Adaptive chaotic noise reduction method based on dual-lifting wavelet

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ABSTRACT

For the subjectivity of lifting wavelet coefficients selection, an adaptive noise reduction method is proposed for chaotic signals corrupted by nonstationary noises. Here, wavelet coefficients including coarse approximation and detail information are obtained by dual-lifting wavelet transform. The coarse parts are handled by the singular spectrum analysis, whereas the detail parts are analyzed combining with gradient decent algorithm in neural networks for the adaptive choice of wavelet coefficients. The chaotic signals generated by Lorenz model as well as the observed monthly series of sunspots are respectively applied for simulation analysis. The experimental results show a dramatic improvement of the proposed method, the advantages of which include the simple of achieving, the small reconstruction error and the efficiency for the noisy chaotic signals.

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1. Introduction

Noise reduction, as an integral part of signal estimation, has been widely studied for many years in diverse fields (Alexander & Robin, 2005; Celka & Cyselfs, 2006; Hsieh & Kuo, 2008). However, recent researches show that noises contaminating chaotic signals limit the performance of many techniques such as identification (Vicha & Dohnal, 2008; Yousef, Reyad, & Wajdi, 2004), parameters estimate (Smirnov, Vlaskin, & Ponomarenko, 2005; Yonemoto & Yanagawa, 2007) and prediction accuracy (Elshorbagy, Simonovic, & Panu, 2002; Han & Wang, 2009). It follows that preprocessing chaotic signals to reduce noises without distorting the dynamics of the underlying signal is therefore highly desirable.

Much work has been carried out on noise reduction methods. Some of them, however, either use linear or nonlinear filter. Linear filtering like low-pass or high-pass filter does not work well for overlapping bandwidths of chaotic signals (Deng, Gao, & Mao, 2005; Wang, Z, Lam, & Liu, 2007). On the other hand, nonlinear filtering such as the median filter fails to remove Gaussian noises since the larger the window size used, the more the characteristic of chaotic signals is removed. Recently, wavelet theory has been extensively applied in image processing (Deng, David, & Marusic, 2007), chaos (Hramov & Koronovskii, 2005; Murguia & Campos, 2006; Soltani, 2002), data compression (Aminghafari, Cheze, & Poggi, 2006), fractals (Mallat & Wen, 1992; Site & Ramakrishnan, 2000), etc. As a powerful time–frequency analysis tool, it can characterize accurately the local features of nonstationary signals. So it is effective for noise reduction of the noisy chaotic signals. The Hard-threshold wavelet (Zhang, Bao, & Pan, 2001) and the Soft-threshold wavelet (David & Donoho, 1995; Han, Liu, & Xi, 2006) are two effective noise reduction methods, but the threshold function of the former is not continuous and that of the latter has a constant deviation in application.

Noise reduction methods based on classical wavelet transform (WT) are superior to linear and nonlinear filtering, but construction of classical WT relies on the fourier transform and needs clumsy mathematical operations. In this work, a novel approach is proposed for chaotic noise reduction. This noise reduction approach mainly uses second generation wavelet (Ercelebi, 2004; Jiang, He, & Duan, 2006; Kuzume, Nijjima, & Takano, 2004; Yilmaz, Subasi, & Bayrak, 2007). Wavelet coefficients are decomposed and constructed by a lifting scheme. The lifting scheme then gradually constructs new wavelets with improved properties. A construction with the lifting scheme is entirely spatial and therefore ideally suited for building second generation wavelets when no fourier transform is available. The lifting scheme allows a faster implementation and a fully calculation of the WT in place. Moreover, no extra memory is needed and the original signal can be replaced with its WT.

The adaptive chaotic noise reduction method proposed in this paper uses dual-lifting wavelet. In order to lower the containing noise, the singular spectrum analysis (SSA) and gradient decent algorithm in neural networks is respectively employed for the analysis of the approximate and the detail coefficients obtained by dual-lifting wavelet transform. At the same time, gradient decent algorithm can extract useful signals adaptively in detail parts. The proposed method is compared with the Soft-threshold wavelet method (Han et al., 2006) and the basic single lifting wavelet method (Ercelebi, 2004). According to the features of chaos, various
evaluation indicators are also introduced for comparing the pros and cons results. The experimental results show that the proposed method is more efficient and faster than that of other methods.

