Recurrence Plot Analysis of GPS Ionospheric Delay
Time Series in Extreme Ionospheric Conditions

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Abstract: With provision of Positioning, Navigation, and Timing (PNT) services, satellite navigation systems have become a pillar of modern society. These services lay the foundations of a growing number of technological and socio-economic systems and constitute a key enabling technology for transportation systems, services and components. Mitigation of disruptions and degradation of Global Navigation Satellite System (GNSS) positioning performance and operation quality become critical issues for satellite navigation advancement and adoption. Ionospheric conditions are the single prime natural cause of GNSS positioning performance disruptions and degradations. Complex, non-linear and random nature of the ionospheric effects on GNSS positioning performance adds to the challenges of the suitable mitigation processes development. Here a contribution to the understanding of the ionospheric effects on GNSS positioning performance is provided through a study of Total Electron Content (TEC) and GNSS pseudorange measurement errors time series in the selected cases of characteristic ionospheric conditions, using the Recurrence Plot Analysis (RPA), a common procedure for studying general time series. Based on experimental GPS observations, this study found good alignment of TEC and TEC-rate time series with several characteristic schemes of dynamical behaviour, thus allowing for classification of ionospheric conditions and related TEC behaviour based on their dynamical properties. Further to this, the study identified several RPA predictors as precursors of developing ionospheric storm and the consequent disruptions and degradation of GNSS positioning performance. The study stressed the importance of TEC time series assessment, and initiates research challenges for consideration of TEC time series RPA predictors for mitigation, correction, and forecasting model development of GNSS pseudorange measurements, and GNSS position estimation errors, thus contributing to GNSS resiliency development against space weather and ionospheric effects.

Keywords: GPS ionospheric delay; Recurrence Plot Analysis (RPA); Total Electron Content (TEC); GPS positioning performance; non-linear time series

Author contributions: K.L. conceived and designed the study, implemented the R-based code and processed the data. R.F. contributed to the problem formulation and assessed the research state-of-the-art. Both authors assessed, inferred from and discussed the results, and drew the conclusion.
1 Introduction

The Global Navigation Satellite System (GNSS) positioning performance varies in relation to positioning environment conditions (space weather, geomagnetic field, ionosphere, local multipath environment), operation of artificial sources of disruptions and degradations that affect GNSS signals and spectrum (GNSS spoofing, jamming, and meaconing), and the selection of signal processing, error correction, and position estimation methods [1,2,3,4,5].

Ionospheric condition is the single major cause of GNSS pseudorange measurement errors and, consequently, the single major contributor to the GNSS position estimation error budget, affecting resilient operation of GNSS [2,4,6]. Ionospheric effects [1,7] introduce ionospheric delay to GNSS signals, with a complex and largely unpredictable nature, rendering mitigation a challenging task [2,6,8,9]. It may be shown that the Total Electron Content (TEC), a predictor of ionospheric conditions due to its relation with the actual vertical ionospheric profile, is directly proportional to the GNSS pseudorange measurement error $\delta \rho$ of a satellite signal transmitted with a radio carrier frequency $f$, as presented with Eq (1) [2,6].

$$\delta \rho = \frac{40.3}{f^2} \cdot \text{TEC}$$  \hspace{1cm} (1)

Dual-frequency GNSS receiver may mitigate the ionospheric effect through ability to determine the amount of the GNSS signal ionospheric delay through utilisation of the pseudoranges measured at two different radio frequencies $\rho_1$ and $\rho_2$ processed with Eq (2) [2].

$$\Delta \rho = 40.3 \cdot \text{TEC} \cdot \frac{1}{\frac{1}{f_2^2} - \frac{1}{f_1^2}}$$  \hspace{1cm} (2)

with:

$$\text{TEC} = \int_{\text{Earth's surface}}^{\text{upper bound of ionosphere}} N(h) \cdot dh$$  \hspace{1cm} (3)

where:

