MEASUREMENT, STATISTICS, AND RESEARCH DESIGN

Education is a Complex Dynamical System: Challenges for Research

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ABSTRACT

The complex dynamical systems (CDS) approach consists of a family of theories emanating largely from the exact sciences. These theories share a common focus on the behavior of systems and their interrelated parts and are concerned with the processes of stability, change, and unpredictability in those systems. This article takes stock of the methods tailored toward the study of complex dynamical systems in education—for example, the interaction between teachers and students in classrooms, educational organizations such as school buildings and districts, and collaborative learning settings. A historical and conceptual background is provided as a context for CDS. Use of the perspective in education is evaluated according to three basic systemic assumptions: Systems behavior is complex, it evolves over time, and the nature of systemic transformation is qualitative. A comprehensive yet incomplete overview is provided of available research methodologies concerned with those assumptions.

KEYWORDS
Complexity; dynamical systems; school reform; educational change; methodology

THE COMPLEX DYNAMICAL systems (CDS) approach represents an interdisciplinary paradigm that concerns itself with the behavior of systems and their interrelated parts and with the processes of stability, change, and unpredictable behavior in those systems. One of the central ideas behind CDS is that any entity can be described in terms of its components, or subsystems, and the interrelationship between them can be called a system. The importance of distinguishing these different levels of description lies in the premise that systemic behavior is often not readily reducible to the interactions between the system components. The designation of something as a system primarily serves the analytical convenience of scholars and practitioners, and it has the advantage of being applicable in a wide variety of situations and across disciplines as divergent as physics, engineering, theoretical biology, management science, econometrics, psychology, education, and art.

The relevance of CDS to education has been readily appreciated for quite some time, and we have systemic descriptions of a wide range of aspects of the educational process as systems, such as teacher-student interactions (Rasmussen, 2005; Steenbeek & van Geert, 2013), interaction between students and learning content (Garner & Russell, 2016), school building leadership (McQuillan et al., 2016), and the policy realm (Osberg & Biesta, 2010). A systemic view has also been put forward to describe the entire educational constellation in terms of the various stakeholders in the educational process, such as students, teachers, parents, principals, school boards, consultants, district-level administrators, and philanthropists (Fullan, 1991). Thus, how systems are conceptualized, in education and elsewhere, is not set in stone, and definitions can vary.

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depending on the question at hand. Thus, a mother and child interacting could be the system of interest in a language acquisition study and the language itself, in a generative grammar study. A school building and its faculty and administrative leadership could be the system of interest in a study of the effectiveness of professional development or distributed leadership initiatives (e.g., Elmore, 2004). What distinguishes a CDS thinking from other paradigms is its focus on the ways in which the interaction between components of the system results in novel higher-level aggregate behaviors that are not reducible to those component interactions (Ashby, 2004/1962; Goldstein, 1999). This irreducibility is one of the defining characteristics of complexity, and thus, a CDS conceptualization relies on the description of interaction at multiple levels and on the relationship between those interactions and their accumulative behavioral expression at higher levels of description of the system.

Some very good early theoretical work has been published to discuss the potentially far-reaching implications of the perspective to the field of education. Doll (1993) describes curriculum and pedagogy such that the emergent transformative processes are the aggregate behaviors that need to be understood over and above fixed entities such as curricular units, lesson plans, or student test scores. As such, education becomes dynamical, with many moving and interlocking parts whose behavior evolves over time and is not necessarily predictable. The idea of learning as an emergent process can also be found in Piaget’s (1967) stage-wise theory of child development, which posits that radical transformation occurs of the child’s cognition upon repeated interactions with the environment, resulting in input that is inconsistent with extant cognitive schemata (accommodation), and in Vygotsky’s zone of proximal development, which describes learning as a dynamical interplay between learners’ capabilities and the learner’s potential to exceed those capabilities under guidance (Vygotsky, 1978). Piaget’s formulation was consistent with the dynamical theories of the time and he made them explicit in his work (e.g., equilibrium, perturbation, transformation).

The compatibility of the dynamical description of the developmental process by Piaget (1967, 1971) and Vygotsky (1978), on the one hand, and contemporary formulations of systemic processes by, for example, Bak (1996) and Guastello and Lievobovitch (2009), on the other, have resulted in a deeper understanding of the developmental process from childhood onward. Extensive work has been done to articulate the dynamical processes underlying Piaget’s stage transitions (van der Maas & Molenaar, 1992) and to generate a meaningful synthesis of Piaget’s adaptation and Vygotsky’s zone of proximal development (van Geert, 1998). Likewise, van Geert and Steenbeek (2005) provide a formulation of Vygotsky’s scaffolding in terms of contemporary CDS theory. Productive simulation studies in both of these latter papers illustrate how the original theories indeed do imply the CDS processes for which we now have the requisite mathematical formulations.

