In this paper, we study the stability of a high speed milling process of nickel superalloys Inconel 713C by methods used in nonlinear dynamics. Stability Lobe Diagram was a result of modal analysis and next verified by recurrence plots, recurrence quantification analysis and classical nonlinear methods. A stability lobes diagram shows the indistinct boundary between chatter-free stable machining and unstable processes. Nevertheless, some recurrence quantification analysis measures give interesting results.

Keywords: Stability lobes; milling process; recurrence plot and quantifications.

1. Introduction

In recent years, manufacturing techniques have progressed rapidly, and machine tools have become more integrated so that they are required to exhibit high productivity and economy in conventional workshops as well as in flexible manufacturing systems. The cutting process is still a very important manufacturing method, therefore new and effective technologies for new constructional materials are nowadays studied. High-speed milling is one of the most preferred and efficient cutting processes. The nickel based superalloys are a special group of materials for modern aircraft, where they are used for extensively high temperature elements in planes (especially in jet turbine engines). They characterize by exhibiting high strength, good fatigue and creep resistance, good corrosion resistance, and the ability to operate at elevated temperatures for extended periods of time. Common trade names for nickel superalloys are: Inconel (often referred as Inco), Nimonic, Rene, Hastelloy, Udiment.

Nickel alloy (specially Inconel) is a difficult metal to shape and machines use traditional techniques due to rapid workpiece hardening. After the first machining pass, a workpiece hardens causing elastic deformation either in a workpiece or faster tool wear occurs on subsequent passes. For this reason, age-hardened Inconels such as 713C are machined using relatively small speeds and hard tool. The productivity of machining is often limited by vibrations that arise during the cutting process. These vibrations cause poor surface finish, increase the rate of tool wear, and reduce spindle lifetime. Therefore, it is a challenging task for researchers to explore its special dynamical properties, including the stability conditions of the cutting process and the nonlinear behavior that may occur near the stability boundaries.

Stability conditions of machining process are visualized by stability charts (called stability lobe diagram SLD) which plot the maximum chatter-free chip width or depth of cut as a function...
of spindle speed. Theoretical stability analysis was done by many researchers, who use a special theoretical model [Insperger & Stepan, 2000; Insperger et al., 2003a, 2003b; Fofana, 2002]. Experimental verification of these models is very difficult because of disturbances coming from the environment and a complicated nonlinear system: miller-tool-workpiece [Mann et al., 2003; Bayly et al., 2001]. Additionally, after several passes of tool cut, the accuracy of chatter stability diminishes due to friction between the tool flank and finish surface.

The stability charts published in the scientific literature are commonly accompanied by frequency diagrams illustrating chatter frequencies at the loss of stability [Stepan, 1989; Insperger et al., 2003a, 2003b]. Recent development in sensors and computers has led to the rise of analytical–experimental methods and analyses via computer simulations. The paper [Altintas, 2000] explains a widely known analytical–experimental method based on the modal analysis where an impact hammer instrumented with a piezoelectric force transducer hits on static mill-tool holder system. The system response as a transfer function is obtained using CutPro software.

This paper focuses on experimentally based results of milling forces during cutting of nickel alloy–Inconel 713C for various technological parameters. Chosen techniques of nonlinear dynamics are proposed to determine the process properties, especially its stability. We conducted an analysis of the SLD designated using modal analysis by commercial software CutPro. Detailed investigations of stability lobes are performed by recurrence plots analysis (RP), recurrence quantifications analysis (RQA), reconstruction of phase space and calculating the Lyapunov and Hurst exponents.

2. Experimental Setup

The measurements are conducted on the experimental setup, presented schematically in Fig. 1. The experimental system is composed of the numerically controlled milling machine DMU 80P duoBlock Deckel Maho, the piezoelectric rotating four-component dynamometer Kistler 9123C, piezoelectric conditioner Kistler 5134 and the analog–digital converter NI 6062E by National Instruments. The rotating four-component dynamometer is used for measuring cutting forces (F_x, F_y and F_z) and torque (M_z) on the rotating tool spindle. The dynamometer consists of a four-component sensor fitted under high preload between a baseplate and top plate. This dynamometer is especially suitable for high speed machining processes.