The remainder of the paper is organized as follows: Section 2 reviews briefly the theory of lifting scheme. Section 3 is devoted to the description of the adaptive chaotic noise reduction method. Simulations of the noisy chaotic signals and the experimental results are shown in Section 4. Finally, Section 5 presents the conclusion.

2. Lifting scheme principle

Lifting scheme is an entirely space domain wavelet construction. It starts with trivial wavelets, which do nothing but hold the formal properties of wavelet. A typical lifting scheme procedure consists of three basic steps: split, predict and update (Ercelebi, 2004). The specific introduction is discussed as follows.

In the space of all square integrable functions \( L^2(\mathbb{R}) \), define the noisy chaotic signal by:

\[
x(n) = y(n) + \eta(n), \quad n = 1, 2, \ldots , N,
\]

where \( y(n) \) is the original chaotic signal, and \( \eta(n) \) represents Gaussian noise with normal distribution.

In the first step of lifting scheme, the noisy chaotic signal \( x(n) \) is divided into the even subset \( x_e(n) \) and the odd subset \( x_o(n) \), the split procedure can be described by:

\[
\text{Split}(x(n)) = (x_e(n), x_o(n)),
\]

where \( x_e(n) = x(2k), x_o(n) = x(2k + 1), k = 1, 2, \ldots \)

If the original signal has a local correlation structure, then the even and odd subsets are highly correlated. In the subsequent step, the odd coefficients \( x_o(n) \) is predicted from the neighboring even coefficients \( x_e(n) \), and the prediction differences \( d(n) \) are defined as the detail signal:

\[
d(n) = x_o(n) - \mathbf{P}(x_e(n)),
\]

where \( \mathbf{P} \) is the prediction operator. The prediction procedure is equivalent to a high-pass filter, we generally select \( \mathbf{P}(x_e(n)) = 1 / 2( x_e(n) + x_e(n+2) ), i = 1, 2, \ldots \)

The last step forms coarse approximate to the original signal. We combine the even coefficients with the linear combination of the prediction differences, and then the approximation signal \( a(n) \) is obtained:

\[
a(n) = x_e(n) + \mathbf{U}(d(n)),
\]

where \( \mathbf{U} \) is the update operator.

Since the design of lifting scheme without reference to fourier techniques, each lifting step is always invertible. Assuming the same prediction operator and update operator are chosen for the forward and inverse transform, the lifting scheme guarantees perfect reconstruction for any \( \mathbf{P} \) and \( \mathbf{U} \). The inverse transform named anti-update, anti-predict and merger can be obtained by:

\[
x_e(n) = a(n) - \mathbf{U}(d(n)).
\]

\[
x_o(n) = d(n) + \mathbf{P}(x_e(n)).
\]

\[
x(n) = \text{Merge}(x_e(n), x_o(n)).
\]

The chart of the forward and inverse lifting transform is related as Fig. 1.

Iteration of Fig. 1(a) on the output \( a(n) \), and then the coarse and detail coefficients at different levels are generated. Scaling function \( \phi(n) \) and wavelet function \( \psi(n) \) of lifting scheme can be derived from \( \mathbf{P} \) and \( \mathbf{U} \) by iteration algorithm. The scaling function and wavelet function are symmetrical and compactly supported. We can also choose the prediction operator and update operator according to the properties of the given signal.

For using factorization program to enhance the realization of wavelet transform in the spatial space, the lifting scheme is effective in the area of chaotic noise reduction. However, the selection of lifting wavelet coefficients is difficult. When the threshold selection is too large, the result is not satisfactory. Conversely, there exists a larger construction error. At the same time, features that are large compared with noise in the detail parts are removed in the processing of treatment, so these flaws limit its application in practice.

3. Adaptive chaotic noise reduction method

Based on second generation wavelet transform, an adaptive chaotic noise reduction method is proposed. In the processing, two lifting wavelets are employed for the transformation and decomposition of chaotic signals. The approximate and detail parts are considered separately by singular spectrum analysis (SSA) and gradient decent algorithm in neural networks. In order to demonstrate the performance of noise reduction method, this paper introduces several evaluation indicators from different perspectives. The simulation verifies that the proposed adaptive chaotic noise reduction method is more straightforward and easier to implement.