- $\text{TEC}$ ... denotes Total Electron Content, an ionospheric conditions predictor,
- $N(h)$ ... denotes vertical ionospheric profile,
- $h$ ... denotes height above the mean Earth’s sea level,
- $f_1, f_2$ ... denotes two different GNSS carrier frequencies used for simultaneous pseudorange measurements (as an example, $f_1 = 1575.42$ MHz, $f_2 = 1227.6$ MHz for GPS).
Eq. (2) may be reversed to serve as the TEC derivation model from experimental observations, as given in Eq. (4) [2].

\[
\text{TEC} = \frac{\Delta \rho}{40.3} \cdot \frac{f_1^2 - f_2^2}{f_1^2 f_2^2}
\]  

(4)

The error sources affect the accuracy of the GNSS pseudorange measurements through introduction of GNSS pseudorange measurement error \( \delta \rho \), which maps onto GNSS position estimation error \( \Delta \mathbf{x} \) as expressed with Eq. (5), utilising the GNSS Geometric matrix \( \mathbf{G} \), and, alternatively using advanced weighting method, matrix of weights \( \mathbf{W} \) to account for different error contributions as expressed with Eq. (6) [2,6,8,10].

\[
\Delta \mathbf{x} = (\mathbf{G}^T \mathbf{G})^{-1} \mathbf{G}^T \cdot \delta \rho
\]  

(5)

\[
\Delta \mathbf{x} = (\mathbf{G}^T \mathbf{W} \mathbf{G})^{-1} \mathbf{G}^T \mathbf{W} \cdot \delta \rho
\]  

(6)

GNSS Geometric matrix \( \mathbf{G} \) is given as expressed in Eq. (7) [2].

\[
\mathbf{G} = \begin{bmatrix}
x_{u} - x_{s1} & y_{u} - y_{s1} & z_{u} - z_{s1} \\
x_{u} - x_{s2} & y_{u} - y_{s2} & z_{u} - z_{s2} \\
x_{u} - x_{s3} & y_{u} - y_{s3} & z_{u} - z_{s3} \\
x_{u} - x_{s4} & y_{u} - y_{s4} & z_{u} - z_{s4}
\end{bmatrix}
\]  

(7)

where:

\[
R_i = \sqrt{(x_u - x_{si})^2 + (y_u - y_{si})^2 + (z_u - z_{si})^2}, \ i = 1, \ldots, 4
\]  

(8)

c … denotes velocity of light (electromagnetic wave) in vacuum,

\((x_u, y_u, z_u)\) … denotes the position of a GNSS user equipment,

\((x_{si}, y_{si}, z_{si})\) … denotes the position of the i-th satellite.

Characterisation of GNSS pseudorange measurement error time series is essential for understanding and modelling the GNSS positioning performance. The GPS ionospheric effects have been recognised as the single most influential source of disruptions and degradations of the GPS positioning performance [4]. Developments so far were mainly focused on identification of the influencing predictors of space weather, geomagnetic, and ionospheric conditions as the potential sources of the GNSS positioning performance disruptions and degradations [2,6].

The accepted additive model of GPS positioning errors [11] assumes the existence of a bias, linear, and random (stochastic) component of the total GPS positioning error, as shown in Eq. (9).
\[ \rho = \rho_R + \delta \rho = \rho_R + \delta \rho_{bias} + \delta \rho_{syst} + \delta \rho_{rand} \]  

(9)

where:

\( \rho \) ... denotes the pseudorange measurement,
\( \rho_R \) ... denotes true distance between the satellite and the user aerials,
\( \delta \rho_{bias} \) ... denotes bias component of pseudorange measurement error,
\( \delta \rho_{syst} \) ... denotes systemic component of pseudorange measurement error,
\( \delta \rho_{rand} \) ... denotes random component of pseudorange measurement error.

