The ADHD effect on the high-dimensional phase space trajectories of EEG signals

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\textbf{A R T I C L E   I N F O}

Article history:
Received 30 September 2018
Revised 8 February 2019
Accepted 10 February 2019
Available online 19 February 2019

Keywords:
ADHD
EEG
Recurrence plot
Complexity

\textbf{A B S T R A C T}

Attention-deficit/hyperactivity disorder (ADHD) as a behavioral challenge, which affects the people’s learning and experiences, is one of the disorders, which leads to reducing the complexity of brain processes and human behaviors. Nevertheless, recent studies often focused on the effects of this disorder in the frequency content of single and multichannel EEG segments and only a few studies that employed the approximate entropy for estimating this reduction. In this study, we provide a different view of this reduction by focusing on the texture of patterns appeared on the auto-recurrence plots obtained from the phase space trajectories reconstructed from the EEG signals recorded under the open-eyes and closed-eyes resting conditions. The outcomes of this analysis generally indicated a significant difference in the texture of recurrence plots, which its reason was the increase of recurrence, parallel and similar behaviors in the trajectories. Evaluating the features extracted from these recurrence plots in the studied children without and with ADHD using the sequential forward selection (SFS) algorithm provided a remarkable accuracy (90.95% for the testing sets), which is a confirmation on changing the texture of recurrence plots relevant to the EEG signals of ADHD children. Nevertheless, evaluating these results and the results of previous researches with each other represented that the volume of statistical population is an important factor for reducing the rate of separability in the classifiers developed by an EEG segment. Therefore, these findings generally proved that although the ADHD averagely leads to the complexity reduction of EEG processes, the classifiers developed by just an EEG segment cannot be applicable in clinical conditions.

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1. Introduction

Attention deficit/hyperactivity disorder (ADHD) is a neurobehavioral developmental disorder \cite{1,2}, which changes the interaction style of brain with the environment \cite{2,3,4}, so that a person with this disorder often experiences misbehaviors such as hyperactivity, impulsivity, and inattention \cite{2,5,6}. Hence, this disorder, which often involves nearly 5\% of the adults \cite{7,8} and 3\%-9\% of school-aged children worldwide \cite{8,9}, is one of childhood behavioral challenges that can endanger the future of the child due to decreasing the child’s adventurous and experimental resources, although 40\%-60\% of these children can naturally reorganize their brain at over time for reducing their ADHD symptoms \cite{2,10,11}. Such conditions, i.e. the significant prevalence rate of attention deficit/hyperactivity disorder and the creativity and self-organizing of brain in reducing the symptoms of this disorder, have accordingly caused that researchers to provide different methods for evaluating and improving the misbehaviors resulted from the ADHD. It is remarkable that the influence of this disorder on the brain activities has also caused that a large part of studies on the ADHD focuses on processes recorded from the brain, especially the EEG signals. This part of studies is usually categorized as three groups \cite{2}. First group that examined the influence of ADHD symptoms on the event-related potentials (ERP) using continuous performance tests and reported a different level of activity on the parietal and frontal lobes \cite{12,14}. Second group that investigated the ADHD effects on the slow cortex potential (SCP), and expressed that the contingent negative variation (CNV) in the ADHD children was lower than that of healthy children \cite{15,16} and Last group in the data mining \cite{17,18} and neurofeedback \cite{20,21} that used the EEG signal, and revealed that the hyperactivity and impulsivity of these children increase the power of $\delta$ and $\theta$ bands and their inattention decreases the power of $\alpha$ and $\beta$ bands. There are also works \cite{20,21,22,23,24} that relied on a neurofeedback course for improving the ADHD symptoms, i.e. hyperactivity, impulsivity and inattention. Pharmaceutical studies on these children also reported

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https://doi.org/10.1016/j.chaos.2019.02.004
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a significant latency at the P200, N200 and P300 of ERP wave [25], which represents the influence of misbehaviors internalized in these children on the attention (alerting, orienting and conflict [26,27]) and sensorimotor networks.

These negative and positive changes in the behavioral indicators and the indictors obtained from the EEG signals not only accordingly prove that the ADHD can create the behaviors by changing the brain activities, which have lesser complexity compared to that of healthy children, but also they show that the brain as a self-organizing and creativity system [2] can control the ADHD by changing the dynamic range of its activities. The brain with changing its dynamic range actually leads to a decrease in the power of $\alpha$ and $\beta$ bands, and an increase in the power of $\delta$ and $\theta$ bands, which is also visible in the baseline EEG signal of ADHD children. Interestingly, such different frequency content in the standard EEG bands according to the Lipsitz and Goldberger studies [28,29], which considered the loss of complexity in the physiologic processes resulted from relative frequency reduction in the high-frequency components and corresponding increase in the relative contribution of lower-frequency components, means decreasing the complexity of brain activities in the ADHD children. In contrast, improving the ADHD symptoms at over time or a neurofeedback course according to the Vaillancourt and Newell studies [30], which considered the observed changes in the complexity of systems dependent on the intrinsic dynamic of system and the changes required to realize a demand local task, means returning the complexity of brain activities in the ADHD children to the normal range due to the ability of self-organizing and creativity in the brain. Therefore, these conditions in the brain activities of ADHD children generally indicate that the ADHD usually reduces the complexity of brain processes for creating the simpler behaviors, i.e. hyperactivity, impulsivity, and inattention.