3.1. The improved noise reduction method

Choose two lifting wavelets \( \psi_1(n) \) and \( \psi_2(n) \) which have the same length and similar properties (near symmetry, highest number of vanishing moments for the given support width), then the lifting wavelet transform of chaotic signal is:

\[
W_{x_0}(n) = \langle x(n), \psi_q(n) \rangle = W_{y_0}(n) + W_{\eta_q}(n)
\]

\[
= a_q(n) + d_q(n), \quad q = 1, 2, \quad j = 1, 2, \ldots ,
\]

where \( j \) is the level of lifting wavelet transform, \( a_q(n) \) is the approximate coefficients mainly composed of the low-frequency signals in the level \( j \), and \( d_q(n) \) represents the detail parts mainly composed of the high-frequency noises.

Basic single lifting wavelet method only takes the coarse parts as the approximation of the original signal without any treatment. However, on the one hand, chaotic signal after lifting wavelet

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Fig. 1. The forward and inverse lifting transform. (a) The forward lifting transform. (b) The inverse lifting transform.
transform still contain a certain noise, and on the other hand, some
effective information in the detail parts is probably lost. For solving
the above problems, the adaptive noise reduction method proposed
in this paper analyzes the approximate and detail coefficients with
different approaches. Firstly, the singular spectrum analysis is
employed for analysis of the approximate parts. Secondly, gradient
decent algorithm is used for the adaptive selection of wavelet
coefficients. The specific processing is discussed in the below.

(1) The approximate parts are handled by SSA, aiming at reserv-
ing the larger singular value representing signal and removing
the smaller singular value representing noise. The specific algorithm is described as follows. Assume \( a_q(n) \),
\( n = 1, 2, \ldots, N \) represents the approximate part in the location
\( n \) and level \( j \), then the lagged covariance matrix of \( a_q(n) \) is
defined by:

\[
T_{a_q} = \begin{bmatrix}
    c(0) & c(1) & \cdots & c(m - 1) \\
    c(1) & c(0) & \cdots & c(m - 2) \\
    \vdots & \vdots & \ddots & \vdots \\
    c(m - 2) & \cdots & \cdots & c(1) \\
    c(m - 1) & \cdots & c(1) & c(0)
\end{bmatrix}
\]

and

\[
c(\tau) = \frac{1}{N - \tau} \sum_{n=1}^{N-\tau} a_q(n)a_q(n + \tau), \quad 0 \leq \tau \leq m - 1,
\]

where \( c(\tau) \) is the covariance with the delay time \( \tau \), and \( m \) is
the embedding dimension.

Because \( T_{a_q} \) is a nonnegative symmetry matrix, there exist
eigenvectors \( e_{q_i} \), \( i = 1, \ldots, m \). The principal component of \( i \) satisfies:

\[
I_{q_i}(n) = \sum_{\tau=1}^{m} a_q(n + \tau)e_{q_i}(\tau), \quad 0 \leq n \leq N - m.
\]

The approximate parts reconstructed by the former principal
eigenvectors are named by:

\[
\hat{a}_q(n) = \sum_{i=1}^{p} I_{q_i}(n)e_{q_i}(\tau), \quad p \leq \tau \leq m.
\]

Then the average of the approximate parts after two lifting
wavelets transform is defined by:

\[
\bar{a}_q(n) = \frac{\hat{a}_d(n) + \hat{a}_g(n)}{2}.
\]

So we can obtain the approximation of the original signal.
SSA is an overall method, so it does very well for the portrayal
of the overall features, and can maintain the smoothness of
the systems.

(2) The detail parts are analyzed combining with gradient
decent algorithm in neural networks, the specific algorithm
is related as follows.

The original chaotic signals will be decomposed into the
approximate and detail parts after dual-lifting wavelet transform.
In classical wavelet transform, there are two approaches for
the treatment of the detail parts, they are the Hard-threshold and
Soft-threshold wavelet method respectively. With the same way,
we can also treat the detail parts obtained by dual-lifting wavelet
transform. However, the value of the function couldn’t change with
parameters. Therefore, we introduce an adaptive treatment for the
detail parts combing with gradient decent algorithm in neural net-
works. The basic idea is related as follows.

Suppose the level of lifting scheme decomposition is \( J \), the
approximate and detail coefficients are \( a_q \) and \( d_q \) respectively after
dual-lifting wavelet transform, then the reconstruct signal is:

\[
\hat{x}(n) = a_q(n) + \sum_{j=1}^{J} d_q(n),
\]

After adding Sigmoid threshold filtering unit to the details, Eq.
(13) can be written by:

\[
\hat{x}(n) = a_q(n) + \sum_{j=1}^{J} d_q(n)f(d_q(n), \theta_q, s_q),
\]

where Sigmoid threshold filtering unit is:

\[
f(d_q(n), \theta_q, s_q) = \frac{1}{1 + \exp(-s_q(d_q(n) - \theta_q))}.
\]

In Eq. (14), \( \theta_q \) and \( s_q \) are the parameters of Sigmoid threshold filtering
unit.