For modal analysis, the hammer PCB model 086C03 which has 8kHz frequency range and 10 mV/lbf sensitivity and, the two-axes piezoelectric accelerometers 352C43 are used. The gauging point of modal hammer is the tool tip. The modal impact hammer delivers impulse forces into the tested structure. Hammer model selection involves determining the size and mass of the hammer, which provide the force amplitude and frequency.
content required for proper excitation of the structure under the test.

In experiment, the end milling cutter (ECH060 B16-6C06) made of solid carbide with diameter of 6 mm and four cutting edges is applied. This tool is designed for the finishing of hard materials, up to 65HRC and high temperature alloys. The sampling rate of data reordered during the test equals 3 kHz.

3. Recurrence Plot and Qualification Analysis

The essential problem in nonlinear time series analysis is to determine whether a given experimental data series is a deterministic or chaotic signal and to measure the complexity of the data. Results estimated from one nonlinear method often can lead to erroneous conclusion because of noise and disturbances, usually present in real signals. In this work we verify real experimental milling data series by using classical nonlinear methods such as Lyapunov and Hurst exponents, phase space reconstruction and relatively new methods like recurrence plot and recurrence quantifications. Due to the existence of excellent papers on nonlinear time series analysis, algorithms of classical methods are not described here. Detailed description of these methods can be found in monographs [Abarbanel, 1996; Kantz & Schreiber, 1997; Sprott, 2003] and paper [Rosenstein et al., 1993].

The standard procedure to perform an analysis is the phase space reconstruction based on the delay coordinates method. A single coordinate in nonlinear time series can be substituted by a specific vector. The corresponding vector elements are defined by the same coordinate with a certain time delay. For the scalar series \( x_i \), the delay vectors are constructed

\[
\mathbf{S}_i = (x_i, x_{i+d}, x_{i+2d}, \ldots, x_{i+(m-1)d})
\]

where parameter \( m \) is the embedding dimension and parameter \( d \) is the time delay. Each unknown point of the phase space at time instant \( i \) is reconstructed by the delayed vector \( \mathbf{s}_i \) in an \( m \)-dimensional space called “the reconstructed phase space”. This vector is useful only if parameters \( m \) and \( d \) are properly chosen by using appropriate methods. Usually the time delay and embedding dimension can be estimated by applying average mutual information (AMI) [Fraser & Swinney, 1986] and false nearest neighbors method (FNN) [Kennel et al., 1992].

The identification of FNN is a popular method of choosing the minimum embedding dimension of a time series for the purpose of phase space reconstruction. The nearest neighbor of every point in a given dimension is searched, next it is checked if there are still close neighbors of one higher dimension. To choose the appropriate time delay, users can compute the average mutual information function. The time delay should be chosen such that the elements in embedding vectors are no longer correlated, thus subsequent analysis would reveal spatial or geometrical structures.

The recurrence analysis is a graphical method designed to locate hidden recurring patterns, nonstationarity and structural changes, introduced by [Eckmann et al., 1987]. A Recurrence Plot (RP) is a graph which shows all those time instants at which a state of the dynamical system recurs. In other words, the RP method reveals the whole times when the phase space trajectory visits roughly the same area in the phase space. A recurrence plot can be described by computing the matrix

\[
M_{ij} = \theta(\varepsilon - |s_i - s_j|)
\]

where \( \theta \) is the Heaviside step function, \( \varepsilon \) is a tolerance parameter (threshold), \( s_i \) is a delay vector of the embedding dimension. This matrix is symmetric by construction. If the trajectory in the reconstructed phase space returns at time \( i \) into the neighborhood of \( \varepsilon \) where it was \( j \) then \( M_{ij} = 1 \), otherwise \( M_{ij} = 0 \). These results are plotted as black and white dots, respectively. Value of chosen parameter \( \varepsilon \) is very important. If \( \varepsilon \) is too small, there may be almost no recurrence points and we cannot learn anything about the recurrence structure of the considered system. On the other hand, if \( \varepsilon \) is chosen too large, almost every point is a neighbor of every other point, that leads to a big number of artificial points.