Accordingly, some researchers have applied different techniques from the information and chaos theory such as entropy and fractal dimension to quantify the ADHD effect on the EEG signals. For example, Khoshnoud and his colleagues [31] applied the distribution of approximate entropy for estimating the complexity of baseline EEG signals, and demonstrated that the irreversibility of the EEG signals in the ADHD children is averagely less than that of the healthy children. Mohammad and his colleagues [32] with analyzing the non-linear characteristics of EEG signals recorded during cognitive visual stimulations also represented that the ADHD reduces the entropy and fractal dimension of EEG signals during the cognitive visual stimulation. There are also classification studies [31–34] that recommend the combination of the entropy with other nonlinear features for separating the ADHD from healthy children. Nevertheless, although these researches have obtained fairly good results, they usually provided a limited amount of information about the complexity of brain activities, because they often investigated the complexity using the entropy and fractal dimension of EEG segments. In this study, we accordingly employed the recurrence plot for evaluating the complexity of high-dimensional phase space trajectories reconstructed from the EEG signals relevant to children with and without ADHD. Since, the EEG is a biotic process [2,35] that the brain as a self-organizing and creativity system generates it, we according to Sabelli’s studies [35], which emphasized on the self-organizing and creativity of biological system, finally focused on the size effect of statistical population in the classification accuracy and evaluated the ADHD effect on the recurrent rate of recurrence plots obtained from the successive EEG segments.

The remainder of this paper is organized as follows: Sections 2.1–2.3 present the procedure of data acquisition. Section 3 represents the effect of ADHD on the auto-recurrence plots obtained from the EEG signals. Section 4 provides features extracted from the auto-recurrence plots to quantify the effect of ADHD on the brain activities. Section 5 illustrates experimental results, and finally, Sections 6 and 7 present the discussion and conclusion.

2. Materials and methods

2.1. Subjects

Our analyses were made on two children groups, which had an age range between 7 and 12 years. These groups, which generally consisted of 30 children diagnosed as ADHD and 30 healthy children, had almost similar condition in terms of gender distribution, so that the ADHD group included 11 females and 19 males and the healthy group included 13 females and 17 males.

2.2. Behavioral and cognitive tests

For distinguishing these children and grouping them into two groups: healthy and ADHD, we got assistance from a professional psychiatrist, which used the detailed history of past and current functioning, and the scales of child behavior checklist (CBCL) [36–38] and integrated visual and auditory (IVA) test [39] for this grouping. Accordingly, we asked the mothers of the children to complete a computerized child behavior checklist through the internet at home before evaluating their children, which includes 113 questions with answers as a three-level Likert scale (never, sometimes and always) [40,41]. We also asked these mothers to consider the last six months behaviors of their children in their answers [38]. Then, we evaluated the children performance using the computerized-IVA test. In this test, we actually asked the children to press a special switch once, if they saw or heard the number “1”, and not to press, if they saw or heard the number “2” [42]. All of the children had also done a training stage before examining the major test, which includes a combination of 500 visual and auditory stimulation with a ratio of target to non-target of 5:25:1.

Fig. 1a illustrates the average value of T scores of DSM (the diagnostic and statistical manual of mental disorders) scales extracted from the child behavior checklist for the ADHD and healthy children. As shown in this figure, the DSM scales except the scale of somatic problems were often along with an increase for the ADHD children. Evaluating the separability of these scales, except the scale of somatic problems, in two groups (ADHD and healthy) using the paired sample t-test analysis and also the Wilcoxon rank-sum analysis (Fig. 1b) provided a significant separation ($p < 0.05$ or $−\log(p) > −\log(0.05))$ in the ADHD children under the test [43], which generally reflects the overcoming of behavioral disorders, especially inattention coupled with hyperactivity and impulsivity, in these children. Measuring the scales of continuous performance of ADHD and healthy children using the integrated visual and auditory test also recorded a reduction in the scales relevant to the ADHD children (Fig. 1c) [44,45], which averagely means a reduction in the continuous performance of children with the mentioned disorder. Interestingly, according to the results of t-test and Wilcoxon rank-sum analysis, this reduction in the scales of IVA test was significant ($p < 0.05$) in most scales. Fig. 1d shows these separations as the logarithm of $p$ values obtained from the t-test and the Wilcoxon rank-sum analysis for the IVA scales recorded from the healthy and ADHD children. As shown in this figure, the results of these tests in all of the IVA scales, except the scales of auditory speed (AS), auditory stamina (AS) and visual prudence (VP), had a logarithmic value greater than $1.3 (−\log(0.05))$, which means a significant difference in the frequency distribution of mentioned scales between the ADHD and healthy children. Based on the results of these tests, the full attention quotient (FAQ) also had the highest logarithmic value, which represents the remarkable inattention in the ADHD children.
2.3. Resting electroencephalographic records