There is also significant work at the level of educational policy research. Davis and Sumara (2006) concern themselves directly with the methodological implications of the CDS perspective for our practices as educational researchers. They argue that our thinking as educational researchers is often confined by the untested assumptions of the linear paradigm, such as that behavioral changes are proportional to the changes in systemic input conditions, that change is gradual, and that cause and effect are sequentially related occurrences. With respect to learning as an emergent situation, for instance, Davis and Sumara note that “one of the difficulties in studying instances of emergence is that the specific conditions and mechanisms of its occurrence can vary dramatically across situations” (2006, p. 81). As a result, the generalization of insights about change and transformation across settings and populations needs to be supplemented by the detailed analysis of the underlying process of transformation in particular cases. CDS is well-equipped to deal with such particularities.

Another of the areas for which the potential of CDS has been appreciated is educational reform, where change is known to be elusive (Fullan, 2016) and very hard to accomplish (Fleener, 2016). Thus, a deeper understanding of the underlying mechanisms of change and
resistance thereto is needed to identify the forces facilitating and inhibiting transformation. The school reform literature is rich with case studies describing reform attempts in urban districts, sometimes explicitly from a complexity perspective (Mirón, Beabout, & Boselovic, 2015). Collectively, these studies show how much we do not know about the underlying mechanisms of stability and change in educational systems. Lemke and Sabelli (2008) propose a framework for the use of complexity research to inform the area of educational reform, arguing that all education is local and that, therefore, focusing on large-scale attempts to change systems and measure the effects of those attempts may not be the best approach (see e.g., Betts, Zau, & King, 2005). Other reform scholars have also come around to this point, realizing that attempting whole-system reform and measuring its effect may be too ambitious, as well as not allowing us to learn enough about why certain educational initiatives are working at the local level (Elmore, 2016; Fullan, 2016). A missing link in research, therefore, is the description of the local conditions under which reform is possible and uncovering the processes that may facilitate or inhibit transformation on a larger scale.

The importance of CDS to educational research is also that it challenges widely cherished notions about data in research design, such as the prevalence of normal probability distributions, regression to the mean, the central limit theorem, and linear cause-effect relationships (West & Deering, 1995). Some probability distributions are highly asymmetric. An example of such a distribution is the power law, which predicts rare occurrences by describing frequency distributions from frequently to infrequently observed events as a linear slope. A well-known example of a power law is the frequency distribution of earthquakes depending on their magnitude: The larger the magnitude of the quake, the less likely it is to occur (Bak, 1996). Such distributions can be fitted based on empirical or simulation data to confirm predictions made in the literature, as is done, for instance, in a study on domain-specific excellence in human performance that is confirmed to be a power law distribution (den Hartigh et al., 2016).

Linear causality is the prevailing model in educational research and policy research arena (Koopmans, 2014). In education, the linear model typically presupposes that the implementation of interventions will be the cause of observed improvements in educational outcomes—or not, depending on their effectiveness. The task for researchers, then, is to establish this link. However, the notion of systemic behavior as an aggregate result of interactions between a system’s constituents implies a different idea about cause and effect that requires an understanding of how the interaction between people produces group behavior that goes over and above the individual components—that is, “social causation” (Sawyer, 2002, 2003).

As the above suggests, the methodological implications of a CDS view of educational research are considerable. The purpose of this article is to come to terms with this challenge and to outline some of the responses that can be found in the research literature on educational systems. A set of methodological priorities is suggested that better accommodates the CDS assumptions about the behavior of systems. The plan for the remainder of this article is as follows: A brief overview is provided of the basic foundations and history of CDS. This is followed by a discussion of the methodological implications of this perspective and a description of the use of CDS-based methods in recent educational research. Finally, the limitations and future prospects of CDS are discussed. The material presented here is not meant to be exhaustive, and some techniques, such as agent-based modeling and evolutionary computation, fall outside of the scope of the article (but see Gilbert, 2008, and Wilenski & Rand, 2015).

**Complex dynamical systems theory: A brief overview**

Complex dynamical systems (CDS) theory is a modern extension of two older models, one is general systems theory (GST), whose original focus is theoretical biology (Von Berthalanffy, 1968), and the other is cybernetics, whose original focus is engineering (Ashby, 1957). GST formulates
an abstract model of systems and their constituent components (subsystems, individuals) and models the dependencies between those components within and across the different levels of description. Thus, teachers and students as individuals constitute one level of description; classrooms and aggregate learning behavior are another. Cybernetics postulates a mechanism of the interaction between systemic components at different levels of description called feedback. The textbook example of a feedback process is the thermostat, a closed system that regulates temperature by switching itself on and off to maintain consistency. An example of a feedback loop in education is Vygotsky’s zone of proximal development (Vygotsky, 1978), which marks the difference between the level of performance that learners are capable of when left on their own and the level they are able to achieve when working under the guidance of adults or more capable peers. The feedback in this case is between the behavior of the learner as an individual and the interaction process that elevates their learning outcomes. A detailed formulation of all aspects of the zone of proximal development in terms of contemporary CDS vocabulary and an informative simulation of the underlying dynamical process can be found in van Geert and Steenbeek (2005).