The Recurrence Quantification Analysis (RQA) is a method of nonlinear data analysis which quantifies the number and duration of recurrences of a dynamical system presented by its state space trajectory. The measures introduced for the RQA were developed in [Zbilut & Webber, 1992; Webber & Zbilut, 1994; Marwan, 2003]. The main advantage of the recurrence quantification analysis is that it can provide useful information even for short and nonstationary data, where other methods fail. RQA can be applied to almost every kind of data [Litak et al., 2010]. The most important RQA measures...
are [Marwan et al., 2007]:

- Recurrence Rate (RR), which is the density of recurrence points in a recurrence plot. The physical meaning of RR is the probability that the system will recur

$$RR = \frac{1}{N} \sum_{i,j=1}^{N} M_{i,j}. \quad (3)$$

- Determinism or Predictability (DET), which is the fraction of recurrence points forming diagonal lines. Diagonal lines represent epochs of similar time evolution of states of the system

$$DET = \frac{\sum_{l=l_{\text{min}}}^{l_{\text{max}}} lP(l)}{\sum_{l=1}^{l_{\text{max}}} lP(l)}. \quad (4)$$

- Laminarity (LAM), which is the fraction of recurrence points forming vertical lines. Vertical lines are typical for intermittency. Therefore, LAM is related to the amount of laminar states in the system

$$LAM = \frac{\sum_{v=v_{\text{min}}}^{v_{\text{max}}} vP(v)}{\sum_{v=1}^{v_{\text{max}}} P(v)}. \quad (5)$$

- Trapping time (TT), which is the mean length of vertical lines. TT measures the mean time that the system is trapped in one state or change only very slowly

$$TT = \frac{\sum_{v=v_{\text{min}}}^{v_{\text{max}}} vP(v)}{\sum_{v=1}^{v_{\text{max}}} P(v)}. \quad (6)$$

In the above frame, RQA provides us with the probability $p(l)$ or $p(v)$ of line distribution according to their lengths in diagonal ($l$) or vertical ($v$) lines. $N$ is number of points on phase space trajectory. The probability is calculated as follows:

$$p(x) = \frac{P(x)}{\sum_{x=x_{\text{min}}}^{x_{\text{max}}} P(x)}. \quad (7)$$

where $x = l$ or $v$ depending on vertical or diagonal structures in the recurrence plot. However, $P(x)$ denotes histogram of $x$ and a fixed value of $\varepsilon$. Based on the probability $p(x)$ the Shannon entropies ($L_{\text{ENT}}, V_{\text{ENT}}$) are defined

$$L_{\text{ENT}} = - \sum_{l=1}^{l_{\text{max}}} p(l) \ln p(l), \quad (8)$$

$$V_{\text{ENT}} = - \sum_{v=1}^{v_{\text{max}}} p(v) \ln p(v). \quad (9)$$

The length of the longest diagonal line can be calculated:

$$L_{\text{max}} = \text{max} \{l_i; i = 1, \ldots, N_l\}. \quad (10)$$

the length of the longest vertical line

$$V_{\text{max}} = \text{max} \{v_i; i = 1, \ldots, N_v\}. \quad (11)$$

The inverse of $L_{\text{max}}$ (Divergence $DIV$) is related to the positive Lyapunov exponents [Marwan et al., 2007].

4. Stability Lobes Diagram and Cutting Forces

The Stability Lobes Diagram (SLD) determined by modal analysis using commercial CutPro 9 module is presented in Fig. 2. This software automatically and accurately identifies the structural dynamic parameters of a machine from Frequency Response Function (FRF). Accurate prediction of natural frequency, damping ratio, residues, modal stiffness and mass from FRF let us identify machine and its

![Fig. 2. Stability Lobe Diagram (SLD) of milling process determined for Inconel 713C.](image-url)
parameters which are crucial in predicting chatter vibration and stable lobes. The stability lobe diagram generated by CutPro 9 incorporates such features as: the length and diameter of the tool, number of cutting edges, tool/toolholder interface, toolholder style, toolholder/spindle interface, size, material and arrangement of the spindle bearings, maximum spindle speed, and torque.

The cutting conditions above blue line areas (Fig. 2) should produce chatter, which can be characterized by heavy vibration, tool damage, poor surface quality and scrapped workpieces. The conditions below blue line are stable without chatter.