Define the Root Mean Square Error (RMSE) as the error criteria
of noise reduction, it is:

\[
\text{RMSE} = \sqrt{\frac{1}{2N} \sum_{n=1}^{N} (\hat{x}(n) - y(n))^2}.
\]

Then we adjust \( \theta_q \) and \( s_q \) according to gradient decent algorithm
until RMSE achieves the minimum value, and the adaptive choice
of detail coefficients is obtained. In the \( k \) step, we adjust:

\[
\theta_q(k + 1) = \theta_q(k) + \Delta\theta_q,
\]

\[
s_q(k + 1) = s_q(k) + \Delta s_q,
\]

where

\[
\Delta\theta_q = -a_{q\theta_q}\frac{\partial\text{RMSE}}{\partial\theta_q},
\]

\[
\Delta s_q = -a_{q_{s_q}}\frac{\partial\text{RMSE}}{\partial s_q},
\]

and \( a_{q\theta_q} > 0, a_{q_{s_q}} > 0 \) are both the adjustment parameters.

Then the average of the detail part after dual-lifting wavelet
transform is defined by:

\[
d_q(n) = \frac{\hat{d}_q(n) + \hat{d}_g(n)}{2}.
\]

So we can obtain the detail of the original signal.
In the end, the signal can be reconstructed though taking the
approximate and detail coefficients having been analyzed. The signal
after noise reduction can be written by:

\[
\hat{y}(n) = \hat{a}_q(n) + \sum_{j=1}^{J} \hat{d}_q(n).
\]

The main steps of adaptive chaotic noise reduction proposed in this
paper are shown as follows:

Step1: Dual-lifting wavelet transform of the actual chaotic sig-
nal observed, and obtain the approximation and the detail coef-
ficients.
Step2: Calculate and analyze the lagged covariance matrix of
the approximation coefficients using SSA for reserving the larger
singular value representing signal.
Step3: Add Sigmoid threshold filtering unit to the detail coef-
ficients.
Step4: Adjust the parameters \( \theta_q \) and \( s_q \), when RMSE achieves
the minimum value, the adaptive choice of detail coefficients
is obtained, and stop the iteration, or go on the iteration.
Step5: Reconstruct the signal with the approximate and detail
coefficients having been analyzed.
3.2. The evaluation indicators of noise reduction method

In order to show the validity of the proposed method, we apply different noise reduction methods to the same noisy chaotic signals, and introduce various evaluation indicators to compare them. In this paper, we consider this problem mainly from two aspects.

3.2.1. Chaotic signals with known model

These chaotic signals are obtained by the numerical sample or iterative of the difference and differential equations. Some evaluation indicators for these chaotic signals are adopted to evaluate the performance of noise reduction methods. The formula of these evaluation indicators are shown in Table 1.

3.2.2. Chaotic signals with unknown model

Chaotic signals of this type can't describe precisely the dynamic equation of system, so we don't obtain the original signal. Because of these, we introduce evaluation indicators from time domain, frequency domain and the structure features of the chaotic signals.  

3.2.2.1. Autocorrelation function analysis. Correlation analysis is an effective way to reflect useful information under the background noise in the time domain. The autocorrelation function value of noise in the time domain. The autocorrelation function value reflects the similar of noise reduction signal and the original signal, but the original signal must be known. So it can be used for the analysis of chaotic signals with known model.

\[
R_x(\tau) = \frac{1}{N-\tau} \sum_{n=1}^{N-\tau} x(n)x(n+\tau).
\]

Based on Eq. (23), we can define the difference of autocorrelation function:

\[
\Delta R(\tau) = R_y - R_x = \frac{1}{N-\tau} \sum_{n=1}^{N-\tau} \bar{y}(n)\bar{y}(n+\tau) - \frac{1}{N-\tau} \sum_{n=1}^{N-\tau} y(n)y(n+\tau),
\]

which reflects the similar of noise reduction signal and the original signal, but the original signal must be known. So it can be used for the analysis of chaotic signals with known model.