In common parlance, complexity is defined as a state of being multifaceted (Merriam-Webster, 1990). Thus, a work assignment can be complex, or an organizational chart can be complex. In CDS circles, use of the term is more specific and originates in cybernetics. Ashby (2004/1962) made the argument that once elements interact, they form a system, and systems can therefore be defined as hierarchical structures, requiring distinction between the behavior of a system in its entirety and that of its constituent components. Complexity, in this context refers to the attribute that the behavior of a system cannot be reduced to that of its individual components, nor does the behavior of those components fully determine the behavior of the system as a whole. Therefore, systems are complex by definition (Thietard & Forgues, 2011), and the choice of the unit of analysis for a data project is therefore not as straightforward as it is sometimes made out to be.

One of the ways in which contemporary CDS models are seen to differ from the “classical” ones discussed above is in the underlying assumptions about stability in the system. It has traditionally been assumed that systems are stable in principle, with interaction between individuals being predictable once the system is known, with occasional turbulence to accompany transformations in the system. The notion that systems are stable unless they are transforming underlies Kurt Lewin’s force field theory (Lewin, 1947), which describes the behavior of systems in terms of centrifugal and dissipative forces that cancel each other out to keep the system’s structure in place. This notion can also be found in catastrophe theory (Thom, 1975), which mathematically models turbulence as an accompaniment to the transition of a system from one stable state to another.

These assumptions about stability and change have been challenged, most notably by Prigogine and Stengers (1984), who use chaos theory (Poincaré, 2007/1914) to demonstrate that systems are not necessarily stable and that their behavior is not necessarily predictable. The impact of this work has reached far beyond the area of physics and chemistry, from which Prigogine and Stengers derived their examples. With respect to human organizations, the implications of the insights from chaos theory were appreciated by Goldstein (1988, 1990), who suggested that the assumption of stability yields too many counterexamples when human interaction is considered. We therefore need to account for the possibility that systems could be in a state of ongoing turbulence—or far from equilibrium—rather than characterizing such turbulence as a temporary transformative state, maintaining just enough stability for their integrity as a system while displaying volatility in its exchanges with the environment. In the context of for-profit environments, it has frequently been argued that the lack of stability may in fact create opportunities for creativity and innovation within those organizations that would not exist if the behavior of individuals were delineated strictly by the parameters generating predictability within those organizations, such as top-down management structures and explicit job descriptions (Dooley,
Instead, a bottom-up approach describes the emergence of new behaviors at the organizational level, in terms of the interaction between lower-level components and the creative possibilities engendered by those interactions (Goldstein, 2014).

**A few essential constructs**

A rich array of scenarios of stability and transformation has been formulated in the dynamical literature (Nicolis & Prigogine, 1989). This richness illustrates how much systems thinking has evolved from the placid equilibrium models that dominated the dynamics field in the mid-twentieth century (Lewin, 1947; von Berthalanffy, 1968) to concern itself more directly with the unstable aspects of systems’ behavior and the identification of the processes and factors that generate potential for transformation in the future. Among the most well-known contemporary dynamical constructs are self-organization, emergence, self-organized criticality, sensitive dependence on initial conditions, attractors, bifurcation, and autopoiesis. Below is a brief description of each.

**Self-Organization** is a process through which apparent random interactions between elements in a system evolve into a coherent systemic whole capable of responding to specific input conditions (Ashby, 2004/1962; von Foerster, 1960). This notion is one of the central pillars of complexity thinking, because of the implication that adaptive systems do not “just exist” but are entities that are actively maintained through the interaction between components and come into being through interactions between previously unrelated elements. A teacher initiating classroom discourse with a new class at the beginning of the school year would be an example of such a process, wherein structured interaction about ground rules for behavior in the classroom might define the interaction as a new system.

**Emergence** is defined as radical novelty in the higher-level behavior of systems resulting from seemingly random fluctuations in the lower-level interactions within those systems (Goldstein, 1999; Sawyer, 2002, 2003). Emergence can be found in educational systems on various levels. In collaborative learning situations in the classroom, for example, seemingly random exchanges between students exploring a science problem can result in the discovery of principles resulting in a higher level of understanding of the underlying principles in all participants and in more-productive problem-solving behavior going forward. Chiu (2008a, 2008b), for instance, shows how group-interaction characteristics at the micro-level (e.g., argumentation, disagreement) can facilitate breakthroughs (watersheds) in the problem-solving process at the macro-level in secondary students solving algebra problems. School reform likewise can be captured in terms of emergent processes—that is, as productive exchanges about effective strategies between teachers and administrators that may result in the institutionalization of better practices (Elmore, 2004).