The red points marked at SLD represent parameters where the cutting forces $F_x$, $F_y$ and $F_z$ are measured. It may be noted that points denoted as 14, 16 and probably 13 (located on the border) are placed in unstable region, where chatter exists. Analysis of the points will be carried out later in this work. Milling at cutting depth below 2 mm should be stable all time regardless of spindle speed. Thus, the unstable lobes are dangerous only for higher depth of cut.

Figure 3(a) shows the cutting forces $F_x$ versus depth of cut relationship and (b) the ratio of amplitude to average value of $F_x$.

Further analysis of the point stability is done with the help of Chaos Data Analyzer software which let us calculate the largest Lyapunov and Hurst exponents [Figs. 4(a) and 4(b) respectively]. The Lyapunov exponent is calculated by using the algorithm of [Rosenstein et al., 1993]. Hurst exponent is applied to evaluate a persistence of time series in long-range dependence. The Hurst exponent can also be interpreted as a measure of the smoothness of a fractal time series based on the asymptotic behavior of the rescaled range of the process.

Analyzing the value of Lyapunov exponent for cutting speed of 2450 rpm can be noted, that for depth of cut 2 mm and 2.5 mm (unstable points 14 and 16 in SLD, Fig. 2) it achieves the highest positive value (about 0.08) compared to points placed before (0.5 mm and 1.0 mm). For easier identification of these points they are marked with circle. Other value of the Lyapunov exponent are also positive but two times lower. For cutting speed 1714 rpm value of Lyapunov exponents are very similar to each other for all depth of cut. Point no. 13 (cutting speed 1714 rpm and 2.5 mm depth of cut) has positive value, but not the highest because it is placed just on the border line. Therefore, this Lyapunov exponent criterion seems to be quite good to distinguish stable and unstable cutting processes.

Interestingly, the Hurst exponent has larger value for smaller cutting speeds. It proves that small speeds give more regular times series. The Hurst exponent value ($H$) between 0 and 0.5 exists for time series with "anti-persistent behavior".
This behavior is sometimes called "mean reversion" which means the future values will have a tendency to return to a longer term mean value. Value between 0.5 and 1 indicates "persistent behavior", that the time series is trending. The larger the $H$ value is, the stronger the trend. Series of this type are easier to predict. The Hurst Exponent value close to 0.5 indicates a random data (a Brownian time series). The points 14 and 16 by Hurst exponent for 2450 rpm are closer to 0.5 than other points.

5. Analysis of Unstable Points by Recurrence Diagram

In order to analyze the dynamics of the process, the signals of cutting forces are subjected to methodological approach by a study of the recurrence plots (RP), used in nonlinear mechanics for a nonlinear signal analysis, also for composite material cutting [Rusinek, 2010]. In Fig. 5 we can see the recurrence plot of the stable point 17 (on the left) and unstable $−13$ (on the right), for parameters: cutting depth $1.5 \text{ mm}$ and $2.5 \text{ mm}$ respectively and cutting speed $1714 \text{ rpm}$. The vertical distance between these lines corresponds to the period of the oscillation. The recurrence diagrams is done using 2000 data points. Diagonal lines clearly show that signal of cutting force $F_x$ is periodic (Fig. 5 on the left). However, in Fig. 5 on the right, we observe diagonal dashed line, especially in the first part of analyzed diagram. This shows that received experimental signals are located on the boundary of the stable and unstable processes (lie on the blue border line in SLD).

The reconstruction of phase space Figs. 6(a) and 6(b) is done by Tisean package [Hegger & Kantz, 1999]. This software is suitable for nonlinear time series analysis. The result shows that both signals are periodic. Interestingly, that phase space consist of “circles”, which probably result from four cutting edges of end mill. This phenomenon is clearly visible in Fig. 6(a), while in Fig. 6(b) this effect is small compared to the attractor size.

Additionally we calculated the maximal Lyapunov exponent of the system, using the algorithm developed independently by [Wolf, 1985]. The algorithm tests the exponential divergence of nearby trajectories directly and thus allows a robust estimation of the maximal Lyapunov exponent. In case of point 17 [Fig. 7(a)] we observe the Lyapunov exponent tends to negative value. But, for point 13 Lyapunov exponent is very close to zero. This, can suggest that we have chosen cutting parameters which are on the boundary stability lobes diagram. These results agree with the recurrence diagram.