3.2.2.2. Power spectrum analysis. The power spectrum analysis is a powerful tool from frequency domain. It can find the frequency components distribution of system based on the limited data. So we can compare the pros and cons results by the power spectrum, which is defined by:

\[
P_x(w) = \frac{1}{\sqrt{2\pi}} \sum_{m} x(n)x(n+\tau)e^{-jnw}.
\]

3.2.2.3. Correlation dimension analysis. Correlation dimension (Scarlat, Stan, & Cristescu, 2007) is an important feature of chaos systems, it will tend to a saturation value as the increasing of the embedding dimension. Its formula can be written by:

\[
C_m(r) = \frac{1}{N^2} \sum_{i,j=1}^{N} \Theta(r - \| \mathbf{X}_i - \mathbf{X}_j \|), \quad i,j = 1,2,\ldots,N,
\]

where \( r \) is the pre-determined distance, \( \mathbf{X}_i \) and \( \mathbf{X}_j \) is the vector points in the phase space, \( \| \cdot \| \) represents Euclidean distance norm, and \( \Theta(\cdot) \) is the Heaviside function, its expression is shown as follows:

\[
\Theta(x) = \begin{cases} 
1, & x \leq 0, \\
0, & x > 0.
\end{cases}
\]

3.2.2.4. Recursive analysis. Recursive phenomenon is a basic feature of the certainty, nonlinear and chaos systems. Recursive map (Marwan, Kurths, & Saparin, 2007; Marwan & Kurths, 2005; Antoniou & Vorlow, 2004; Gao, Cai, Antoniou, & Constantinou, 2000) is a tool mainly based on the analysis of phase space attractor structure, researching on the recursive characteristic of adjacent vector in the attractor orbit. Its formula can be written by:

| Table 1 | \begin{tabular}{|c|c|c|} 
| Indicator & Equation & Meaning \\
\hline | Signal–Noise ratio & SNR & Reflect the ability of noise reduction, the greater the value, the better the effect \\
| Signal–Noise ratio gain & SNRG & The function of SNRG is the same as SNR \\
| Gain parameter & GP & The ratio of the original noise and the remainder noise \\
| Root mean square error & RMSE & Reflect the average deviation degree of noise reduction signal and the original signal \\
| Bias & BIAS & Reflect the average systematic deviation of noise reduction signal and the original signal \\
| Noise reduction accuracy & NRA & Reflect the correlation of signal, where \( yr \) is the average of \( y \), \( S_y \) and \( S_x \) is the Standard deviation \\
| Decision coefficient & DC & Reflect the average deviation degree of noise reduction signal and the original signal average \\
\hline
\end{tabular} |

| Table 2 | \begin{tabular}{|c|c|c|} 
| RQA & Equation & Meaning \\
\hline | Recurrence rate & \( \text{RR} = \frac{1}{N} \sum_{i=1}^{N} R_i \) & Reflect the aggregation degree and the correlation among the points in the phase space trajectories \\
| Determinism & \( \text{DET} = \frac{1}{N} \sum_{i=1}^{N} \frac{y(i)}{R_i} \) & Describe the extent of recursive cycle. Stochastic data cause none or only short diagonals, whereas deterministic systems cause longer diagonals \\
| Laminarity & \( \text{LAM} = \frac{1}{N} \sum_{i=1}^{N} \frac{y(i)}{R_i} \) & Reflects the relative speed of system state changing \\
| Maximum diagonal length & \( L_{\text{max}} = \max(\{ |i| \geq 1 \ldots N \}) \) & Reflect the separation rate between the adjacent track and the sensitivity of the initial state from another angle \\
| Recurrence Trend & \( \text{RT} = \frac{1}{N} \sum_{i=1}^{N} \frac{(i-1)\text{RR}_i}{\text{RR}_i} \) & Reflect the stability of system \\
\hline
\end{tabular} |
\begin{align}
R_{ij} &= \Theta(r - \|X_i - X_j\|), \quad i,j = 1,2,\ldots,N. \tag{28}
\end{align}

The recursive map of chaotic sequence is composed of some bands paralleling to the main diagonal, and the value of these bands are the same in the symmetry location of the main diagonal. However, it is no recursive rules for noise sequence. So we can compare the effect of noise reduction though the RP.