**Self-organized criticality** is a critical state occurring in a system such that due to its repeated exposure to certain input conditions, change becomes imminent (Bak, 1996). This state is also called the *edge of chaos* (Waldrop, 1992). A prototypical example of self-organized criticality is the sand pile on a flat surface over which additional sand is poured constantly until the friction between the grains triggers avalanches that reorganize the pile (Bak, 1996; Jensen, 1998). In the educational system, one might similarly argue that repetitive cycles of input and response within the system create a friction between the systems’ components that ultimately results in transformation. An example is the aforementioned process of accommodation in Piaget’s stage transitions, wherein repeatedly unproductive exchanges between a child’s schemata and the environment create the momentum that is needed for a restructuring of those schemata to fit the new input. Van Geert (1998) successfully simulated processes of assimilation and accommodation as instances of self-organized criticality and discusses the broader potential of the construct for our understanding of transitions in developmental science (van Geert, 2009).
Sensitive dependence on initial conditions is one of the basic insights from chaos theory, and it refers to a process wherein states of turbulence and transformation in a system originate in infinitesimal variations in the behavior of the system at an earlier point in time (Poincaré, 2007/1914). While chaos theory lay dormant for a good part of the twentieth century, the work of MIT meteorologist Edward N. Lorenz revived interest in the theory when he incidentally was faced with an example of sensitive dependence on initial conditions. Lorenz was skeptical about the use of the linear approaches to predict weather patterns over the long term, and he noticed that small rounding errors in the recorded values in a simulated time series resulted in qualitatively different trajectories of predicted values in his time series in the long run. This discovery gave rise to the development of a new set of mathematical models to describe the impact of small fluctuations on the long-range stability in time series trajectories (Palmer, 2008), thus providing new impetus to the use of chaos theory as a method to describe and predict patterns of unstable behavior (Sprott, 2003). The problem with the application of this concept to the behavior of adaptive systems is that the adaptive process generates ongoing variability in the system, making the detection of such infinitesimal fluctuations extremely difficult (McSharry, 2005), in addition to which there are limitations to the description of an adaptive system purely in terms of its previously observed behavior, without considering the behavior of other systems with which the adaptive system may interact.

Attractors are the preferred coordinates in the space of possible behaviors to which a system tends to gravitate in the course of its interactions with fluctuating environmental and internal demands (Guastello & Liebovitch, 2009). Attractors can be seen as behavioral precedents. At each point in time, there is a degree to which behavior deviates from these precedents and displays behavior that can be close to or far removed from this attractor state. Systems’ responsiveness to incidental fluctuations depends on the how much behavior deviates from these attractors at a given point. Close to the attractor state, systems are stable and unlikely to be perturbed, while behavior that is removed from these attractor states is said to be very susceptible to transformation from one attractor state to another. The points of least stability are called repellors, points from which a system is apt to move away to ease back into an attractor state and where behavioral variability tends to be at its highest. In unstable systems such as chaotic ones, there may not be one but several attractors to which systemic behavior orients itself, resulting in a high degree of turbulence. The shift in a system from a one- to a two-attractor regime is called a bifurcation, a shift from one to two attractor points, resulting in greater complexity.

Autopoiesis is defined as the process of ongoing self-realization and self-production of a system through interaction among its components (Maturana & Varela, 1973), leading it to prioritize its own longevity. In its original formulation, the theory of autopoiesis was primarily concerned with biological and neurophysiological processes in which systemic interactions are purportedly geared toward survival. Consistent with the notions of complexity forwarded by cybernetic theory, autopoiesis views the interactions within systems as an ongoing process through which systems (re)assert themselves as such, and autopoietic theory postulates that the self-perpetuation of the system is the main driver of the systems interactive behavior and can thus be seen as an attractor state. This view does not preclude that interactive behavior in the system can be oriented toward consolidation (negative feedback) or transformation (positive feedback, Ashby, 1957), nor does it preclude that systems can be dissolved in the face of environmental demands as new systems get created in their stead.

**CDS priorities and methods**

CDS and related models are often presented as a much-needed alternative to existing research paradigms that are broadly characterized as the *linear model* (West & Deering, 1995) or *simplicity* (Morin, 2008). Three popular assumptions defining the linear model are (a) that cause and effect
are sequentially ordered (Pearl, 2009), (b) that observed variability in the distribution of outcomes between cases generalizes to an understanding of the variability in repeated measurements within cases (Molenaar, 2004), and (c) that observed changes in behavioral outcomes are proportional to changes in the input conditions (West & Deering, 1995). CDS takes an interest in processes that typically do not conform to these assumptions, and therefore, it is often presented as a distinct paradigm (e.g., Fleener & Merritt, 2007; Jörg, 2011; Morin, 2008) operating under its own set of methodological priorities. These priorities roughly fall into three broad areas: (a) the study of complex processes (i.e., the irreducibility of the behavior of systems to that of their constituents), (b) the study of stability and change in behavior over time (i.e., the temporal dimension), and (c) the mathematical expression of qualitative transformations for theory building and empirical confirmation (West & Deering, 1995). Below is an enumeration of the suitability of some of the research methodologies that are available to rigorously study (some combination of) these processes.

