Intermittent and chaotic vibrations in a regenerative cutting process

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\textbf{A R T I C L E  I N F O}

\textbf{A B S T R A C T}

We have examined a cutting process with dry friction and contact loss effects. To model the second pass of the tool through the workpiece, we used a harmonic function representing the surface corrugations. The mathematical model consists of a single differential equation of second order. By numerically solving the governing differential equation, we obtained the time series for the cutting depth for different excitation frequencies. The time series are analyzed using the methods of recurrence plots, recurrence quantification analysis and wavelet analysis. The results show an intermittent character of the system dynamics with transition to chaotic motion.

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1. Introduction

In a cutting process, chaotic vibrations are known to develop under certain operating conditions. Chaos can be eliminated by suitable choices of the cutting parameters such as the cutting width and cutting velocity. The technology demand is to improve the final surface properties of the workpiece and to minimize the production time with higher cutting speeds. This can be achieved with a better understanding of the physical phenomena associated with a cutting process. A cutting process is inherently nonlinear and may exhibit a wide range of complex behaviour due to frictional effects [1–11], delay dynamics [12–17], or structural nonlinearities [18–21]. It may also involve loss of contact between the tool and the workpiece [17,22]. Various experiments have confirmed the appearance of chaos in a cutting process [23–28].

In this paper, we examine a simple one-degree-of freedom model of a cutting process. This model was used by Litak [22] and has also been recently examined by Wang et al. [17]. It incorporates the important mechanics of interaction between the tool and the workpiece involving dry friction and contact loss effects. Our presentation is organized as follows. In Section 2, we present the mathematical model and perform a numerical simulation of the governing differential equations to derive the time series of cutting depth. This is followed in Section 3 by an analysis of the cutting depth time series using the methods of recurrence plots and recurrence quantification analysis. In Section 4, we carry out a wavelet analysis of the cutting depth time series. A summary of the main results is given in the final section (Section 5) along with a few concluding remarks.

2. The model

The physical model of the cutting process examined here is shown in Fig. 1. Here the symbols $r$, $k$, and $\Omega_0$ denote, respectively, the radius, effective spring constant and angular velocity of the workpiece; $h_0$ is the uncut depth, $h$ is the actual
cutting depth, and $v_0$ is the relative velocity between the tool and the workpiece tangent to the workpiece surface. The horizontal displacement of the workpiece symmetry axis at time $t$ is denoted by $y(t)$.

After the first pass of the tool, the cutting depth can be expressed as

$$h(t) = h_0 - y(t) + y(t - T),$$

where $y(t - T)$ corresponds to the position of the workpiece during the previous pass, and $T = 2\pi / \Omega_0$ is the period of revolution of the workpiece. The cutting depth $h(t)$ can be determined from the model proposed by Stepan and Kalmar-Nagy [12]

$$\ddot{h} + 2n\dot{h} + p^2 h = -\frac{1}{m} \text{sgn}(v_0 + \dot{h}) F_y(h).$$

where $p$ is the frequency of free vibration, and $2n = c/m$ is a dimensionless damping coefficient. Note that $F_y$ is the thrust force, which is the horizontal component of the cutting force, and $m$ is the effective mass of the workpiece.

The above equation of motion can be written in terms of the current position, $y(t)$, of the tool as

$$\ddot{y}(t) + 2ny(t) + p^2 y(t) = \frac{1}{m} \text{sgn}(v_0 - \dot{y}(t))(F_y(h(t)) - F_y(h_0)).$$

The thrust force $F_y$ is based on dry friction between the tool and a chip, and is assumed to have a power law dependence on the actual cutting depth

$$F_y(h) = \Theta(h) K w (h)^{3/4},$$

Here $K$ is the coefficient of friction, $w$ is the chip width, and $\Theta$ is a Heaviside step function. To proceed with our analysis, we assume that in the first approximation, $y(t - T)$ can be described by a periodic function [17, 22]

$$y(t - T) = a \cos(\omega t - \phi) = a \cos \left( \frac{\omega x}{v_0} - \phi \right),$$

where $\omega$, $a$, and $\phi$ are the frequency, amplitude and phase of the surface shape modulation, respectively, and $x(t)$ denotes the relative distance passed by the tool on the cylindrical surface of the rotating workpiece.