There are also some Recurrence Quantification Analysis (RQA) indicators for quantitative characterization of the RP. The main quantitative parameters are shown in Table 2.

where \( P(l) \) is the distributions probability of the diagonal line lengths \( l \), \( P(v) \) is the distributions probability of the vertical or level line \( v \), \( l_{\text{min}} \) and \( v_{\text{min}} \) are the least analysis length, \( RR_k \) is the

![Fig. 2. The phase space map of Lorenz signal. (a) The Lorenz signal with 20% noise. (b) Denoised data with the Soft-threshold wavelet method. (c) Denoised data with the basic single lifting wavelet method. (d) Denoised data with the adaptive dual-lifting method in this paper.](image)

### Table 3
The performance comparison of three noise reduction methods.

<table>
<thead>
<tr>
<th>Method</th>
<th>SNR</th>
<th>SNRG</th>
<th>GP</th>
<th>RMSE</th>
<th>BIAS</th>
<th>DC</th>
<th>NRA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soft-threshold wavelet method (Han et al., 2006)</td>
<td>23.1800</td>
<td>9.1054</td>
<td>9.1054</td>
<td>0.3840</td>
<td>0.0021</td>
<td>1.0057</td>
<td>0.9979</td>
</tr>
<tr>
<td>Basic single lifting wavelet method (Ercelebi, 2004)</td>
<td>22.2232</td>
<td>8.1486</td>
<td>8.1486</td>
<td>0.4231</td>
<td>0.0011</td>
<td>1.0122</td>
<td>0.9970</td>
</tr>
<tr>
<td>Adaptive dual-lifting method in this paper</td>
<td>24.6631</td>
<td>10.5884</td>
<td>10.5884</td>
<td>0.3195</td>
<td>0.0033</td>
<td>1.0006</td>
<td>0.9983</td>
</tr>
</tbody>
</table>

![Fig. 3. The SNRG and RMSE curve with different noise levels. (a) The SNRG curve. (b) The RMSE curve.](image)
4. Simulations of dynamic systems

In this paper, we verified our method by the real data: the noisy Lorenz chaotic signal and the monthly sunspots signal from January of 1756 to December of 2005.

4.1. The noisy Lorenz signal

Consider the Lorenz dynamic equation:

\[
\begin{align*}
\dot{x} &= -\sigma(x - y), \\
\dot{y} &= \gamma x - y - xz, \\
\dot{z} &= xy - bz,
\end{align*}
\]

with \(\sigma = 10, \gamma = 28, b = 8/3\). The equation is integrated using a fourth order Runge–Kutta method with a fixed step size of 0.01 s. We take 2000 data points from the X-coordinate into consider and this serves as noise free series. Zero mean uncorrelated unpredictable noise with normal distribution is then added to obtain the noisy series which the signal-to-noise ratio is set to 14.0746 dB and the noise level equals to 20%.

First of all, we get the lift framework of two wavelets, and obtain two lifting wavelets with the same length and similar properties. Then we give the lifting wavelet transform of the noisy Lorenz signal. In order to check the effectiveness of the proposed noise reduction algorithm, we use the Soft-threshold wavelet method, the basic single lifting wavelet method and the adaptive dual-lifting wavelet method for noise reduction of noisy Lorenz signal respectively, the phase space map of Lorenz signal before and after noise reduction are shown as Fig. 2.

Table 3 lists the SNR, SNRG, GP, RMSE, Bias, DC and NRA comparison of the above three noise reduction methods.

According to Table 3, we can obtain the SNRG and RMSE curve with different noise levels. When the noise level is 5%, 10%, 20%, 30%, 50%, 70%, 90% and 100% respectively, the SNRG and RMSE curve are shown in Fig. 3.

From Table 3 and Fig. 3, the extremely improvement performance of adaptive dual-lifting wavelet method can be seen obviously. Compare with Soft-threshold wavelet method and basic single lifting wavelet method, adaptive dual-lifting wavelet method further enhances the SNR, SNRG, GP and NRA of Lorenz system, and reduces the RMSE, BIAS and DC.

To thoroughly illustrate the effectiveness of adaptive method based on dual-lifting wavelet for chaotic signals with known model, the difference of autocorrelation function are employed for the analysis from time domain. When the delay time is 0, 1, 2, 3, ... 20 s, the autocorrelation difference map of Lorenz signal after noise reduction is shown in Fig. 4. From Fig. 4, we can see that the autocorrelation difference of Lorenz signal after adaptive noise reduction is lower than those of the other methods.

Fig. 4. The autocorrelation difference map of Lorenz signal after noise reduction.