After grouping the children into two groups, the resting EEG signals of the Fp1, Fp2, F3, F4, C3, C4, P3, P4, O1, O2, F7, F8, T3, T4, T5, T6, F2, Cz and Pz channels located on the scalp of each child were recorded under the eyes-open and eyes-closed conditions at 240 s according to the 10–20 international system. The average of A1 and A2 electrodes was also considered as the reference value. The specialists of a psychiatric clinic sponsored by the Semnan University had also helped to us in collecting these EEG signals using a Mitsu–EEG–202 system, which its sampling rate and band pass filter were set on 500 Hz (resolution 24 bit) and DC–100 Hz, respectively.

In the preprocessing phase, we also filtered the signals of nineteen recorded EEG channels by a 6th-order low pass Butterworth filter with the 40 Hz cutoff frequency to remove high frequencies and power line noises. Then, we partitioned these EEG signals into segments of 10 s without any overlap. The following subsection provides the ADHD effect on the texture of patterns appeared on the recurrence plots of these EEG segments.

3. The auto recurrence plot in the ADHD children

The EEG signal is generally a homeo–biotic process [2,35], so that its oscillations in the phase space are usually as the sustainable and reversible behaviors with the high recurrence rate. Therefore, according to the Poincaré recurrence theorem [46], we can estimate the amount of complexity of an EEG segment by using the amount and the style of reversibility of trajectories reconstructed from this EEG segment to a certain Poincaré section and area.

The recurrence plot (RP) as one of the techniques of nonlinear analysis introduced by Eckmann and colleagues in 1987 [47], which can estimate the amount and the style of reversibility of phases to the old (other) phases in the higher-dimensional phase spaces, is...
one of the methods of nonlinear dynamic domain, which usually provides useful information about the complexity of processes generated by a system. In this study, we accordingly applied this technique to quantify the recurrence style of trajectories reconstructed from the EEG signals of ADHD and healthy children.

The following equations is the mathematical formula that is often used for computing the recurrence plots [48].

\[ R_{i,j} = \Theta(\varepsilon - D_{i,j}) \quad \text{i.e. } i, j = 1, 2, ..., N \]
\[ D_{i,j} = \|x_i - y_j\| \quad \text{(1)} \]

where the \(x\) and \(y\) vectors are trajectories generated from the variations of state or phase variables of one system or two different systems. Therefore, \(R\) can be the auto- or cross-recurrence values. In this equation, \(\Theta\) is also a Heaviside function and \(\varepsilon\) is a threshold.

The \(\varepsilon\) threshold is an important parameter in this equation, because it determinants the recurrence basin of \(y\) trajectory to the \(x\) trajectory. Therefore, the incorrect selection of this threshold (for example a constant threshold) can lead to the extraction of dissimilar information from the EEG segments relevant to a class.

In this research, we used the following equation to determine this threshold, i.e. the \(\varepsilon\) threshold.

\[ \varepsilon = D_{\text{min}} + \alpha(D_{\text{max}} - D_{\text{min}}) \quad 0 \leq \alpha \leq 1 \quad \text{(2)} \]

As seen in this equation, the \(\varepsilon\) threshold is determined based on the largest \((D_{\text{max}})\) and smallest \((D_{\text{min}})\) Euclidean distance of two trajectories \(x\) and \(y\) with a new decisive parameter named \(\alpha\), which effects on area defined for the recurrence (Eq. (1)).

Fig. 2a averagely shows the effect of this parameter on the difference of recurrence rate (RR) of recurrence plots obtained from the EEG segments relevant to the healthy and ADHD children based on the 19 EEG channels (the FP1, FP2, F3, F4, C3, C4, P3, P4, O1, O2, F7, F8, T3, T4, T5, T6, Fz, Cz and Pz). As shown in these curves, the most difference between the RR parameter obtained from the recurrence plots of healthy and ADHD children averagely occurred around \(\alpha = 0.25\). Therefore, we used this value to calculate the auto-recurrence plot of EEG segments. Fig. 3 typically indicates the auto-recurrence plots calculated from two EEG seg-

![Fig. 2a](image1)

**Fig. 2.** The effect of \(\alpha\) parameter and embedding dimension \((m)\) on the average value of difference obtained from the recurrence rate of recurrence plots reconstructed from the 19 EEG channels recorded from the ADHD and healthy children.