Note that the frequency $\omega$ is proportional to the rotational velocity $\Omega_0$. Our model includes also the contact loss between the tool and the workpiece. Contact loss phenomenon may be particularly important at high cutting speeds. The impact between the tool and the workpiece can be modeled by the equation

$$\dot{y}(t^+) = -\beta \dot{y}(t^-),$$

where $t^+$ and $t^-$ denote time before and after impact, respectively, and $\beta < 1$ is the coefficient of restitution.

Note that the present model is a simplified one but is relevant for analyzing a chatter effect based on complex nonlinear behaviour including bifurcations from regular vibrations to chaotic motion. By solving Eqs. (1)–(5) numerically, we have calculated the time series of the cutting depth. The following parameters [22]:

- $K = 1.25 \times 10^6 \text{ N/m}^2$,
- $w = 3.0 \times 10^{-3}$ m,
- $h_0 = 0.3 \times 10^{-3}$ m,
- $p = 816 \text{ rad/s}$,
- $m = 17.2 \text{ kg}$,
- $a = 0.2 \times 10^2 \text{ m}$,
- $\phi = 2\pi$,
- $\beta = 0.75$.

Fig. 2 depicts the cutting edge time series for $\omega = 800$, 1200, 1600, and 2600 rad/s. For a small excitation frequency ($\omega = 800 \text{ rad/s}$, Fig. 2a), relative vibrations are small and consequently appear with a small amplitude. For larger frequencies, Fig. 2b and c illustrate the contact loss phenomenon which can be of a nonperiodic ($\omega = 1200 \text{ rad/s}$, Fig. 2b) or periodic ($\omega = 1600 \text{ rad/s}$, Fig. 2c) type [17, 22]. Finally, for a large enough frequency ($\omega = 2600 \text{ rad/s}$, Fig. 2d), the system responds with a periodic motion of a smaller amplitude. In this case, full continuous contact between the tool and the workpiece is established again.
To analyze the time series of cutting depth depicted in Fig. 2, we will use the methods of recurrence plots (RPs) and recurrence quantification analysis (RQA) [29–33]. A recurrence plot is a graphical device for visualizing the time correlation structure of a process. The idea of a recurrence plot was introduced by Eckmann et al. [29] and has been used for understanding system dynamics in a wide variety of applications [33]. For a quantitative understanding of the various features of a recurrence plot, the method of recurrence quantification analysis (RQA) has been developed. The first step in the RP/RQA procedure is to embed the time series into a high dimensional phase space using an appropriate time delay [34–37]. For the present purpose, we construct from the cutting depth data, a time-delayed vector
\[ \mathbf{h}_i = \left[ h_i, h_{i-D}, h_{i-2D}, \ldots, h_{i-(M-1)D} \right], \]
where \( D \) is the time delay in units of the sampling time, and \( M \) is the embedding dimension. The values of \( D \) and \( M \) can be estimated from the average mutual information [34–36], and the false nearest neighbor fraction [36,38], respectively. The optimal time delay corresponds to the minimum value of average mutual information, and the optimal value of \( M \) is obtained when the fraction of false nearest neighbors tends to zero. For the sake of brevity, the details of their calculations are not given here. Table 1 lists the values of \( D \) and \( M \) estimated for the cutting depth time series at various speeds.

Using the time-delayed vector obtained in the manner described above, a recurrence plot is constructed from the matrix \( \mathbf{R} \) with its element \( R_{ij} \) given by [29–33]
\[ R_{ij} = \Theta(\epsilon - |h_i - h_j|), \]
where \( \epsilon \) is a threshold value, and \( \Theta \) is a Heaviside step function. In a recurrence plot, the elements 0 and 1 of the matrix \( \mathbf{R} \) are represented by an empty space and a black dot, respectively. By examining the structure of a recurrence plot, the dynamics of the system under consideration can be easily visualized [33].

Fig. 3a–d illustrates the recurrence plots for the time series of the cutting depth depicted in Fig. 2. Consider first Fig. 3a and d. Each of these figures exhibits a regular structure composed of diagonal lines indicating a simple periodic motion. The distance between the diagonal lines represents the characteristic period. Fig. 3c also consists of diagonal lines along with regularly spaced points distributed parallel to the diagonal. The distance between a pair of diagonal segments in this figure

| Table 1 |
|-----------------|--------|--------|--------|--------|
| Embedding parameters | \( \omega \) = 800 | 1200 | 1600 | 2600 |
| \( D \) | 5 | 3 | 2 | 1 |
| \( M \) | 2 | 4 | 3 | 2 |
represents a period-2 motion. On the other hand, Fig. 3b has a structure with lines of different lengths. Note that the distances between lines in the horizontal and vertical directions can be interpreted as characteristic periods. In Fig. 3b, these distances are changing with the time index $i$. The stripes along $i = 90, 270, \text{ and } 380$ are associated with some phase shifts in motion.