**Ethnography**

In education, studying behavior in systemic context has typically been relegated to ethnographic research (e.g., Bogdan & Biklen, 1982), which, by providing thick descriptions of behavior, enables the capture of systemic interdependencies more rigorously, including an assessment of the impact of the investigator on the system of interest. The cross-sectional research that relies on the statistical aggregation of the behavior of nonrelated individuals into group averages is less conducive to the investigation of these interdependencies, because the “groups” over which the averages are taken in the latter type of designs do not constitute a system of interacting elements as defined above. In the systemic sense, groups are particular as are the interaction processes that constitute these groups, and therefore, research needs to be able to address this particularity.

Despite the historical affinity between ethnographic research and systems thinking (e.g., Bateson, 1972/1935), the qualitative research tradition in education has generally not shown a substantive commitment to study complex dynamical processes in education. Nevertheless, the potential of ethnographic research to make a significant contribution to the study of the behavior of systems is considerable and has been appreciated in some detail by Bloom and Volk (2007). In addition, important work has been done using the approach to examine complex processes in education, focusing on the classroom (Hudson & Casbergue, 2016; Laidlaw et al., 2013), the peer group (Anderson et al., 2016), the school building (Combes & Patterson, 2013; McQuillan et al., 2016) and the school district (White & Levin, 2013). McQuillan et al. (2016) and Marion et al. (2012) particularly extend the ethnographic viewpoint of participant observation into the CDS realm by collaborating with practitioners in the field to uncover the dynamical processes and systemic descriptions of interest within the school setting.

**Orbital decomposition**

Traditionally CDS has relied heavily on mathematical models to define complexity (e.g., Ashby, 1957; von Bertalanffy, 1968) and complex transformation (Nicolis & Prigogine, 1989; Thom, 1975), and it evolved from scientific disciplines with strong quantitative traditions such as physics, engineering, and econometrics (Mandelbrot, 1997). One methodology used to quantify systemic features that carries great promise in education is orbital decomposition (Guastello, Hyde, & Odak, 1998), a method for quantifying the dynamics underlying turn-taking patterns in the communication in small groups. Consistent with GST and cybernetics, the method takes the system as a unit of analysis to study the behavior of its interrelated components. The technique relies on verbatim recorded segments of verbal exchange that are coded such that utterances are classified into different types that have substantive meaning in the context of the research question. For example, Stamovlasis (2016a) analyzes exchange patterns in collaborative-learning situations of
students solving science problems and, thus, one part of his coding system includes such categories as “reflection on the problem,” “explanation with a physical law,” “argument,” “hypothesis,” and “recall of a physical law.” The analysis proceeds by statistically deriving an “optimal string length” (e.g., three turns or four turns) for the exchanges and then analyzing the strings containing the classifications of individual utterances (e.g., “argument—recall—hypothesis”). The strings thus analyzed form the basis for the detection of the complexity and predictability of the interaction patterns. Complexity is defined here as a fractal dimension (i.e., strings replicating within larger strings), with predictability defined in terms of the observed entropy in the exchanges (i.e., the extent to which certain types of strings are not favored over others).

**Social network analysis**

Another example of a technique used to capture the systemic complexity of behavior is social network analysis (Borgatti, Everett, & Johnson, 2013), an analytical approach that is based on the theory that the behavior of individuals can be understood in terms of the structure of the social networks that form the context of those behaviors. Social network analysis quantifies such systems in terms of nodes (individuals, units) and links (connections between units) and visualizes these relationships in sociograms that describe the presence or absence of interactions between elements and the centrality of the position of individual elements within the network. Such information can be used in turn to quantify such constructs as network density, cohesion, and reciprocity, which characterize the interactions between network components in terms of the overall system. A prototypical example of the use of social network approaches in education was provided by Moreno (1934), who created sociograms of classrooms linking students who wanted to sit next to each other (see Grandjean, 2015). Social network analysis has been used productively by Marion et al. (2012) to study education leadership patterns in the context of conflicting pressures of bureaucratic stability in the public school system and by Ugurlu (2016) to establish the degree of connectedness between faculties in an international student exchange program. The approach has also been used to describe professional learning networks in school settings (Rodway, 2018).

Some recent work in this area provides a more general formulation of network behavior and concerns itself specifically with the evolution of networks as an emergent process that includes their growth over time, the formation of preferential attachment patterns between nodes (Barabási & Albert, 1999), and the presence of power-law distributions in the clustering of those networks (Watts & Strogatz, 1998). It is also possible to conceptualize the interaction of variables within individuals and across individuals within dyads as a network. Steenbeek and van Geert (2013), for example, postulate as a network the interplay between concerns about relatedness, autonomy, appraisal, empathy, and action across two children interacting in play situations; den Hartigh et al. (2016) propose a network of associations of high achievement in individuals with ability-growth measures and environmental support components.