![Fig. 3](image2)

**Fig. 3.** The recurrence plots calculated from two EEG segments of healthy and ADHD children recorded under the eyes-open conditions.
ments of healthy and ADHD children recorded under eyes-open condition for the Cz channel. As seen in the figure, there is a remarkable difference between the auto-recurrence plots of healthy and ADHD children, so that the recurrence rate (RR) in the auto-recurrence plot of EEG segment without ADHD (42.50%) was lower than that of EEG segment with ADHD (57.93%). Of course, in order to having such difference in most auto-recurrence plots, the lag ($\tau$) and the embedding dimension ($m$) should be determined, because we reconstructed the $x$ trajectory based on the following equation [49]:

$$\tilde{x}(n) = [x_{\text{EEG}}(n), x_{\text{EEG}}(n - \tau), ..., x_{\text{EEG}}(n - (m - 1)\tau)]$$

Based on this equation, the embedding dimension ($m$) plays an important role in reconstructing the phase space trajectories. Hence, we determined this parameter based on the difference of RR parameters obtained from the auto-recurrence plots of healthy and ADHD children. Fig. 2b averagely provides the effect of this parameter, i.e. the embedding dimension, on the difference of recurrence rate obtained from the 720 EEG segments of ADHD and healthy children recorded from the 19 EEG channels under the open-eyes and closed-eyes conditions. As shown in this figure, the curves indicated that the most difference averagely occurred around $m = 5$.

We also utilized the autocorrelation time to determine the lag value, when the autocorrelation approaches $e^{-1}$ [50–53]. In other words, we first estimated the proper lag value for each of the EEG segments using the stated method. Then, we calculated the probability of estimated lags. Fig. 4 depicts these estimated lags and their probabilities. As shown in this figure, the probability of lag of 0.034 s (sample/sampling frequency = 17/500) was more than that of other lags. Therefore, we applied this value to compute the auto-recurrence plot of EEG segments in the different channels.

Fig. 5 shows two brain maps that were generated from the logarithm of $p$-values obtained from the $t$-test analysis of recurrence rate computed from the recurrence plots of EEG segments relevant to the ADHD and healthy children. As seen in these maps, the logarithm of $p$-values for all of the EEG channels (19 channels) was more than two ($p < 0.01$), which means significant separation in the distribution of recurrence rate extracted from the EEG segments of two groups: ADHD and healthy children. Therefore, this significant difference in the recurrence rate of trajectories of EEG phase space indicates that the features extracted from the EEG recurrence plots are informative. In the following subsections, we accordingly quantified the ADHD effect on the auto-recurrence plot using the recurrence quantification analysis (RQA) developed by the values of $\alpha$, lag and embedding dimension ($m$) determined in this subsection.

4. The feature extraction and selection

As shown in the previous section, the auto-recurrence plots of EEG segments obtained from the ADHD children had significant difference with that of healthy children, so that the recurrence rate of EEG signals in these children were averagely more than that of healthy children. Therefore, we quantified the auto-recurrence plots of EEG segments in the ADHD and healthy children by using the features introduced in the recurrence quantification analysis [48]:

![Fig. 4. The estimated lags using the autocorrelation function and their probabilities.](image)

![Fig. 5. The logarithm of $p$-values obtained from the $t$-test analysis of recurrence rate computed from the recurrence plots of EEG segments relevant to the ADHD and healthy children in the 19 EEG channels.](image)
1) **The recurrence rate (RR):** This parameter measures the density of recurrence points in a recurrence plot as following:

\[ RR = \frac{1}{N^2} \sum_{i,j=1}^{N} R_{i,j} \]  

where \( N \) is the length of EEG segments.

2) **The amount of recurrence points that forms diagonal lines with minimal length \( l_{\text{min}} \) (Determinism):** This parameter measures the volume of parallel convergence behaviors of the \( x \) and \( y \) trajectories as following:

\[ DET = \frac{\sum_{l_{\text{min}}}^{l_{\text{max}}} P(l) \cdot l_{\text{min}} = 15}{\sum_{l=1}^{l_{\text{max}}} P(l)} \]  

where \( P(l) \) is the frequency distribution of diagonal lines with length \( l \) and \( l_{\text{min}} \) is the length of smallest diagonal line. We considered this length equal to 15 according to the curves of Fig. 6a, which averagely represent the effect of \( l_{\text{min}} \) on the difference of DET parameter in the recurrence plots obtained from the EEG segments of healthy and ADHD children in the 19 EEG channels recorded under the eyes-open and eyes-closed conditions. This value could actually create most difference in the mentioned parameters for separating two classes.

3) **The amount of recurrence points that forms vertical lines with minimal length \( l_{\text{min}} \):** This parameter measures the amount of convergence behaviors of \( x \) trajectory to the \( y \) trajectory in a special time. Hence, this measure is a method for evaluating the amount of laminar phases in a system. We used following equation for estimating this measure:

\[ LAM = \frac{\sum_{l_{\text{min}}}^{l_{\text{max}}} P(l) \cdot l_{\text{min}} = 23}{\sum_{l=1}^{l_{\text{max}}} P(l)} \]  

where \( P(l) \) is the frequency distribution of vertical lines with length \( l \) and \( l_{\text{min}} \) is the length of smallest vertical line. We considered this length equal to 23 according to the curves of Fig. 6b. As shown in this figure, this value created the most difference in the mentioned parameters for separating the two classes.