A vector of the instantaneous GPS pseudorange measurements may be defined to contain all the individual pseudorange measurements from visible satellites that are used in GPS position estimation process, as presented with Eq (10).

\[ \hat{\rho} = \begin{bmatrix} \rho_1 \\ \vdots \\ \rho_n \end{bmatrix}, n \geq 4 \]  

(10)

Here the hypothesis of the existence and the nature of a non-linear deterministic component of the total GPS pseudorange measurement error is addressed with the non-linear analysis of experimental GPS ionospheric delay observations taken during selected cases of space weather and ionospheric conditions. Characterisation of the GNSS pseudorange measurement error time series in relation to ionospheric conditions [2] offers prospects for advancement of GNSS user equipment design, as well as contribution to resilient GNSS development. The research study presented here aimed at: (i) methodology development for studying the non-linear behaviour of TEC, as the major contributor to GNSS pseudorange measurement error and, hence, to GNSS positioning error, (ii) characterisation of non-linear nature of the GNSS ionospheric delay time series in selected scenarios of extreme and ordinary ionospheric conditions, and (iii) inference from the Recurrence Plot Analysis (RPA) that may advance understanding, description, and modelling the ionospheric effects on GNSS positioning performance.

2 Method

This section provides data description and methodology outline, both used in the research.

2.1 Original data description

GNSS pseudorange observations used in the study were taken from the International GNSS Service [12], an open-access vault of voluntary provided high-quality GNSS data.
serving the purpose of data source for research. Access was performed through NASA repository [13]. The Matera, Italy IGS reference station (Figure 1) was selected as a mid-latitude GPS observations source, situated in an area with the intense air and maritime traffic. The Matera, Italy reference station has a long-standing record of high-quality GNSS data collection.

Data sets of GPS dual-frequency raw (i.e. uncorrected) pseudorange observations (RINEX o files) and the received GPS navigation messages (RINEX n files) were used for selected days of characteristic classes of ionospheric conditions. Selection of characteristic ionospheric conditions scenarios was performed based on examination of ionospheric conditions data provided by NOAA Space Weather Prediction Center, with the structured access arranged by the UK Government Office for Science [14]. Table 1 outlines essentials of the selected cases of characteristic classes of ionospheric conditions used in the research, while Table 2 summarises space weather, geomagnetic, and ionospheric conditions indices in the observed period for the selected cases from Table 1 as follows:

- $Dst_{min}$ denotes Disturbance Storm Time index minimum time value,
- $Dst_{max}$ denotes Disturbance Storm Time index maximum time value,
- $Kp_{min}$ denotes Planetary geomagnetic $K$ index minimum time value,
- $Kp_{max}$ denotes Planetary geomagnetic $K$ index maximum time value,
- $SSN$ denotes Sunspot number,
- $Solar\ Flux$ denotes Solar radio emission at 10.7 cm wavelength.
Dst and Kp indices are collected on 1 min and 3 hours bases respectively. SSN and Solar Flux are measured on daily bases. Since Dst and Kp indices extend the rate of change and intensity of geomagnetic and ionospheric disturbances, it is of importance to assess the ranges of their values instead of their variance throughout the day.

<table>
<thead>
<tr>
<th>Date</th>
<th>Day of Year (DoY)</th>
<th>Space weather conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>29 October, 2003</td>
<td>302</td>
<td>A massive space weather storm</td>
</tr>
<tr>
<td>17 March, 2015</td>
<td>076</td>
<td>Fast developing ionospheric storm</td>
</tr>
<tr>
<td>8 September, 2017</td>
<td>251</td>
<td>Fast developing geomagnetic storm</td>
</tr>
<tr>
<td>10 May, 2018</td>
<td>130</td>
<td>Quiet space weather conditions</td>
</tr>
<tr>
<td>28 October, 2003</td>
<td>301</td>
<td>Quiet space weather conditions on Earth immediately before the onset of a massive ionospheric storm the next day</td>
</tr>
</tbody>
</table>

Table 1 Selected cases of characteristic classes of ionospheric conditions

**2.2 Derivation of TEC/GPS pseudorange measurement errors from GPS observations**

TEC estimations were derived with 30s sampling rate from dual-frequency GPS pseudoranges collected at Matera, Italy IGS reference station using the model (4). GPS pseudorange observations are generally compromised with the satellite and receiver biases [2]. Data cleaning from satellite bias was conducted through utilisation of Differential Code Bias (DCB) [15]. GPS pseudorange observations were not corrected for receiver bias.