Fig. 5. Time Diagrams of sunspots. (a) The raw sunspots data. (b) Denoised data with the Soft-threshold wavelet method. (c) Denoised data with the basic single lifting wavelet method. (d) Denoised data with the adaptive dual-lifting wavelet method.
reduction method is smaller than other methods in the same delay
time. It fully explained that Lorenz signal after this method is more
similar to the original signal, and more noises are removed.

Through the above analysis, we can also see that the adaptive
dual-lifting noise reduction method has better results in maintain-
ing signal dynamic features and the overall smoothness, it im-
proves the whole performance of the system.

4.2. The monthly sunspots series

Sunspot is a slowly changing phenomenon of solar activity. It
has a direct impact on the phenomenon change of the Earth’s cli-
mate and hydrology, so it is of great significance to research them.
With the development of chaos theory and fractal technology, re-
search on the chaotic features of sunspots series is also increasing
(Christiano, Chaos, & Harrison, 1999; George & Xenophon, 2007).
Since the observation sunspots are always mixed with a certain de-
gree of noise, it is necessary to denoise for the observation
sunspots.

A total of 3000 sunspots data are chose from January of 1756 to
December of 2005 for the analysis in the below simulation. We use
the Soft-threshold method, the basic single lifting method and the
adaptive method for noise reduction of sunspots data, the time dia-
grams of sunspots signal before and after noise reduction are listed
as Fig. 5. Fig. 6 shows the corresponding 3D phase space map of
sunspots signal before and after noise reduction.

By comparing the time diagrams and the 3D phase space maps
before and after noise reduction, we can find the adaptive method
is more effective for noise reduction of the sunspots data, it not
only maintains the system’s smooth, but also demonstrates the
inherent dynamic characteristics of system.

In order to further illustrate the effectiveness of this adaptive
method for chaotic signals with unknown model, autocorrelation
function, power spectrum, correlation dimension and the RP are

Fig. 6. The 3D phase space maps of sunspots. (a) The raw sunspots data. (b) Denoised data with the Soft-threshold wavelet method. (c) Denoised data with the basic single
lifting wavelet method. (d) Denoised data with the adaptive dual-lifting wavelet method.

Fig. 7. The autocorrelation coefficient maps of sunspots data. (a) Raw data and signal after noise reduction. (b) The removing noise after noise reduction.
employed for the analysis from time domain, frequency domain and the structure of the signals. The analysis processing is shown as follows.

Autocorrelation function analysis: Choose the delay time is 1, 2, 5, 10, 20 and 50 years respectively. Then, the autocorrelation function maps of the sunspots data and the removal noise after noise reduction are illustrated in Fig. 7(a) and (b).

By contrast, it can be found that the autocorrelation function value of the sunspots data is larger than that of the removal noise. From Fig. 7(a), we can see that the autocorrelation function value of data considered by the adaptive dual-lifting wavelet method is larger than other methods. Tables 4, 5 lists the specific autocorrelation function value of sunspots data and the removal noise after noise reduction. The results indicate that most of noise is removal, further reflect the effectiveness of the proposed method.

Table 4
The autocorrelation coefficient value of raw data and signal after noise reduction.

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>1</td>
<td>0.9321</td>
<td>0.9718</td>
<td>0.9683</td>
<td>0.9745</td>
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<td>2</td>
<td>0.8742</td>
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<td>0.6331</td>
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<td>20</td>
<td>0.5960</td>
<td>0.6213</td>
<td>0.6193</td>
<td>0.6233</td>
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<tr>
<td>50</td>
<td>0.5194</td>
<td>0.5414</td>
<td>0.5399</td>
<td>0.5432</td>
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Table 5
The autocorrelation coefficient value of the removing noise.

<table>
<thead>
<tr>
<th>Delay time (year)</th>
<th>Soft-threshold method (Han et al., 2006)</th>
<th>Basic single lifting method (Ercelebi, 2004)</th>
<th>Adaptive dual-lifting method</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0063</td>
<td>0.0035</td>
<td>0.0084</td>
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<tr>
<td>2</td>
<td>0.0024</td>
<td>0.0014</td>
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<tr>
<td>5</td>
<td>0.0006</td>
<td>0.0007</td>
<td>0.0009</td>
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<tr>
<td>10</td>
<td>0.0001</td>
<td>0.0002</td>
<td>0.0013</td>
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<tr>
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</tr>
<tr>
<td>50</td>
<td>0.0000</td>
<td>0.0001</td>
<td>0.0007</td>
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</tbody>
</table>