4) **The averaged diagonal line length:** This parameter averagely provides information about the parallel convergence behaviors of the \( x \) and \( y \) trajectories. Hence, this parameter is an important index for the predictability of a system.

\[ L = \frac{\sum_{l_{\text{min}}}^{l_{\text{max}}} P(l) \cdot l_{\text{min}} = 15}{\sum_{l=1}^{l_{\text{max}}} P(l)} \]  

where \( P(l) \) is the frequency distribution of diagonal lines with length \( l \).

5) **Trapping time:** This parameter is related with the laminarity time of the system, i.e. how long the system remains in a specific state. It usually is measured as following:

\[ TT = \frac{\sum_{l_{\text{min}}}^{l_{\text{max}}} P(l) \cdot l_{\text{min}} = 23}{\sum_{l=1}^{l_{\text{max}}} P(l)} \]  

where \( P(l) \) is the frequency distribution of vertical lines with length \( l \).

6) **Divergence:** This parameter, which indirectly depends on the maximal diagonal line length (\( l_{d_{\text{max}}} \)), is an estimation of divergence in two trajectories of \( x \) and \( y \).

\[ DIV = \frac{1}{l_{d_{\text{max}}}} \]  

7) **The Shannon entropy of the probability that a diagonal (or vertical) line has exactly length \( l \):** These parameters measure the structural complexity of recurrence plots as following:

\[ EDL \text{ or } EVL = - \sum_{l_{\text{min}}}^{l_{\text{max}} \text{ or } l_{d_{\text{max}}}} p(l) \cdot \ln p(l) \]  

where \( p(l) \) is the probability of diagonal or vertical lines with length \( l \).

8) **The linear regression coefficient between the density of recurrence points in a line parallel to the LOI (line of identity) and its distance to the LOI:** This parameter provides information about the stationarity of recurrence plots. We formulated it as following:

\[ C_{\text{Trend}} = \frac{\sum_{i=1}^{N} \left( i - \frac{N}{2} \right) (P_i - \bar{P})}{\sum_{i=1}^{M} \left( i - \frac{N}{2} \right)^2} \]  

\[ R_k = \frac{1}{N-k} \sum_{i=j+k}^{N} R(i, j) \]  

Fig. 7 averagely illustrates the density of recurrence points in the lines parallel to the LOI for the ADHD and healthy children in the Cz channel. As shown in this figure, the linear regression slope in the ADHD children under the eyes-open and
eyes-closed conditions was more than that of healthy children. Therefore, we considered this parameter as a feature.

9) The averaged Poincaré recurrence time (PRT): This parameter is the length of time elapsed until the recurrence. Therefore, it is an index for measuring the complexity of signal. In this research, we used the average length of vertical lines (\(l_{vl}\)) resulted from the non-recurrence points to estimate this parameter \([54]\).

\[
PRT = \frac{1}{N} \sum_{l=1}^{l_{vl}} l_{vl}
\]

We also added the Shannon entropy (\(En\)) \([55]\) and the power (\(P\)) of EEG signals to our feature vector, because these features extract other part of the EEG information, which consists of the information stored in the EEG amplitude and the time complexity of EEG segments, respectively. We also used the following SFS algorithm \([2,4]\) to select the optimal features, because the utilization of all features (19 × 12), in addition to creating a high computational cost, can lead to decreasing the classification accuracy due to the redundant information in some of the features.

SFS algorithm
If \(X = \{x_1, x_2, ..., x_N\}\) is the input feature vector

1. Start with an empty set: \(Y = []\)
2. Select the next best feature using vector \(Y, x_{\text{new}} \in X\) and the accuracy of validating set
3. Go to 5. If the accuracy of selection criteria is larger than the previous step or desired accuracy. The selection criteria in this step was the accuracy of RBF-SVM classifiers developed by three sets obtained from the holdout method with the division rate of 50%, 20% and 30% for the training, validating and testing set.
4. Select \(Y\) for features. Go to 6
5. Replace \(x_{\text{best}}\) in \(Y\) and remove \(x_{\text{best}}\) in \(X\). Go to 2
6. End

In next section, we provided the results obtained from evaluating the features for distinguishing the healthy from ADHD children.

5. Experimental results

The patterns of Fig. 8, which we obtained the logarithm of \(p\)-values obtained from the \(t\)-test analysis of features extracted from 19 EEG channels relevant to the ADHD and healthy children under the eyes-open and eyes-closed conditions, depicted a significant difference \((\text{a logarithmic value over } \log(0.05))\) in the distribution of features (except the DIV feature) extracted from the EEG segments of ADHD and healthy children, which means changing the texture of recurrence plots obtained from the EEG signals of ADHD children compared to that of healthy children. According to this analysis, the EVL feature also had the most separation in both of the eyes-open and eyes-closed conditions. It was remarkable that the logarithm of \(p\)-values relevant to the Shannon entropy (\(En\)) of EEG signals had a high value unlike the power (\(P\)) of EEG signals, which represents the ADHD effect on the complexity of brain activities. Therefore, such conditions in the mentioned features prove that the recurrence plots extracted from the EEG signals are informative for diagnosing the ADHD. In other words, these conditions represent that information extracted from the oscillation basin of trajectories around the attractors of nervous system in phase space can be effective for identifying the ADHD in a child.