In Fig. 8 we observe recurrence diagram of point 18 (on the left) and 14 (on the right), for cutting depth $1.5 \text{ mm}$ and $2.5 \text{ mm}$ and cutting speed $2450 \text{ rpm}$. Comparing both results, we can conclude that recurrence plot of point no. 14 consist of diagonal lines as well as separate points. This may be the result of vibration type. The reconstructed phase space is much more complicated [Fig. 9(a)] than reconstructed phase space of point 18 [Fig. 9(b)]. The cutting force is larger comparing to point located in stable area of SLD [Fig. 9(a)].

The Lyapunov exponent calculated for point no. 18 is nearly zero, but in the case of point 14, it
Fig. 5. (a) Recurrence plot of point 17, the time delay $d = 6$, the embedding $m = 5$ and threshold $\varepsilon = 30$ and (b) recurrence plot of point 13, the time delay $d = 13$, the embedding $m = 6$ and threshold $\varepsilon = 30$.

Fig. 6. Reconstruction of phase portrait of (a) point 17, the embedding $m = 5$ and threshold $\varepsilon = 30$ and (b) point 13, the time delay $d = 13$, the embedding $m = 6$ and threshold $\varepsilon = 30$. 

Fig. 7. Lyapunov exponent calculating from data series of point (a) 17 and (b) 13 by Wolf algorithm.

Fig. 8. (a) Recurrence plot of point 18, the time delay $d = 4$, the embedding $m = 5$ and threshold $\varepsilon = 30$ and (b) recurrence plot of point 14, the time delay $d = 3$, the embedding $m = 6$ and threshold $\varepsilon = 30$. 
Fig. 9. Reconstruction of phase portrait of (a) point 18, the embedding $m = 5$ and threshold $\varepsilon = 30$ and (b) point 14, the time delay $d = 3$, the embedding $m = 6$ and threshold $\varepsilon = 30$.

Fig. 10. Lyapunov exponent calculating from data series for point (a) 18 and (b) 14 by Wolf algorithm.
6. Recurrence Quantifications
Analysis
In order to go beyond the visual impression yielded by RPs, several measures of complexity which are known as “Recurrence Quantification Analysis” (RQA) are calculated. These measures are based on the recurrence point density and the diagonal and vertical lines of RPs. In Table 1 the results or recurrence quantifications for all points of stability lobes diagram are presented.

Different line structures can be associated with different values of parameter RR. Note, that parameter RR indicates the fraction of recurrence for the cutting force $F_x$. RR quantifier of points 13, 14 and 23 is much less compared to other points. The quantifications DIV in case of points 13, 14 and 23 (13, 14 are unstable points in SLD) has value much higher than the other remaining points. Therefore, this parameter can be used to identify the kind of motion. The same information give quantifications $L_{max}$ which achieve much lower value. These quantifications are the best for analyzing the unstable behavior. Interestingly, that point 23 is also unstable while it is placed on the stable region in SLD. The reason for this situation is nonstationary time series of force $F_x$ coming from tool wear. This trial (point 23) was completed with Catastrophic Tool Damage (CTD). The effect of CTD will be considered a bit later.

Table 1. Results of Quantification Analysis (RQA) for $\varepsilon = 30$ and 2000 data points.