**Fractional differencing**

Fractional differencing is an extension of time series analysis, the statistical approach that is used to model dependencies in observations ordered on a time scale and that, in turn, uses this information to make predictions (Box, Jenkins, & Reinsel, 2008). Ashby (1957) already noted that a system’s behavior evolves over time by definition, and the empirical study of systemic behavior therefore requires that information be collected over time to enable the detection of the determinants of the observed temporal patterns. Conventional time series are an effective way to model predictable cycles; that is, observations are correlated according to some fixed time scale, such as the days of the week or the months in a year. Finding a replication pattern according to such
cycles is substantively important because it characterizes behavior that is nonrandom and seasonally dependent. This knowledge may assist with planning and policy and with predicting a system’s behavior in the future. But what if the time scale of the repeat fluctuations is variable and unpredictable? Fractional differencing, also known as autoregressive fractional integrated moving average (ARFIMA) modeling, addresses such unpredictability (Beran, 1994; Sowell, 1992; Stadnitski, 2012).

Fractional differencing estimates dependencies across the entire spectrum of the time series irrespective of specific lag sizes, thus providing an overall measure of dependency in the series that covers the short run, as is also done in conventional time series analyses, but also includes the long run—that is, correlations between observations that are very far apart on the time scale. A parameter estimate \( d \) is generated (as well as a related measure, the Hurst exponent, or \( H \)). The value of \( d \) can range from \( d = -0.5 \) to \( d = +0.5 \). If \( d = 0 \), we conclude that there are no significant long-range dependencies. If \( d \) is greater than zero, we conclude a positive correlation between data points including the remote ones, or persistence, and if \( d \) is less than zero, we conclude a negative correlation or antipersistence. A statistically significant differencing parameter suggests complexity in the system in the sense of self-similarity—that is, patterns replicating themselves within patterns (see Koopmans, 2016, p. 309, for an example). Self-similarity suggests a more complex adaptation to environmental demands whose influence cannot be predicted from the behavior within the system (Beran, 1994; Stadnitski, 2012).

Koopmans (2015, 2018) used the approach to try to determine whether daily high school attendance rates fluctuate in predictable or unpredictable cycles, and found that there is unpredictability in those rates in many of the high schools included in his sample. While school personnel may be knowledgeable about the predictable fluctuations in the attendance in their schools—that is, weekly or monthly cycles—the unpredictable ones are less readily discernible and therefore harder to respond to effectively. They are nonetheless relevant because they may point to the adaptiveness of the system to fluctuating external circumstances (Stadnitski, 2012), such as perhaps shifts in the students’ schooling situation due to family, community, socioeconomic, and other factors.

This approach makes it possible to determine whether there is fractality in the behavior of the system—that is, fluctuation patterns on a time scale that replicate within themselves numerously over smaller fractions of that same time scale; hence they are sometimes referred to as scale invariance. Fractal patterns have been found in the fluctuation of self-evaluation recordings in undergraduate students in a course or recording a three-minute narrative about themselves (Wong, Vallacher, & Novak, 2014) and in daily high school attendance counts in some schools in the course of a multiyear time span (Koopmans, 2018).

The drawback of the fractional approach is that a very large set of data points is needed to be able to statistically detect such processes reliably: \( N = 512 \) has been said to be the very lowest number to obtain a sufficient degree of resolution in the data for reliable estimation (Delignières, Torre, & Lemoine, 2005), and researchers often work with a manifold of this. Furthermore, while finding persistence or antipersistence allows us to conclude that self-organized criticality in the system cannot be ruled out, it does not firmly establish its presence either (Beran, 1994).

**Multivariate time series and recurrence quantification analysis (RQA)**

The behavior of adaptive systems typically does not depend just on its own past behavior but also on its interaction with other systems, and those interactions can be measured in a time sensitive manner as well. Multivariate time series and recurrence quantification analysis allows us to model this aspect of systemic behavior. In these models, fluctuations in one measure across a time scale is evaluated in terms of fluctuations of another measure across the same time scale. Molenaar et al. (2009), for example, measured the arousal of adolescents communicating with
their (step-)fathers as well as the arousal of those (step-)fathers in the course of the same interactive episodes, thus allowing for fluctuations in the behavior of one system to be contingent upon those in the other.

RQA offers another way of looking at the dependencies in serially ordered data. The technique is used to determine to what extent certain observed patterns recur across the series (Eckmann et al., 1987). Recurrence is a fundamental property of dynamical systems, because it describes the extent to which its behavior is predictable (Marwan et al., 2007). Such predictability can manifest itself in behavior that is cyclical, such that patterns recur on a fixed time scale, or it can be non-cyclical when repetitions are manifest without falling onto a fixed time scale.

To describe recurrence, RQA relies on scatterplots of time against itself, such that the data points in the body of the graph mark recurrence at given time coordinates of an outcome matrix. Periodicity is marked by a clustering of recurrences along diagonal lines—that is, outcomes continuing to recur at predictable lags within the series. A clustering of recurrence along vertical lines indicates recurrence that is noncyclical. Vertical lines tend to show up in patches in these plots showing pockets in the outcome space in which observations are closely correlated with others in the vicinity, indicating areas of short-range dependency in the series (see Reuzel et al., 2014, p. 381, for an example of an RQA plot).