The 10-fold cross-validation of sequential forward selection (SFS) algorithm for classifying the children with and without ADHD in Fig. 9 is other confirmation for the stated issues, because first features in all of the folds belong to the features extracted from the recurrence plots of EEG signal. Of course, the optimal combination of features (10 features) extracted from 19 EEG channel recorded under the eyes-open and eyes-closed conditions (2 × 19 × 12 feature), which could averagely provide an accuracy about 97.86%, 96.34% and 90.95% for the training, validating and testing sets, respectively, included the power of EEG signals in the some of the folds. Therefore, these conditions in the optimal combination of features not only indicated that the features extracted by the recurrence quantification analysis can provide the significant classification accuracy for separating the ADHD from healthy children, but also it highlighted that a feature with the lesser separation in the \(t\)-test analysis (the power of EEG signal) can be placed in the optimum combination of features.

This ranking of features in this figure also indicated that the features extracted from the EEG signals recorded under the eyes-closed condition had a special role in separating the EEG segments of ADHD and healthy children, so that the features selected by this algorithm, i.e. the SFS algorithm, often belong to the EEG signals recorded under the eyes-closed condition. The mentioned ranking also represented the features extracted from the frontal lobe, especially the F8 and F7 channels, plays an effective role in the search trajectory of SFS algorithm. Comparing these classification results with the results reported in the previous researches (Table 1), which is provided in Fig. 10a as the effect of statistical population on the results of ADHD diagnosis, also indicated that the accuracy obtained for the ADHD diagnosis using the RBF-SVM developed by the extracted features was similar to the result of reports that the number of subjects \((N=60)\) was equal to present research \([32,56]\). Therefore, these results generally prove that the recurrence plot of EEG signal is a useful informational resource for the ADHD evaluation and its features can be used for improving the diagnostic assistance techniques.

Nevertheless, the results of this 10-fold cross-validation show that the change of statistical population in the testing set could create a change about 1.25% in the accuracy of support vector machines (SVM) with the radial basis function (RBF) kernel. It was interesting that there is such change in the classification accuracy reported by previous researches (Fig. 10a). The line fitted (red line) on the data of this figure with displaying an indirect relationship between the number of subjects \((N)\) in the statistical population...
Fig. 8. The logarithm of p-values obtained from the t-test analysis for the features of two classes: ADHD and healthy in the 19 EEG channels recorded under the eyes-open and eyes-closed conditions.