<table>
<thead>
<tr>
<th></th>
<th>302/2013</th>
<th>076/2015</th>
<th>251/2017</th>
<th>130/2018</th>
<th>301/2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dst min</td>
<td>-350</td>
<td>-222</td>
<td>-124</td>
<td>-30</td>
<td>-32</td>
</tr>
<tr>
<td>Dst max</td>
<td>-10</td>
<td>+56</td>
<td>-44</td>
<td>-17</td>
<td>-1</td>
</tr>
<tr>
<td>Kp min</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Kp max</td>
<td>9</td>
<td>8</td>
<td>4</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>SSN</td>
<td>330</td>
<td>60</td>
<td>23</td>
<td>11</td>
<td>230</td>
</tr>
<tr>
<td>Solar Flux</td>
<td>279</td>
<td>138</td>
<td>124</td>
<td>070</td>
<td>274</td>
</tr>
</tbody>
</table>

Table 2 Ionospheric conditions predictors for the Table 1 cases
An individual GNSS satellite TEC estimate obtained using (4) from raw GNSS pseudoranges contains dependency on the elevation angle of reception, thus named as the Slant TEC (STEC). STEC estimates were transformed to more objective and mutually comparable Vertical TEC (VTEC) estimates using the transform (11). VTEC estimates were averaged across a set of visible satellites for every time instance, thus yielding the TEC estimate for the given time instant [2]. A set of TEC estimates defined TEC time series that served as the input variable for the RPA method, outlined in the following Section.

\[
VTEC_i = \sqrt{1 - \left(\frac{R_E}{R_E + z} \cos \Theta\right)^2} \cdot STEC_i
\]  

where:

- \(VTEC_i\) … denotes individual satellite Vertical TEC estimate,
- \(R_E\) … denotes Earth’s radius (for GPS \(R_E = 6378.137\) km),
- \(z\) … denotes height of the concentrated ionospheric layer (for GPS = 350 km),
- \(\Theta\) … denotes elevation angle of the satellite signal path [°],
- \(STEC_i\) … denotes individual satellite Slant TEC estimate.

### 2.3 Recurrence Plot Analysis (RPA) methodology for non-linearity detection and characterisation in GPS pseudorange measurement errors

The approach taken in this study involved the utilisation of the RPA method, which allows for non-linearity detection and characterisation in time series under consideration [9,16]. Recurrence plot is an advanced method for the analysis of non-linear time series. It examines the phase space trajectory of a dynamical system described by time series of its variables and detects its unfolding back (recurs) to the state space region already visited in the past [9,16]. The phase space of a dynamical system with \(n\) variables (predictors) is constructed as an \(n\)-dimensional space. A set of predictor values taken at the same time instant \(t_i\) forms a space state vector. Phase space trajectory is a set of space state vectors [9]. Complex systems may extend phase space trajectories that recur (return) to the state space regions previously visited (Figure 2). Characterisation of different kinds of recurrences may reveal additional information on the dynamical nature of time series and contribute to its proper description and modelling.
The roots of RPA may be found in the work of Henri Poincare, a French mathematician [18]. A method for visualisation of the recurrence of states in a phase space was introduced by Eckmann et al. [19], thus establishing the RPA framework.

Table 3 presents the main classes of recurrence plots. Plot examples were created within this study using simulations of related processes in the R open-source environment for statistical computing.

A qualitative analysis of geometrical properties of the recurrence plot and the patterns within reveal the nature of the underlying time series [20], as listed below.

