We investigate the time series of a torque applied to the ripping head in the process of a cutting concrete rock with sharp and blunt tools. By applying nonlinear embedding methods and the recurrence plots technique to the corresponding time series, we indicate the changes in nonlinear dynamics lying behind the ripping process, and we propose a test of the ripping machine efficiency and a way of monitoring the state of tools.

*Key words:* ripping head, nonlinear vibrations, recurrence plot

1. Introduction

The dynamics of a cutting process, in the case of metals, was described by Merchant several decades ago (Merchant, 1945). His proposals, involving such phenomena as friction, impacts, and chips breaking, are still inspiring in the current research of metal cutting with higher speeds (Litak, 2002; Sen *et al*., 2007; Warmiński *et al*., 2003). They were also extended to the problem of cutting of brittle materials including cutting of rocks. In this paper, we discuss experimental results obtained using the technology of multi-tool ripping heads (Eichbaum, 1980; Fowell, 1993; Gajewski and Jonak, 2006; Jonak and Gajewski, 2008). We will show time histories of the ripping head torque applied to a
standard rock and later examine them by means of embedding and recurrence theories.

The aim of the present paper is to show how recurrence plots (RP) and recurrence quantification analysis (RQA) can help in the assessment of states of the ripping head tools.

2. Considered problem

Multi-tool ripping torque time histories $T(i)$ versus the sampling index $i$ for sharp and blunt tools are presented in Figs. 1a and 1b, respectively. All other working conditions besides the states of tools were the same. Note, that the times series of sharp tools is characterized by slightly larger peak values. On the other hand, blunt time series shows a characteristic time interval $i \in [3700, 7100]$ of a fairly smaller amplitude, which can indicate on possible intermittency in the examined system.

Fig. 1. Multi-tool ripping torque time histories $T(i)$ versus the sampling index $i$ for sharp (a) and blunt (b) tools. The sampling time was 0.005 s while the rotational velocity was 42 rev/min

To go further with the standard methods of nonlinear time series analysis (Fig. 1), one should investigate the embedding properties. This approach is based on properties of a phase space which can be reconstructed by supplementing missing coordinates with time delayed ones (Takens, 1981). Thus, the
examined torque $T_i$ on the ripping head at the sampling time given by $i$ can be expressed as the following vector

$$T_i = [T_i, T_{i-\Delta i}, T_{i-2\Delta i}, \ldots, T_{i-(M-1)\Delta i}]$$ (2.1)

where $\Delta i$ is the time delay in sampling time units, while $M$ is the embedding dimension. Now we explain how to obtain proper values of $\Delta i$ and $M$ by examining the average mutual information (AMI) (Fraser and Swinney, 1986; Hegger et al., 1999; Kantz and Schreiber, 1997) and the false nearest neighbour fraction (FNNF) (Abarbanel, 1996; Hegger et al., 1999; Kennel and Brown, 1992; Sen et al., 2007). AMI is defined via conditional probabilities of the sequence events

$$\text{AMI}(\delta i) = -\sum_{kl} p_{kl}(\delta i) \ln \frac{p_{kl}(\delta i)}{p_k p_l}$$ (2.2)

where, for some partition (16 equal parts) of the ripping head, torque values are in an interval $T \in [T_{min}, T_{max}]$, $p_k$ is the probability of finding a time series value in the $k$-th interval, and $p_{kl}$ is the joint probability that an observation falls later into the $l$-th element, and the observation time is given by $\delta i$. The optimal time delay $\Delta i = \delta i$ is to be determined by the first AMI minimum for which the examined events are independent enough to define a new coordinate. Note, that AMI is positively defined and its smallest value (theoretically AMI = 0) can be reached when $p_{kl}$ can be factorized to individual probabilities $p_k$ and $p_l$ ($p_{kl} \approx p_k p_l$) for any $k$ and $l$ far from each other by $\delta i$.

On the other hand, to get FNNF one has to choose a point indicated by $T_i$ and calculate the distance to its nearest neighbour point $T_j$ in the $m$-dimensional space. For an Euclidean distance, which is typically used here, it is $|T_i - T_j|_m$.

By iterating both points along the time series, we compute the control parameter $Q_{i,m}$ defined as

$$Q_{i,m} = \frac{|T_i - T_j|_{m+1}}{|T_i - T_j|_m}$$ (2.3)

By comparing the above value to a chosen threshold $Q_c$, we calculate a fraction of cases for which $Q_{i,m}$ exceeds the threshold value $Q_c$. The FNNF can be then estimated from the following expression

$$\text{FNNF}(m) = \frac{1}{N} \sum_i \Theta(Q_{i,m} - Q_c)$$ (2.4)
where $N$ is the number of vector elements in the vector time series, $\Theta(x)$ is the Heaviside step function. This, so called, fraction analysis is repeated by choosing different values of the dimension $m$. The optimal value $M = m$ is defined when the fraction of false nearest neighbours tends to zero (note, in some cases, depending on $Q_c$ in respect to the standard square deviation of examined time series $T(i)$, some points are omitted and FNNF reaches the minimum for the optimal dimension $m = M$)

$$\lim_{m \to M} FNNF(m) \to 0$$ (2.5)

3. Results of numerical analysis

Using the above definitions for AMI and FNNF, we have estimated the embedding for the time series of sharp tools. In Figs. 2a,b one can easily find that the optimal values are $\Delta i = 3$ and $M = 7$ representing the time delay and the embedding dimension.