Table 1

<table>
<thead>
<tr>
<th>Author</th>
<th>Year</th>
<th>Number subjects</th>
<th>N</th>
<th>age</th>
<th>Measures</th>
<th>Accuracy (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nazari [12,57]</td>
<td>2011</td>
<td>16 ADHD and 16 control</td>
<td>32</td>
<td>7–13</td>
<td>Multi-channels, $\theta/\beta$ ratio</td>
<td>(p &lt; 0.05)</td>
</tr>
<tr>
<td>Williams [57,58]</td>
<td>2010</td>
<td>169 ADHD and 167 control</td>
<td>336</td>
<td>7–19</td>
<td>Fz/Cz, $\theta/\beta$ ratio</td>
<td>(p &lt; 0.05)</td>
</tr>
<tr>
<td>Loo [57,59]</td>
<td>2013</td>
<td>390 ADHD and 100 control</td>
<td>490</td>
<td></td>
<td>Cz, $\theta/\beta$ ratio</td>
<td>38</td>
</tr>
<tr>
<td>Liechti [57,60]</td>
<td>2013</td>
<td>54 ADHD and 51 control</td>
<td>105</td>
<td>8–16</td>
<td>Cz, $\theta/\beta$ ratio</td>
<td>53</td>
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<tr>
<td>Helgadóttir [62]</td>
<td>2015</td>
<td>90 ADHD and 90 control</td>
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<td>Multi-channels, optimal features extracted from the EEG band</td>
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<td>67 ADHD and 50 control</td>
<td>117</td>
<td>18–50</td>
<td>Multi-channels, absolute power ($\delta\theta\alpha\beta\gamma$)</td>
<td>82.3</td>
</tr>
<tr>
<td>Ogrin [57,63]</td>
<td>2012</td>
<td>62 ADHD and 36 control</td>
<td>98</td>
<td>7–16</td>
<td>Cz, absolute $\theta$ and $\beta$ power ($\theta/\beta$) ratio</td>
<td>85</td>
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<td>Sadatnezhad [57,64]</td>
<td>2011</td>
<td>21 ADHD and 12 BMD</td>
<td>43</td>
<td>10–22</td>
<td>Multi-channels, fractal dimension, AR model, band power ($\delta\theta\alpha\beta\gamma$)</td>
<td>86.4</td>
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<td>Magee [57,65]</td>
<td>2005</td>
<td>75 ADHD and 75 control</td>
<td>150</td>
<td>7–13</td>
<td>Absolute and relative power</td>
<td>87</td>
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<td>Ahmadian [57,66]</td>
<td>2010</td>
<td>12 ADHD and 12 control</td>
<td>24</td>
<td></td>
<td>Inter-electrode synchronization ($\delta\theta\alpha\beta\gamma$)</td>
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<td>Simoska [56]</td>
<td>2016</td>
<td>30 ADHD and 30 control</td>
<td>60</td>
<td>6–14</td>
<td>Cz, $\theta/\beta$ ratio</td>
<td>87.9</td>
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<tr>
<td>Snyder [57,67]</td>
<td>2008</td>
<td>97 ADHD and 62 control</td>
<td>159</td>
<td>6–18</td>
<td>Cz, $\theta/\beta$ ratio</td>
<td>89</td>
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<tr>
<td>Monztra [57,68]</td>
<td>2001</td>
<td>96 ADHD and 33 control</td>
<td>129</td>
<td>6–20</td>
<td>Complement plots</td>
<td>91</td>
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<td>This Work</td>
<td>2013</td>
<td>30 ADHD and 30 control</td>
<td>60</td>
<td>7–12</td>
<td>RQA, power and entropy</td>
<td>90.95 ± 1.25</td>
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<td>Mohammadi [32]</td>
<td>2014</td>
<td>30 ADHD and 30 control</td>
<td>60</td>
<td>7–14</td>
<td>Multi-channels, nonlinear optimal features</td>
<td>93.65</td>
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<td>Abuliuav [57,69]</td>
<td>2012</td>
<td>7 ADHD and 3 control</td>
<td>10</td>
<td>7–12</td>
<td>Multi-channels, relative $\theta$ and $\beta$ power, $\delta\theta$ and $\theta/\beta$ ratios</td>
<td>97</td>
</tr>
<tr>
<td>Yaghoobi [4]</td>
<td>2017</td>
<td>20 ADHD and 20 control</td>
<td>40</td>
<td>7–10</td>
<td>Continuous Wavelet Transform (CWT) and standalone classifier</td>
<td>98.07</td>
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<td>Yaghoobi [2]</td>
<td>2018</td>
<td>20 ADHD and 20 control</td>
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<td>7–10</td>
<td>Complement plots</td>
<td>98.25</td>
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and the classification accuracy, not only generally represents the stated issue, but also it indicates that increasing the number of subjects can significantly decrease the accuracy of ADHD diagnosis. Therefore, it seems that diagnosing the ADHD using a classifier developed by an EEG segment cannot be a suitable method for the large EEG databases such as the clinical databases, which its statistical population is changing every day due to the exceptions appeared on the brain activities of patients. Fig. 10b, which typically provides the average and standard deviation of recurrence rate in the recurrence plots of 24 successive EEG segments (F8 channel) relevant to the 30 ADHD children and 30 healthy children under the eyes-closed condition, generally depicted two bands of variations (blue and red band) around the average values (blue and red points) of recurrence rates, which have averagely significant different ($p < 0.05$) with each other. For extracting these curves, we actually sorted the 24 successive EEG segments relevant to each of the subjects based on the recurrence rate of recurrence plots computed for these successive segments. Then, we computed the average and standard deviation of these recurrence rates and finally plotted them based on the EEG segments, which had different recurrence rates from small to large. Therefore, these bands generally represent that the child without and with ADHD can create the similar values for the indicators such as the recurrence rate, although these indicators in the ADHD children can have averagely different values than that of healthy children. This status in the EEG features such as the recurrence rate according to Sabelli’s studies [2,35], which focus on the BISOS theory in the non-linear dynamic systems, also means the possible of creativity of similar or same states in the brain activity of healthy and ADHD children. In other words, the brain as the non-linear dynamic systems, which can generate the biotic behavior, can create similar activities in different states (Health or disease). Therefore, this ability of brain, which is a symbol of creativity in the human nervous system, is a factor that can lead to reducing the classification accuracy against the increase of the statistical population (Fig. 10a) or in other words, against the exceptions of new statistical population. In the next section, we presented a brief discussion and conclusion about the results obtained in this study.