A bivariate version of RQA, cross-recurrence quantification analysis, or CRQA, is specifically concerned with the dependencies among coupled time series to describe the dependency between two systems (Marwan & Webber, 2015). The analysis of dependencies across systems is very suitable for the analysis of social interaction between dyads, an option that has been taken advantage of by Guevara et al. (2017), who capture the interactive behavior of seven dyads of young children who are engaged in a problem solving task. CRQA’s ability to describe behavioral dependencies across a detailed time scale makes it possible to determine the extent to which the behavior of the participants in the interaction is coupled, who is leading and who is following, and to what extent the two systems behave relatively independent of one another. This coupled-versus independent character of behavior is an important aspect in the description of learning situations, as it concerns both teacher-student and student-student interactions (Guevara et al., 2017; Steenbeek & van Geert, 2013). Likewise, the underlying dynamics of the interaction between adult clients with intellectual disabilities and service providers have been quantified by Reuzel et al. (2014), who used CRQA to establish patterns of dominance and attunement in the iterative exchanges between these actors to describe the quality and effectiveness of those interactions.

**State space grids**

Attractors and repellors represent stable and unstable behavior, respectively, in terms of strings of repeated observations that are confined within a bounded region called attractor basin (Guastello & Liebovitch, 2009). Within this basin, the variability of observations over time could gravitate toward a fixed point, display cyclical patterns within this basin, or show more-complex patterns. Hence, these three scenarios are referred to, respectively, as fixed point attractors limit cycle and strange or chaotic attractors (Abraham, Abraham, & Shaw, 1990). The dynamical systems literature contains a wide array of complex attractor structures, which are interesting but beyond the scope of this paper. Abraham, Abraham, and Shaw (1990) offer a lucid overview with some vivid illustrations. To examine the behavior of dynamical systems in education, it is productive to define the coordinates of the attractor basin substantively, so that the dynamical process and the attractor structure uncovered can be used to address specific substantive questions.

Pennings and Mainhard (2016) use an approach called state space grids (Hollenstein, 2007, 2013) to identify the attractors underlying teacher behavior and student perceptions of that behavior, measured based on interpersonal theory (Wubbels et al., 2006), which defines two dimensions for characterizing teacher classroom behavior over time: agency, the degree to which
the teacher is individuated, dominant and controlling in the classroom, and communion, the degree to which the teacher is engaged and showing affiliation with the student. Over 600 behavioral instances were coded according to these two dimensions within ten-minute periods of teacher-student interactions, thus, yielding a grid characterizing the underlying attractor structure of the interaction measurements (see Pennings & Mainhard, 2016, p. 262). Finding such attractors is relevant because they tell us to what combination of behavioral attributes teachers tend to gravitate in the course of their interactions with students. Such analyses also give rise to such interesting questions as the extent to which teachers tend to have similar attractor structures underlying their interactive behavior and if not what the determinants are of the differences. State space grids have also been successfully used to describe the attractor dynamics in the range of expressed emotions in the interaction between sixth-grade girls and their mothers interacting under different stress conditions (Hollenstein & Lewis, 2006) and to characterize the relationship between the angle of infants’ gaze and the intensity of distress in lab-induced separation-reunion situations (Lewis et al., 1999).

**Catastrophe theory and other qualitative transformations**

Nicolis and Prigogine (1989) provide an extensive overview of the mathematical expression of processes of gradual and qualitative transformation, which all require a quantitative definition of qualitative information. Catastrophe theory (Thom, 1975) conceptualizes such transformations in terms of a set of mathematical definitions of systemic equilibrium behavior in the face of mathematically impossible regions (e.g., a denominator of zero) in the coordinate space, creating conditions for local turbulence in the system (Gilmore, 1981; Stamovlasis, 2016b). In its original formulation, catastrophe theory consisted of a set of seven deterministic models (Thom, 1975), modeling the conditions of such turbulence. The most well-known of these is the cusp catastrophe. The other six are very rarely considered in the social sciences and therefore beyond the scope of this paper (but see Gilmore, 1981, and Guastello, 1982).

The cusp is a mathematical function that models changes based on two parameters, an asymptotic parameter predicting discontinuity in an outcome plane and a bifurcation parameter describing a splitting pattern in the distribution of outcomes from unimodal to bimodal, as the impossible region is approached (see Stamovlasis, 2016b, p. 150). As part of the same set of functions, cusps also model delay effects (hysteresis), a process wherein gradual changes in the asymptotic variable yield a delayed response in the outcome surface in the form of a sudden jump or fall. The substantive interest in cusps lies in the fact that it models the nature of changes in the system as gradual or qualitative, depending on the values of the bifurcation parameter.