Fig. 8. The power spectrum maps of sunspots data. (a) Raw data and signal after noise reduction. (b) Gaussian white noise and the removing noise after noise reduction.
Power spectrum analysis: the power spectrum of sunspots data before and after noise reduction is shown in Fig. 8. Fig. 8(a) shows the power spectrum maps of the raw sunspots data and the signal after noise reduction. Fig. 8(b) lists the power spectrum maps of Gaussian noise and the removing noise after noise reduction. By contrast, it can be seen that the low-frequency characteristics of sunspots is basically unaffected after noise reduction while the corresponding high-frequency sequence is filtered. It is also found

Fig. 9. Correlation dimension curves of sunspots. (a) Raw sunspots data. (b) Denoised data with the Soft-threshold method. (c) Denoised data with the basic single lifting method. (d) Denoised data with the adaptive method.

Fig. 10. Recursive maps of sunspots data. (a) The raw sunspots data. (b) Denoised data with the Soft-threshold wavelet method. (c) Denoised data with the basic single lifting wavelet method. (d) Denoised data with the adaptive dual-lifting wavelet method.
that the removal series have the approximately properties with the Gaussian white noise, so it can reflect the effective of the noise reduction method.

Correlation dimension analysis: When the embedding dimension is 1, 3, 5, 7 and 9, Correlation dimension curves of sunspots data before and after Soft-threshold noise reduction method, basic single lifting noise reduction method and adaptive dual-lifting noise reduction method are listed in Fig. 9.

From Fig. 9, we can see that the stability speed of sunspots correlation dimension after three noise reduction methods are all faster than that of raw sunspots data, and the speed with adaptive dual-lifting wavelet method is the fastest. So it can reflect the effective performance of adaptive dual-lifting wavelet method.

Finally, we analyzed from the aspect of Recursive map. We select the sunspots data from January of 1973 to December of 2005 added up to 396 sunspots data in a total of 33 years. Due to the sunspot cycle is about 11 years, so that 396 data is equivalent to three sunspots cycles. When embedding dimension is $m = 5$, delay time is $\tau = 1$, and the space distance is $r = 5$ between two points, the recursive maps of the sunspots data are shown in Fig. 10. The abscissa and ordinate are the subscript with the space distance less than $r$. The recurrence quantification analysis indicators of sunspots are shown in Table 6.

From the recursive maps and its RQA indicators, we can see that the RP after noise reduction with the increasing aggregation, the enhance stability and the sensitivity decreased for the initial state is more regular than that before noise reduction. From the analysis of DET we can also see, the recursive cycles on both sides of diagonal is approximately 3, in line with the cycle from sunspot data. By contrast, we can see the advantages of the adaptive dual-lifting wavelet method proposed in this paper.

5. Conclusion

Combined with SSA and gradient decent algorithm, this paper introduces an adaptive dual-lifting chaotic noise reduction method. This method maintains the singularity and smooth of the signals by using singular spectrum analysis, and gradient decent algorithm adaptively chooses detail coefficients. At the same time, the signal after noise reduction is reconstructed by extracting the average of the approximate coefficients and the detail parts. This method further improves the signal–noise ratio of the system, and reduces the system's reconstruction error. Chaotic signals generated by Lorenz model and sunspots are respectively applied for simulation analysis, the numerical experiment results confirm the advantages of adaptive dual-lifting wavelet method.

References


Table 6

<table>
<thead>
<tr>
<th>Method</th>
<th>RR</th>
<th>DET</th>
<th>LAM</th>
<th>Lmax</th>
<th>RT</th>
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<tr>
<td>Raw sunspot data</td>
<td>0.0026</td>
<td>0.9851</td>
<td>0.0498</td>
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<tr>
<td>Soft-threshold wavelet method (Han et al., 2006)</td>
<td>0.0204</td>
<td>0.9944</td>
<td>0.9785</td>
<td>218</td>
<td>0.0021</td>
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<tr>
<td>Basic single lifting wavelet method (Ercelebi, 2004)</td>
<td>0.0194</td>
<td>0.9901</td>
<td>0.9730</td>
<td>210</td>
<td>0.0020</td>
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<tr>
<td>Adaptive dual-lifting wavelet method</td>
<td>0.0217</td>
<td>0.9977</td>
<td>0.9871</td>
<td>228</td>
<td>0.0021</td>
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