(i) Homogeneous recurrence plots are indicative of stationary and autonomous systems.
(ii) Recurrence plots with periodic diagonal lines are typical for oscillating systems. The distance between the diagonal lines reveals the oscillation period.
(iii) Fading patterns indicate non-stationary behaviour of the underlying time series. Systems with a drift tend to fade towards the upper-left and lower-right corners.
(iv) White disrupted areas are indicative of abrupt transitions in the system dynamics. Clear borderlines within a recurrence plot represent chaotic transitions.
(v) Plots containing single recurrence points represent systems with fluctuation, containing states that are rare.
(vi) The states that evolve similarly at different times generate diagonal lines. These are indicative of deterministic processes.
(vii) Vertical lines in a recurrence plot are present when there are states that remain the same for certain time or that change very slowly.
<table>
<thead>
<tr>
<th>Case</th>
<th>Description</th>
<th>RR</th>
<th>DET</th>
<th>NRLINE</th>
<th>L</th>
<th>ENTR</th>
<th>rENTR</th>
<th>LAM</th>
<th>TT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>Oscillator (periodic oscillatory behaviour), sine function, N = 1000</td>
<td>1.286</td>
<td>27.04</td>
<td>687</td>
<td>5.075</td>
<td>0.6207</td>
<td>0.3464</td>
<td>0.31</td>
<td>2</td>
</tr>
<tr>
<td>Case 2</td>
<td>White noise sequence</td>
<td>0.759</td>
<td>14.47</td>
<td>51</td>
<td>21.56</td>
<td>0.0965</td>
<td>0.1392</td>
<td>0.78</td>
<td>2</td>
</tr>
<tr>
<td>Case 3</td>
<td>Auto-regressive time series with negative correlation, AR(1), corr = -0.99, sd = 0.5, N = 1000</td>
<td>0.724</td>
<td>21.21</td>
<td>265</td>
<td>5.80</td>
<td>0.1770</td>
<td>0.1277</td>
<td>0.69</td>
<td>2</td>
</tr>
<tr>
<td>Case 4</td>
<td>Brownian motion, data generated artificially using a dedicated R script, N = 1000</td>
<td>0.914</td>
<td>25.12</td>
<td>621</td>
<td>3.71</td>
<td>0.3437</td>
<td>0.2479</td>
<td>23.67</td>
<td>2.14</td>
</tr>
<tr>
<td>Case 5</td>
<td>Logistic map defined as Xk+1=r Xk (1-Xk) at the onset of chaos (r = 3.56995)</td>
<td>5.756</td>
<td>80.96</td>
<td>677</td>
<td>11.07</td>
<td>1.7660</td>
<td>0.6233</td>
<td>0.00</td>
<td>NaN</td>
</tr>
</tbody>
</table>

Table 3 Five characteristic classes of recurrence plots and their RPA predictors
A more objective analysis can be obtained with a quantitative approach. The RPA algorithmic procedure for a recurrence plot formation was defined by Witt [21], with a set of algorithms for quantified assessment of a recurrence plot performance, thus allowing for understanding and interpretation of the underlying non-dynamical process. The same reference outlined a comprehensive recurrence plot interpretation method. Those are lately modernised in a sense of application in modern computing environments [13]. The recurrence plot predictors defined in [13] and the fundamental establishment of RPA as given in [21] and [22] were used in this study, applied on time series of GPS ionospheric delays observed during rapid development of an ionospheric storm of a significant scale.

The RPA predictors (indicators) used for the quantitative analysis are summarised in Table 4 and their formal definitions are given afterwards.

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>RR</td>
<td>recurrence rate: the percentage of recurrent points falling within the specified radius (range between 0 and 100)</td>
</tr>
<tr>
<td>DET</td>
<td>determinism: proportion of recurrent points forming diagonal line structures.</td>
</tr>
<tr>
<td>NRLINE</td>
<td>The total number of lines in the recurrent plot</td>
</tr>
<tr>
<td>L</td>
<td>The average length of line structures</td>
</tr>
<tr>
<td>ENTR</td>
<td>Shannon entropy: Shannon information entropy of diagonal line lengths longer than the minimum length</td>
</tr>
<tr>
<td>εENTR</td>
<td>Entropy measure normalized by the number of lines observed in the plot.</td>
</tr>
<tr>
<td>LAM</td>
<td>laminarity: proportion of recurrent points forming vertical line structures</td>
</tr>
<tr>
<td>TT</td>
<td>trapping time: the average length of vertical line structures</td>
</tr>
</tbody>
</table>