One area in education in which cusp catastrophe modeling has been particularly productive is the mathematical description of Piagetian and neo-Piagetian stage-wise intellectual development in children and young adults as a basis for a more general theory of development (Stamovlasis, 2016b). In these models, the transition from one stage of intellectual development to the next is modeled as a cusp catastrophe, in the sense that qualitative transformation in behavioral outcomes that can be predicted based on the extent to which the developing child refrains from using irrelevant problem-solving strategies. The child’s increased reliance on relevant strategies marks a stage-wise transition—for instance, from concrete operations to reflective abstraction (van der Maas & Molenaar, 1992). The value of applications such as these lies in the fact that it provides a mathematical basis for a well-known theory of developmental processes and helps attain a deeper understanding of the underlying dynamics of cognitive changes due to learning, development, and the ongoing interaction between the organism and the environment (see also van Geert, 1998).

The application of cusp catastrophes to education includes the modeling of arousal levels of college students in high-achievement situations in terms of subjects’ inclinations toward approach
and avoidance of task difficulty (Stamovlasis & Sideridis, 2014) and the prediction of science achievement in college freshmen based on their information processing capacity and degree of field independence—that is, their ability to disembed information (Stamovlasis, 2006, p. 45 shows the cusp of this relationship). In the former instance arousal levels in high achievement situations are strongly dependent on subjects’ avoidance tendencies. In the latter case, qualitative change is predicated on high levels of cognitive field dependence, in which case outcomes for subjects with low and high working memory capacity are clearly discrepant.

Catastrophe theory is but one of many mathematical models of complex transformation, not all of which require us to assume such things as mathematically unacceptable regions and bifurcations to explain qualitative transformations. Bassano and van Geert (2007) investigated the transition in a child from producing simple one-word to more complex four-word sentences in the course of her development from 14 through 37 months. Consistent with what CDS theory would predict, as the child transitioned from one to two-word, from two to three-word, and from three to four-word sentences, high degrees of variability are observed in the word count; whereas, once the girl settles into a more complex pattern, this variability diminishes.

**Conclusion**

As has been the case for the complexity literature in other disciplines, there is a growing need in the field of education to move from the broad generality of the question of whether CDS is applicable to the more specific but less glamorous task of building empirical models that can be tested, refuted, and replicated (Koopmans & Stamovlasis, 2016; Stamovlasis & Koopmans, 2014). As complexity researchers in education and elsewhere encounter specific challenges in their attempts to address complex questions through research, there will be an ongoing development of methodologies to capture such dynamical processes in specific settings and under specific conditions. The methodologies discussed here are a small sample of an ever-growing repertoire of approaches to investigate the underlying dynamics of stability and transformation. Insofar as education is concerned, we are still at the early stages of this work. However, given the cohesion of CDS as a conceptual framework, the alignment of our research methodologies with it is a promising road forward, as it has been in other disciplines.

Considerable progress has been made toward those aims in child developmental research as it interconnects with the education field, and we now have good conceptualizations of Piaget and Vygotsky’s theories as examples of CDS (van Geert, 1998). New developmental theories have been developed that are explicitly based on the principles of CDS, such as dynamical skill theory (Fischer & Bidell, 1997). Likewise, productive research is taking place in other related fields, such as language acquisition (Bassano & van Geert, 2007; de Bot et al., 2007; Larsen-Freeman, 2015) and motor development (Tani et al., 2014; Thelen & Smith, 1996). In addition, there is a burgeoning tradition of research in the areas of educational leadership and school reform (Fleener, 2016; Goldspink, 2007). Furthermore, the constructive use of state space grids and orbital decomposition have yielded important additional detail about the interaction between students and teachers and students amongst themselves, such as the underlying attractor structure to the degree of individuation and engagement in the interaction between teachers and students. We now also have a set of mathematically grounded models of transformation, such as catastrophe theory and the fractal patterning of repeated observations. The confirmation of these models can form the basis for subsequent work.

We need research to supplement sweeping generalizations about behavior based on the statistical relationship between variables with more-detailed idiographic analyses that looks in greater detail at the behavior of individuals (Molenaar, 2004) and their unique circumstances (van Geert, 2011). Such an approach should have resonance in the field of education, where the interactive exchanges between teachers and learners are inherently particular (Passmore, 1980). Hopefully, a
renewed interest in the analysis of the single case, coupled with the development of increasingly sophisticated methods of analyzing within-subject data can serve as an antidote to the chronic loss of information that comes with our habitual focus on the use of aggregated measures for statistical inference (Molenaar, 2004).

In the end, however, the outlook of any research is decided by a problem-based methodology instead of a methodology-based problem formulation. Our inclination to do otherwise extends, of course, from the world of linear policy research to the world of complexity, wherein the urge to demonstrate the existence of such things as complexity, self-organized criticality, and chaos may overshadow more-substantive concerns about how complex processes can help us better understand how the world works—education in particular. The increased availability of methodologies that are specific to the detection and analysis of nonlinear complex processes may facilitate this endeavor, and put CDS research effectively into the service of the search for a deeper understanding of the dynamics underlying the educational process in all its possible expressions.

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