6. Discussion

As stated in the introduction, some of researchers such as Vaillancourt and Newell [30] believe that the observed increase or decrease in complexity with aging and disease is dependent on
the intrinsic dynamics of the system and the short-term change required to realize a demand local task. In other words, these researchers and also Lipsitz and Goldberger [28] with displaying the negative effects of aging and diseases on the complexity of physiological systems have generally indicated that the loss of complexity in the physiological processes is resulted from relative frequency reduction in the high-frequency components and corresponding increase in the relative contribution of lower-frequency components. Actually, these changes in the frequency components generally prove that the more complex processes have a broader active frequency pattern, while the loss of complexity in a process is usually along with a narrowing of the frequency spectrum. It is interesting that the ADHD as a neurobehavioral developmental disorder, which often appears as inattention and distractibility with or without accompanying hyperactivity, is a change in the brain activities, which its effect on the EEG signals is usually a more activity in low frequency components (δ and θ bands) and a less activity in high frequency components (α and β bands) [17–21]. Therefore, such status in the behavioral indicators and the indicators obtained from the EEG signals of ADHD children generally represents that the ADHD can reduces the complexity of processes generated by the brain, especially processes (the vertical electric current inside the skull resulted from the inhibitory and excitatory post synaptic potentials [2,70]) stored on the EEG signals. The results reported by previous researches such as Khoshnoud’s report [31] on the entropy of EEG signals are a confirmation for this reduction in the complexity of brain activities relevant to the ADHD children. Increasing the recurrence rate in the recurrence plots reconstructed from the EEG signals of ADHD children compared to that of healthy children in the 19 EEG channels studied in this research, especially the EEG channels located on the frontal lobe under the eyes-open condition and the frontal and occipital lobes under eyes-closed condition are also another confirmation for the mentioned reduction in the brain activities of children with ADHD. This increasing in the recurrence rate actually based on the Poincaré recurrence theorem, which usually estimates the complexity of a process generated by a system using the recurrence rate of phase space trajectories to the Poincaré section and area, proves that the ADHD often leads to decreasing the complexity of behaviors and brain activities in the human. In addition, it was also remarkable that computing the DET and LAM indexes in the recurrence plots obtained from the trajectory of phase space reconstructed from the EEG signals related to the ADHD children depicted a significant difference (increase), which its reason was the parallel and similar behaviors in the above-mentioned trajectory. Increasing these behaviors in the phase space trajectories reconstructed from the EEG signals of ADHD children was also to some extent that the noted indexes, especially the DET index, had appeared in combination of optimal features selected by the SFS algorithm.

Therefore, these changes in the texture of patterns appeared on the auto-recurrence plots obtained from the trajectory of phase space reconstructed from the EEG signals recorded under the open-eyes and closed-eyes resting conditions not only indicated that the ADHD could lead to a decrease in the complexity of brain processes in terms of the rate of recurrence, parallel and similar behaviors, but also these changes were new conformations for the results of Vaillancourt’s and Lipsitz’s reports [28,30]. Nevertheless, the influence of statistical population on the accuracy of classifiers indicated that there is an intrinsic limitation for diagnosing the ADHD using an EEG segment, although the discussed results provided a significant difference for the ADHD children. This influence in the accuracy of classifiers in the present research and previous researches (Fig. 10a) actually represents a decrease in the accuracy of ADHD diagnosis compared to increasing the number of subjects in the statistical population, which means the inefficient of classifiers developed based on a clinical databases. Evaluating the recurrence rate obtained from the recurrence plots of 24 successive EEG segment relevant to the ADHD and healthy children (Fig. 10b) also represented that the information of an EEG segment cannot always guarantees the ADHD diagnosis, because the human brain due to its creativity can provide same information in different conditions (Health or disease). In other words, the recording time of EEG segments effects on the results of ADHD diagnosis using the supervise classifiers. Therefore, this proves that the ADHD have different grades in the different times for the people. Hence, this grading are another reason that the ADHD diagnosis using an EEG segment cannot be suitable in the large databases, which are extended every day.

7. Conclusion

As discussed, evaluating the ADHD effects on the recurrences of phase space trajectories reconstructed from the EEG segments generally indicated that the ADHD by changing the brain functions leads to reducing the complexity of trajectories, which are usually generated by the transfer of phases or states around the brain attractors. This evaluating also indicated that the above-mentioned reducing was to some extent that the SFS algorithm based on the RBF-SVM classifier developed by an EEG segment could provide a significant separation for distinguishing the ADHD from healthy children. Nevertheless, the outcomes of this analysis not only proved that the recording time of EEG segments effects on the results of ADHD diagnosis, but also they depicted that the volume of statistical population extremely effects on the accuracy of classifier developed by an EEG segment. Therefore, these conditions show that the ADHD diagnosis in the clinical applications, which its statistical population is changing every day due to the exceptions appeared on the brain activities of patients, requires to a diagnosis technique based on several EEG segments. Accordingly, the development of diagnostic methods based on several EEG segments, the study of reliability in the accurate diagnosis of ADHD using several EEG segments, the comparison of mentioned reliability with the reliability of self-reports and continuous performance tests can be the interesting start points for the future works. Since, the ADHD is an effective disorder on the frequency content of EEG signals, it seems that subsequent studies must also consider the evaluation of trajectories resulted from filtering the EEG signals in the standard EEG bands using the recurrence quantification analysis. These studies also need to examine the cross-recurrence plots of EEG signals in the different EEG channels for ensuring a fuller investigation and provide more information about the ADHD effects on the activities of nervous system.

References