Fig. 2. AMI versus time delay $\delta i$ (a), FNNF versus embedding dimension $m$ (b) for sharp tools

These results would be a frame for the RP and RQA techniques. Here we assumed that these quantities were slowly evaluating during cutting of rock. To simplify the procedure of monitoring of the tools state, we fixed the embedding in the initial stage (for sharp tools). This can be additionally justified by the existence of important embedding invariants including the Shannon information entropy (Thiel et al., 2004).

The recurrence plot is usually defined by the following matrix form $R^{m,\epsilon}$ with the corresponding elements $R_{ij}^{m,\epsilon}$ (Casdagli, 1997; Eckmann et al., 1987;
having 0 and 1 elements to be translated into the recurrence plot as an empty place and a black dot, respectively. In this method (RP), patterns showing the diagonal and vertical or horizontal structure of lines are examined. Having obtained such a structure, one can easily classify the dynamics of a studied system (Marwan et al., 2007).

In Fig. 3a and 3b, we mapped the corresponding matrix elements $R_{ij}$ into recurrence plot graphs. One can see that the lines in Fig. 3a are more dense than those in Fig. 3b. As the space between lines corresponds directly to the characteristic period of torque variations, we can conclude that sharp tools variations possess a higher frequency component in their spectrum.

The other possibility is stronger appearance of random oscillations which usually fill the space between lines uniformly (see the most dense region in the middle of Fig. 3a). On the other hand, a finite length in some of the vertical lines visible in Figs. 3a and 3b invoke the intermittent character of torque changes in both cases (Litak et al., 2007; Marwan et al., 2007; Wendeker et al., 2004).

For quantitative analysis, we define the recurrence rate $RR$

$$RR = \frac{1}{N^2} \sum_{i,j=1}^N R_{ij}^{m,\epsilon} \quad \text{for} \quad |i - j| \geq w$$

(3.2)
which determines the black dots fraction in the RP graph. \( w \) denotes the Theiler window used to exclude identical and neighbour points (in our case \( w = 1 \)) from the above summation, Eq. (3.2).

Furthermore, the RQA can be used to identify vertical or diagonal lines through their lengths up to \( L_{\text{max}} \), \( V_{\text{max}} \) for diagonal and vertical lines, respectively. In its frame, the RQA enables one to perform probability \( p(l) \) or \( p(v) \) distribution analysis of lines according to their lengths \( l \) or \( v \) (for diagonal and vertical lines). In practice, they are calculated as

\[
p(y) = \frac{P^e(y)}{\sum_{y=y_{\text{min}}}^N P^e(y)}
\]

where \( y = l \) or \( v \), depending on the diagonal or vertical structures in a specific recurrence plot.

For various collections of diagonal and vertical lines with respect to their lengths distributions, Shannon information entropies \( L_{\text{entr}} \) and \( V_{\text{entr}} \) can be defined via (Marwan, 2003)

\[
L_{\text{entr}} = - \sum_{l=l_{\text{min}}}^N p(l) \ln p(l) \quad \quad V_{\text{entr}} = - \sum_{v=v_{\text{min}}}^N p(v) \ln p(v)
\]

Other properties of RP as the determinism \( DET \) and laminarity \( LAM \) as well as the trapping time \( TT \) are also based on the probabilities \( P^e(x) \)

\[
DET = \frac{\sum_{l=l_{\text{min}}}^N l P^e(l)}{\sum_{i,j=1}^N R_{i,j}^{m,e}} \quad \quad LAM = \frac{\sum_{v=v_{\text{min}}}^N v P^e(v)}{\sum_{v=1}^N v P^e(v)}
\]

\[
TT = \frac{\sum_{v=v_{\text{min}}}^N v P^e(v)}{\sum_{v=v_{\text{min}}}^N P^e(v)}
\]

In the above equations, \( l_{\text{min}} \) and \( v_{\text{min}} \) (\( l_{\text{min}} = v_{\text{min}} = 2 \) in our case) denote minimal lengths of the diagonal and vertical lines which should be chosen for a specific dynamical system. The determinism quantity \( DET \) is a measure of the predictability of the examined time series and gives the ratio of recurrent points formed in diagonals to all recurrent points. Note that in a periodic system all points would be included in the lines. On the other hand, the laminarity
LAM is a similar measure which corresponds to points formed in vertical lines. For small point-to-point changes (laminar), consecutive points form a vertical line. These measures tell about the dynamics behind sampling point changes, and are strictly connected to the point fraction spanning the diagonal (DET) and vertical (LAM) patterns, respectively. These diagonal and vertical line patterns are skeletons of deterministic features, while any singular point corresponds to randomness in the examined system. Note that for random numbers, the recurrence plot is filled uniformly without any pattern. Finally, the trapping time TT refers to the average length of vertical lines measuring the time scale (in terms of sampling intervals) of these small changes in the examined time history.

