data are used to test with 25 data being used for each condition respectively. As the relationship between the mean degree of all vertices \( <k> \) and global recurrence rate (edge density) \( \rho \), the global clustering coefficient \( C \), assortativity \( A \), and network transitivity \( T \) are used as the features of the vibration signals. In Table II, features of four different conditions are listed.
nine data are wrongly classified using KNN and the accuracy of results is 91% ,while only two data are wrongly classified using SVM, and the accuracy of results is 98%. The comparison shows that SVM has better performance than KNN. The results also indicates recurrence network can extract the features of nonlinear time series effectively.

### TABLE III. FOUR GEAR FAULT TYPES IN DDS SYSTEM USING KNN

<table>
<thead>
<tr>
<th>Faults</th>
<th>Normal</th>
<th>Slight worn</th>
<th>Medium worn</th>
<th>Tooth broken</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>25</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Slight worn</td>
<td>0</td>
<td>24</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>Medium worn</td>
<td>0</td>
<td>1</td>
<td>20</td>
<td>0</td>
</tr>
<tr>
<td>Tooth broken</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>22</td>
</tr>
</tbody>
</table>

### TABLE IV. FOUR GEAR FAULT TYPES IN DDS SYSTEM USING SVM

<table>
<thead>
<tr>
<th>Faults</th>
<th>Normal</th>
<th>Slight worn</th>
<th>Medium worn</th>
<th>Tooth broken</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>25</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Slight worn</td>
<td>0</td>
<td>25</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Medium worn</td>
<td>0</td>
<td>0</td>
<td>25</td>
<td>0</td>
</tr>
<tr>
<td>Tooth broken</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>23</td>
</tr>
</tbody>
</table>

### IV. Conclusion

Recurrence network theory is introduced in this paper for gear fault diagnosis. The mean value of all vertices, global recurrence rate, the global clustering coefficient, assortativity and network transitivity are used to show the characteristics of the vibration signal. To apply this theory to fault diagnosis, vibration signal from four different conditions of gears are analyzed. By comparing the results of KNN and SVM, SVM has better classifying accuracy. With the help of support vector machine as a classifier, the results of the classification show that recurrence network can extract the features of the gear vibration signal effectively. Recurrence network is a good mathematical tool to nonlinear time series analysis for gear fault diagnosis.

### ACKNOWLEDGMENT

This work has been supported by the National Natural Science Foundation of China (51575102).

### REFERENCES