The percentage of recurring points encompassed with the radius specified \( \varepsilon \) (the RR predictor) is given with Eq (12).

\[
RR = 100 \cdot \frac{\text{number of points}}{\varepsilon(\varepsilon - 1)^{1/2}} \quad (12)
\]

The proportion of recurring points that form diagonal line structures (the DET predictor) is defined with Eq (13).

\[
DET = 100 \cdot \frac{\text{number of points with diagonal lines}}{\text{number of recurrent points}} \quad (13)
\]

The total number of lines in the observed recurrent plot is counted and denoted the NRLINE predictor. The average length of line structures is determined throughout the recurrence plot and denoted the L predictor.
The Shannon entropy of the number $P_{\text{bin}}$ of diagonal line lengths exceeding the minimum length value, denoted as ENTR, is determined using Eq (14).

$$\text{ENTR} = -\Sigma (P_{\text{bin}}) \log_2 (P_{\text{bin}})$$

(14)

The proportion of recurrent points that establish vertical line structures, denoted as LAM, is determined using Eq (15).

$$\text{LAM} = 100 \cdot \frac{\text{number of points with vertical lines}}{\text{number of recurrent points}}$$

(15)

The average length of the observed vertical line structures, denoted as TT, is determined using constraint given in Eq (16).

$$\text{TT} = \text{average length of vertical lines} \geq \text{parameter line}$$

(16)

Derived TEC estimates were processed with a dedicated software developed in the R open-source environment for statistical computing, using additional R libraries nonlinearTSeries and tseriesChaos. Following the initial examination, processing parameters were selected, with the most relevant to RPA set as follows: delay = 1, embed = 1, radius = 1.

3 Research results

Daily recurrence plots of the estimated TEC and TEC rate-of-change sets, respectively, for selected cases of characteristic classes of ionospheric conditions are first presented in sub-section 3.1. Time series of $K_p$ and $Dst$ geomagnetic conditions predictors are presented with every recurrence plot for the purpose of assisted interpretation. The results of the quantitative analysis of the TEC RPA, according to the methodology presented in Section 2, are presented and summarised in sub-section 3.2. Finally, a qualitative interpretation and the discussion of the presented quantitative results are given in sub-section 3.3.

3.1 TEC and TEC rate-of-change recurrence plots

This section contains $K_p$ (upper) and $Dst$ (middle) time series diagrams as predictors of geomagnetic conditions, and TEC (lower left) and TEC-rate (lower right) recurrence plots as results of those activities for selected cases of the ionospheric conditions scenarios, as detailed in Section 2.1.
3.1.1 A massive space weather storm - Day 302 in 2003

Figure 3 Kp (upper) and Dst (middle) time series, and TEC (lower left) and TEC-rate (lower right) recurrence plots for Day 302 in 2003
3.1.2 Fast developing ionospheric storm - Day 076 in 2015

Figure 4 Kp (upper) and Dst (middle) time series, and TEC (lower left) and TEC-rate (lower right) recurrence plots for Day 076 in 2015
3.1.3 Fast developing geomagnetic storm - Day 251 in 2017

Figure 5 Kp (upper) and Dst (middle) time series, and TEC (lower left) and TEC-rate (lower right) recurrence plots for Day 251 in 2017.
3.1.4 Quiet space weather conditions - Day 130 in 2018 (control scenario)

Figure 6 Kp (upper) and Dst (middle) time series, and TEC (lower left) and TEC-rate (lower right) recurrence plots for Day 130 in 2018.
3.1.5 Quiet space weather conditions on Earth immediately before the onset of a massive ionospheric storm the next day - Day 301 in 2013

Figure 7 Kp (upper) and Dst (middle) time series, and TEC (lower left) and TEC-rate (lower right) recurrence plots for Day 301 in 2003

3.2 Quantitative analysis of the TEC RPA

Tables 5 and 6 present the RPA results for TEC and TEC rate-of-change estimates derived from experimental GPS observations at Matera, Italy in selected cases of characteristic classes of ionospheric conditions.
Table 5 RPA results for daily TEC estimates time series in selected cases of ionospheric conditions.