We performed calculations (using the numerical code provided in Marwan (2006)) of all the specified quantities for our time series (Fig. 1) and included them into Table 1.

**Table 1.** Summary of recurrence quantification analysis (RQA) for 'sharp' and 'blunt' tools, respectively; \( w = 1 \) (Theiler's window), \( M = 7 \), \( \Delta i = 3 \) and \( l_{\text{min}} = 2 \), \( v_{\text{min}} = 2 \) recurrence threshold \( \epsilon = 4 \)

<table>
<thead>
<tr>
<th>Type</th>
<th>RR</th>
<th>DET</th>
<th>LAM</th>
<th>( L_{\text{max}} )</th>
<th>( V_{\text{max}} )</th>
<th>( L_{\text{entr}} )</th>
<th>( V_{\text{entr}} )</th>
<th>TT</th>
</tr>
</thead>
<tbody>
<tr>
<td>'sharp'</td>
<td>0.016</td>
<td>0.341</td>
<td>0.323</td>
<td>72</td>
<td>26</td>
<td>0.811</td>
<td>0.710</td>
<td>2.36</td>
</tr>
<tr>
<td>'blunt'</td>
<td>0.025</td>
<td>0.819</td>
<td>0.849</td>
<td>318</td>
<td>43</td>
<td>1.918</td>
<td>1.564</td>
<td>3.30</td>
</tr>
</tbody>
</table>

One can see that for the same threshold value \( \epsilon = 4 \) (Eq. (3.1)), we have a larger \( RR \) for the 'blunt' data. This could be connected with higher correlations of these experimental data. The same effect of correlations is also visible in \( DET \) and \( LAM \) parameters which tell about the deterministic structure of the data. Note that this structure is much more transparent in the 'blunt' data. On the other hand, the entropies \( L_{\text{entr}} \) and \( V_{\text{entr}} \) show that the distribution of length of particular diagonal and vertical lines is complex and broader in the case of the 'blunt' data. Note also the differences in \( L_{\text{max}} \) and \( V_{\text{max}} \) (Table 1). These quantities are fairly larger for the 'blunt' data (\( L_{\text{max}} = 318 \) for 'blunt' data while \( L_{\text{max}} = 72 \) for 'sharp' data and \( V_{\text{max}} = 43 \) for 'blunt' data while \( V_{\text{max}} = 26 \) for 'sharp' data) because of the lack of correlations in the 'sharp' data.

4. Concluding remarks

In summary, we carried out recurrence analysis of the dynamical time series of the multi-tool ripping head. By examining the RP results, we found that oscillations in the case of sharp tools have two additional dynamical compo-
ments in the spectrum in the case of blunt tools (Fig. 3a and 3b). The first one is higher frequency, while the second important factor corresponds to random noise.

Comparing the RQA results for both examined cases, one can also notice fundamental differences in the deterministic structure. This structure, expressed by both $DET$ and $LAM$ being close to 1 (see in Table 1 $DET = 0.819$, $LAM = 0.849$), is fairly well defined in the case of blunt tools, while in the case of sharp tools, $DET$ and $LAM$ are much smaller (see in Table 1 $DET = 0.341$, $LAM = 0.323$). This could be an indicator of the effect of wear in tools and used in an automated monitoring procedure of condition of tools.

In a rock cutting process, more attention is paid to the expected size of loosening rock elements, which is strictly connected with energy consumption. For larger pieces, smaller energy is needed. It is obvious that wear in tools directly changing characteristics of friction and impacts could be responsible for different system responses. For instance, the wear effect on the sintered carbides considerably changes their rake angle of the cutting edges. In a consequence for blunt tools, the supplied power increases by about 50% in respect to sharp tools (Jonak and Gajewski, 2008). However, to tell if the recurrence plots provide a useful criterion, we need to perform more systematic experiments, especially with different rotational velocities of the ripping head and feed rates. These results will be reported in next publications.

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References


**Szacowanie ilościowe wpływu stępienia narzędzia na pracę głowicy urabiającej za pomocą wykresów rekurencyjnych**

**Streszczenie**

W artykule przedstawiono wyniki badań nad zastosowaniem nieliniowych metod zanurzenia oraz wykresów rekurencyjnych do oceny zmian dynamicznych zachodzących w procesie urabiania. Przebadano przebiegi czasowe momentu urabiania głowicą uzbrojoną w ostre oraz stępione narzędzia górnicze. Proponowane metody mogą okazać się przydatne w kontekście oceny wydajności oraz monitorowania stanu narzędzi głowicy urabiającej.

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