Next we perform a recurrence quantification analysis (RQA) of the cutting depth time series. In RQA, a few measures are defined, based on the structural properties of an RP. In our treatment, we shall use the following RQA measures: recurrence rate (RR), determinism (DET), laminarity (LAM), maxline ($L_{\text{max}}$) and trapping time (TT). The recurrence rate has the definition

$$RR = \frac{1}{N^2} \sum_{i,j=1}^{N} R_{ij}, \text{ for } |i-j| \geq z.$$  \hfill (9)

This quantity determines the fraction of black dots in an RP. Here $z$ denotes the Theiler window which is used to exclude identical and neighboring points (in our case $z = 1$) from the above summation (Eq. (9)).

Maxline ($L_{\text{max}}$) is the length of the longest diagonal line in an RP. The other three measures, DET, LAM, and TT are defined in terms of the distributions, $P(l)$ or $P(v)$, of the lengths of the diagonal or vertical lines.

$$\text{DET} = \frac{\sum_{l-l_{\text{min}}}^{N} l P(l)}{\sum_{l=1}^{N} l P(l)},$$  \hfill (10)

$$\text{LAM} = \frac{\sum_{v=v_{\text{min}}}^{N} v P(v)}{\sum_{v=1}^{N} v^{1} v P(v)},$$  \hfill (11)

$$\text{TT} = \frac{\sum_{v=v_{\text{min}}}^{N} v P(v)}{\sum_{v=v_{\text{min}}}^{N} P(v)}.$$  \hfill (12)
In these equations, $l_{\text{min}}$ and $v_{\text{min}}$, denote, respectively, the minimal lengths of diagonal and vertical lines; we have taken $l_{\text{min}} = v_{\text{min}} = 2$.

The recurrence rate (RR) is a measure of the density of recurrence points in an RP; it is simply the total number of black dots in an RP and corresponds to the probability that a specific state will occur. DET is given by the ratio of recurrence points along the diagonal lines to all recurrence points, and is a measure of predictability of the time series. The number of recurrence points which form vertical lines in an RP is estimated by LAM. This parameter indicates the extent of laminar phases or intermittency in the time series. Finally, TT quantifies the average length of the vertical lines and is thus related to LAM; in other words, TT describes how long the system remains in the laminar phase. Note that TT reaches its maximum at $\omega = 1200 \text{ rad/s}$ indicating that the system tends to stay in one of possible states more often than at any other investigated frequencies. This type of behaviour can be explained by transition to chaotic vibrations through an intermittency scenario. In this scenario the key role is played by a contact loss. To clarify this point we have examined the two-dimensional reconstructed phase portraits (Fig. 4). Fig. 4a shows, once again, the differences between regular motions for $\omega = 800, 1600,$ and $2600 \text{ rad/s}$ and chaotic motion for $\omega = 1200$. In the last case of a chaotic motion we extracted parts of trajectories for $\omega = 2.000; 2.049$ and $2.123; 2.149$ (compare with Fig. 2). One can clearly see that the contact loss can lead to a characteristic phase shift in system vibration. Apparently phase shifts are appearing irregularly in time series (Fig. 2) leading to intermittent motion between such events. By comparing Fig. 4b to Fig. 2b ($\omega = 1200 \text{ rad/s}$), it is clear that phase shifts are accompanied by an escape of the tool from the cutting material by a smaller distance.

4. Wavelet analysis

In this section we perform a wavelet analysis of the cutting depth time series using a continuous wavelet transform (CWT). Wavelets have been used for time series analysis in diverse applications including dynamic analysis of structural systems [9,40–48]. First, we briefly describe the procedure of CWT. A wavelet is a small wave with zero mean and finite energy. A continuous wavelet transform $W(s, \tau)$ of a function $h(t)$, with respect to a mother wavelet $\psi(t)$, is defined as the convolution of the function with a scaled and translated version of the mother wavelet.

$$W(s, \tau) = \int_{-\infty}^{\infty} h(t) \psi_{{s,\tau}}(t) \, dt,$$

where

$$\psi_{{s,\tau}}(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t-\tau}{s}\right).$$

The symbols $s$ and $\tau$ represent the scale and translation parameters, respectively, and the asterisk in Eq. (14) denotes a complex conjugate. The scale parameter controls the dilatation whereas the translation parameter indicates the location of the wavelet in time.