<table>
<thead>
<tr>
<th></th>
<th>302/2013</th>
<th>076/2015</th>
<th>251/2017</th>
<th>130/2018</th>
<th>301/2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>RR</td>
<td>1.633</td>
<td>1.700</td>
<td>1.519</td>
<td>0.956</td>
<td>2.071</td>
</tr>
<tr>
<td>DET</td>
<td>96.03</td>
<td>95.27</td>
<td>94.70</td>
<td>77.41</td>
<td>98.11</td>
</tr>
<tr>
<td>NRLINE</td>
<td>3287</td>
<td>4021</td>
<td>4773</td>
<td>3101</td>
<td>4473</td>
</tr>
<tr>
<td>ENTR</td>
<td>2.599</td>
<td>2.529</td>
<td>2.018</td>
<td>1.707</td>
<td>2.584</td>
</tr>
<tr>
<td>rENTR</td>
<td>0.5965</td>
<td>0.5913</td>
<td>0.4702</td>
<td>0.4484</td>
<td>0.5757</td>
</tr>
<tr>
<td>LAM</td>
<td>98.94</td>
<td>98.72</td>
<td>98.31</td>
<td>89.79</td>
<td>99.83</td>
</tr>
<tr>
<td>TT</td>
<td>10.066</td>
<td>8.286</td>
<td>6.946</td>
<td>4.219</td>
<td>10.849</td>
</tr>
</tbody>
</table>

RPA predictor values relative to control scenario in %

Figure 8 RPA results for daily TEC estimates time series relative to those of the control scenario (130/2018)
<table>
<thead>
<tr>
<th></th>
<th>302/2013</th>
<th>076/2015</th>
<th>251/2017</th>
<th>130/2018</th>
<th>301/2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>RR</td>
<td>0.908</td>
<td>1.113</td>
<td>1.164</td>
<td>0.712</td>
<td>0.713</td>
</tr>
<tr>
<td>DET</td>
<td>28.33</td>
<td>33.91</td>
<td>33.49</td>
<td>21.14</td>
<td>22.46</td>
</tr>
<tr>
<td>NRLINE</td>
<td>1459</td>
<td>2427</td>
<td>2263</td>
<td>713</td>
<td>761</td>
</tr>
<tr>
<td>ENTR</td>
<td>1.061</td>
<td>1.060</td>
<td>1.236</td>
<td>0.734</td>
<td>0.913</td>
</tr>
<tr>
<td>rENTR</td>
<td>0.4022</td>
<td>0.4263</td>
<td>0.4199</td>
<td>0.3341</td>
<td>0.4693</td>
</tr>
<tr>
<td>LAM</td>
<td>38.22</td>
<td>46.70</td>
<td>47.74</td>
<td>28.47</td>
<td>25.99</td>
</tr>
<tr>
<td>TT</td>
<td>2.993</td>
<td>3.022</td>
<td>3.235</td>
<td>2.700</td>
<td>2.788</td>
</tr>
</tbody>
</table>

Table 6 RPA results for daily TEC rate-of-change estimates time series in selected cases of ionospheric conditions

### RPA predictor values relative to control scenario in %

**TEC rate of change**

![Graph showing RPA predictor values relative to control scenario in %](image)

Figure 9 RPA results for daily TEC rate-of-change estimates time series relative to those of the control scenario (130/2018)
3.3 Interpretation of the results and discussion

The analysis of selected case studies revealed good compliance of the TEC and TEC rate-of-change recurrence plots with those of particular characteristic classes of non-linear behaviour (Table 3), opening room for a more efficient GNSS pseudorange measurement and GNSS position estimation errors modelling and corrections.