<table>
<thead>
<tr>
<th>Point Number</th>
<th>RR</th>
<th>DET</th>
<th>LAM</th>
<th>TT</th>
<th>$\ell_{ENTR}$</th>
<th>$V_{ENTR}$</th>
<th>$V_{max}$</th>
<th>$L_{max}$</th>
<th>DIV</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>0.0035</td>
<td>0.5997</td>
<td>0.2717</td>
<td>2.0062</td>
<td>1.5490</td>
<td>0.2509</td>
<td>5</td>
<td>52</td>
<td>0.0192</td>
</tr>
<tr>
<td>14</td>
<td>0.0027</td>
<td>0.5559</td>
<td>0.1119</td>
<td>2.0086</td>
<td>0.9901</td>
<td>0.0496</td>
<td>3</td>
<td>12</td>
<td>0.0033</td>
</tr>
<tr>
<td>15</td>
<td>0.0019</td>
<td>0.8734</td>
<td>0.8982</td>
<td>2.4147</td>
<td>1.9169</td>
<td>0.7732</td>
<td>5</td>
<td>463</td>
<td>0.0021</td>
</tr>
<tr>
<td>16</td>
<td>0.0162</td>
<td>0.7083</td>
<td>0.5728</td>
<td>2.1904</td>
<td>1.5347</td>
<td>0.5073</td>
<td>4</td>
<td>222</td>
<td>0.0045</td>
</tr>
<tr>
<td>17</td>
<td>0.0016</td>
<td>0.8474</td>
<td>0.7521</td>
<td>2.1357</td>
<td>1.6755</td>
<td>0.4031</td>
<td>4</td>
<td>1402</td>
<td>0.0007</td>
</tr>
<tr>
<td>18</td>
<td>0.0121</td>
<td>0.7548</td>
<td>0.4407</td>
<td>2.0479</td>
<td>1.3774</td>
<td>0.1898</td>
<td>3</td>
<td>205</td>
<td>0.0048</td>
</tr>
<tr>
<td>19</td>
<td>0.0230</td>
<td>0.7777</td>
<td>0.8915</td>
<td>2.6743</td>
<td>1.5603</td>
<td>1.0147</td>
<td>6</td>
<td>355</td>
<td>0.0028</td>
</tr>
<tr>
<td>20</td>
<td>0.0173</td>
<td>0.6905</td>
<td>0.6073</td>
<td>2.1804</td>
<td>1.6156</td>
<td>0.6873</td>
<td>4</td>
<td>339</td>
<td>0.0029</td>
</tr>
<tr>
<td>21</td>
<td>0.0150</td>
<td>0.7379</td>
<td>0.6433</td>
<td>2.5738</td>
<td>1.4671</td>
<td>1.0123</td>
<td>7</td>
<td>398</td>
<td>0.0025</td>
</tr>
<tr>
<td>22</td>
<td>0.0138</td>
<td>0.5662</td>
<td>0.2829</td>
<td>2.0874</td>
<td>1.6179</td>
<td>0.3040</td>
<td>5</td>
<td>185</td>
<td>0.0054</td>
</tr>
<tr>
<td>23</td>
<td>0.0049</td>
<td>0.5663</td>
<td>0.1167</td>
<td>2.0024</td>
<td>1.2461</td>
<td>0.2157</td>
<td>3</td>
<td>34</td>
<td>0.0294</td>
</tr>
</tbody>
</table>

Fig. 11. Quantifications versus depth of cut for cutting speed (a) 1714 rpm and (b) 2450 rpm.
In Fig. 11 we observe the dependence of quantifications versus depth of cut for two cutting speeds: 1714 rpm [Fig. 11(a)] and 2450 rpm [Fig. 11(b)]. Interestingly all quantifications have practically the same behavior. For depth of cut 2.5 mm all quantifications suddenly reduce their value. But for depth of cut from 0.5 mm and 1.5 mm all value of quantifications are nearly the same. Cutting speed of milling process generally does not change the value of quantifications if not coming from the unstable area of SLD.

Figure 12 shows an influence of catastrophic tool damage on quantifiers of RP. CTD was in the tenth time interval of the whole recorded time series then every RQA measure demonstrates a sudden jump. Additionally, CTD is very clearly observable as a black square in the corner of the recurrence plot (Fig. 13).

7. Conclusions

This paper focuses on the verification of the milling process stability in the case of Inconel 713C. Process stability is analyzed by recurrence plot, recurrence quantifications and another classical methods engaged in nonlinear dynamics. Stability Lobes Diagram (SLD) determined by commercial software CutPro reveals stable and unstable points...
during milling process which are practically hard to distinguish by looking only on the force time histories and the magnitude. Therefore, the analysis of unstable points is extended to more advanced tools. The Lyapunov exponents have slightly higher positive value and patterns of RP differ a bit in comparison with stable points. Very important advantage of RP and RQA method is a possibility of analysis for a short time series application. RP and RQA methods can also be used for damage detection of cutting tool or dull of tool. RQA analysis shows that the best parameters to classify the kind of motion and stability border are DIV and $L_{\text{max}}$.

In future work, we would like to confirm RQA results for other superalloys and introduce our own original quantifications which let us exactly distinguish unstable lobes.

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