The wavelet power spectrum $P(s, \tau)$ of the signal $h(t)$ is defined as the squared modulus of the CWT.

$$P(s, \tau) = |W(s, \tau)|^2.$$  

In the case of a signal $h(t)$ described by a time series

$$h(t) \rightarrow h_i, \quad i = 1, 2, \ldots, N,$$

a discretized version of Eq. (13) should be used. Following Torrence and Compo [46], we may write

<table>
<thead>
<tr>
<th>$\omega$ [rad/s]</th>
<th>DET</th>
<th>LAM</th>
<th>$l_{\text{max}}$</th>
<th>TT</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>0.9923</td>
<td>0.1253</td>
<td>379</td>
<td>2.000</td>
</tr>
<tr>
<td>1200</td>
<td>0.9042</td>
<td>0.6970</td>
<td>174</td>
<td>2.047</td>
</tr>
<tr>
<td>1600</td>
<td>0.9049</td>
<td>0.04343</td>
<td>380</td>
<td>2.000</td>
</tr>
<tr>
<td>2600</td>
<td>0.5754</td>
<td>0.0000</td>
<td>370</td>
<td>0.000</td>
</tr>
</tbody>
</table>
The corresponding power spectrum is given by

\[ P_n(s) = |W_n(s)|^2. \]  

(18)

In our analysis we used a Morlet wavelet as the mother wavelet. A Morlet wavelet consists of a plane wave modulated by a Gaussian function and is described by

\[ \psi(\eta) = \pi^{-1/4}e^{i\omega_0\eta}e^{-\eta^2/2}, \]  

(19)

Fig. 4. (a) Two-dimensional reconstructed phase portraits for the cutting process with \( \omega = 800, 1200, 1600, \) and 2600 rad/s. (b) Trajectories extracted for the case with \( \omega = 800 \) rad/s (Fig. 4a): ‘1’ for the first 69 consecutive points \( t \in [2.0000, 2.0345]; \) ‘2’ next 39 consecutive points \( t \in [2.0345, 2.0540]. \) The horizontal arrow shows the difference between the examined parts of the trajectory.

Fig. 5. Wavelet power spectra of the cutting depth time series for \( \omega = 800 \) rad/s (a), 1200 rad/s (b), 1600 rad/s (c), and 2600 rad/s (d), respectively.
where $\omega_0$ is the center frequency, also referred to as the order of the wavelet. Morlet wavelets have been used in a wide variety of applications for feature extraction in time series data. We have used a Morlet wavelet with $\omega_0 = 6$ which offers good balance between time and frequency localizations.

The wavelet power spectra of the cutting depth time series shown in Fig. 2 are depicted in Fig. 5. In this figure, the panels (a–d) correspond to frequencies $\omega = 800$, 1200, 1600, and 2600 rad/s, respectively. It is apparent from Fig. 5a and d that at speeds $\omega = 800$ and 2600 rad/s, the cutting depth undergoes a regular simple periodic motion. These results are consistent with those shown in the recurrence plots of Fig. 3a and d. When $\omega = 1600$ rad/s, Fig. 5c indicates a period-2 motion, analogous to that depicted in the recurrence plot of Fig. 3c. Finally, for $\omega = 1200$ rad/s, we observe from Fig. 5b that the motion is chaotic. The chaotic behavior can be analyzed more precisely by calculating the instantaneous frequencies based on extraction of ridges of the wavelet transform [49,50]. The ridges are the local maxima with respect to the scale $s$.

5. Summary and conclusions

We have modeled the cutting process by a single-degree-of-freedom mass-spring system with dry friction. By increasing the cutting speed, we observed transition from regular vibrations to chaotic behavior and back to regular motion.

The different types of motion have been described by RP, and RQA, and confirmed by wavelet analysis. Note that the system is nonsmooth, with a discontinuous vector field (Eqs. (2) and (3)), and this is the reason that some methods of chaos identification based on derivatives of vector fields [51–53] may fail.

The nature of transition to chaotic motion has been examined by embedding methods including a phase portrait reconstruction. The intermittent character is manifested by a particular phase shift associated with the lack of higher-than-usual escape of the tool from the workpiece. This is a consequence of impacts between the returning tool and the workpiece implying that an intermittent grazing mechanism [54,55] could play an important role in the transition to chaos in the cutting process. To be more specific, one needs to perform a more detailed investigation including the statistics of the intervals between the phase changes. These results will be reported in a future publication.

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References