The TEC time series generated recurrence plots that generally resemble those of Brownian motion. There is a marked tendency of transition towards the Auto-Regressive time series during quiet ionospheric conditions. Disturbed ionospheric conditions increase the values of all the recurrence plot predictors. Most of them achieve their local maxima prior to massive ionospheric disturbance events commence (the recurrence rate and the Shannon information entropy of diagonal line lengths longer than the minimum ones, the total number of lines and their average length in the recurrence plot). The fact is to be researched further in terms of utilisation for forecasting and now-casting of GNSS positioning performance degradation due to space weather, geomagnetic and ionospheric disturbances.

The TEC rate-of-change time series tends to form more uniformly filled recurrence plots during quiet and modestly disturbed ionospheric conditions. Disturbed ionospheric conditions raise the recurrence rate and the determinism, the total number of lines in the recurrent plot, the Shannon information entropy of diagonal line lengths longer than the minimum ones and the laminarity. A significant increase in the total number of lines in the recurrent plot is observed during fast developing ionospheric and geomagnetic storms. In an interesting twist of development, TEC recurrence rate and trapping time, and the Shannon entropy measure of TEC rate-of-change recurrence plot normalised by the observed number of lines in a plot, extended their largest value observed in the period preceding the largest ionospheric storm recorded in modern history.

4 Conclusions and future work

This manuscript presents results of the research in GNSS pseudorange measurement error and TEC time series assessment in classes of positioning (ionospheric) environment conditions, using the RPA method. Recurrence plots of the GPS-derived TEC/GPS pseudorange measurement errors were RPA-analysed for identification of daily time series patterns, with the aim to identify common classes of dynamical properties that would improve the understanding and allow for modelling of GNSS pseudorange measurement errors and the over-all GNSS positioning performance, and for a contribution to the development of the GNSS that is more resilient to ionospheric and space weather effects.

The generalised RPA methods were demonstrated as successful in identification and characterisation of TEC/GNSS pseudorange measurement error time series in relation to ionospheric conditions. Selected cases of ionospheric conditions were examined,
ranging from quiet to a massive ionospheric storm. Obtained recursive plots of TEC/GNSS pseudorange errors and TEC/GNSS pseudorange error rates were compared with the recurrence plots of typical classes of time series models. Additionally, the recurrence plot predictor values were analysed in relation to characteristic ionospheric condition scenarios. A sharp increase of the number of lines was identified during fast developing ionospheric and geomagnetic storms. The TEC recurrence rate and trapping time, and the Shannon entropy measure of TEC rate-of-change were identified as potential predictors and precursor indicators of an approaching TEC/GNSS pseudorange measurement quality degradation. These prospects will be examined further and exploited for GNSS user equipment design and the resilient GNSS developments, in the forthcoming research activities.

Research presented in this manuscript contributes to the subject, as follows. (i) A methodology was developed during the research study execution for studying the non-linear patterns of TEC and TEC rate-of-change, with objective quantitative predictors. (ii) TEC and TEC rate-of-change time series were assessed for non-linear performance in selected representative ionospheric conditions scenarios using RPA. (iii) Results of the assessment in (ii) were inferred for identification of statistically significant predictors and regularities, which may serve further purpose in refinement and improvement of methods for detection, modelling and forecasting GNSS ionospheric delay and, consequently, GNSS positioning performance degradation.

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Data availability

The original data used in this study is described in Section 2.1 and is available through [12][13]. The scripts used to produce the presented results are available at https://github.com/klenac/RPA-ionosphere.git.

References


Highlights

- Understanding the ionospheric effects on GNSS positioning performance
- Development of GNSS positioning that is more resilient to space weather effects
- Assessment of GNSS pseudorange measurement errors using the Recurrence Plot Analysis
- Classification of ionospheric conditions and TEC behaviour based on their dynamical properties
- Identification of RPA predictors as precursors of developing ionospheric storm
Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐ The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: