INVESTIGATION OF SELECTED MECHANICAL SYSTEMS BY RECURRENCE PLOTS METHOD

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Recurrence of states, in the meaning that states are arbitrary close after some time, is a fundamental property of deterministic dynamical systems and is typical of nonlinear or chaotic systems. The main idea of the recurrence plots (RPs) method consists in revealing all the times when the phase space trajectory of the dynamical system visits roughly the same area in the phase space. Such recurrence of states is marked within a two-dimensional matrix with ones and zeros. Graphical representation of such a matrix is known in the literature as a recurrence plot. The paper concerns results of research into dynamic properties and determinism of selected mechanical systems. In order to analyze measured responses (time histories) of the considered systems with the use of the recurrence plots, the phase space of these systems was reconstructed by delay embedding. The smallest sufficient embedding dimensions were estimated with the use of the false nearest neighbors algorithm. For the purposes of time delay determining the method of mutual information function was used. All the necessary computations were performed with the use of the CRP Toolbox for MATLAB.

Keywords: Recurrence plots; phase space; delay embedding.

1. Introduction

The paper concerns issues related to research into system determinism with the application of the recurrence plots (RPs) method. Historically, the method was used for the purposes of visualization of system trajectories, especially in the higher-dimensional phase spaces. Nowadays it is regarded as an advanced method of nonlinear data analysis, making it possible to detect qualitative changes in dynamic behavior of mechanical systems. Applications of the discussed method can be found in numerous fields of science, such as engineering, astrophysics, biology, chemistry, geology, cardiology, neuroscience and economy.

In this paper, the RPs method was used in the course of the research into dynamic properties of damped mechanical system, undamaged and cracked aluminum plates.
excited to vibrations with the application of harmonic signals of frequencies cor-
responding to the frequencies of plates consecutive natural mode shapes and rotational
machine with misalignment and bearing slackness introduced.

2. RPs Method
The main idea of the RPs method, which was introduced by Eckmann, Oliffson and
Ruelle,\textsuperscript{2} consists in revealing all the times when the phase space trajectory of the
dynamical system visits roughly the same area in the phase space. Such recurrence of
a state $x$ at time $i$ at a different time $j$ is marked within a two-dimensional square
matrix $[R]$ with ones and zeros:

$$[R_{i,j}] = \Theta(\varepsilon - \|x_i - x_j\|), \quad i, j = 1, \ldots, N$$

where $N$ is the number of considered states $x_i$, $\varepsilon$ is a threshold distance, $\| \|$ denotes a
norm and $\Theta( )$ the Heaviside function.

Graphical representation of such a matrix is known in the literature as an RP. In
other words, under the term RP, visualization of matrix $[R_{i,j}]$:

$$[R_{i,j}] = \begin{cases} 
1 & \{x_i\} \approx \{x_j\} \\
0 & \{x_i\} \neq \{x_j\} 
\end{cases}, \quad i, j = 1, \ldots, N$$

is understood, where $\{x_i\} \approx \{x_j\}$ are the points from the neighborhood of radius $\varepsilon$
(according to the applied norm).

In the original definition of the method, the neighborhood is specified with the
use of the $L_2$ norm and its radius is selected in such a way that it contains a fixed
amount of states. With such a neighborhood, the radius $\varepsilon_i$ changes for each $x_i$
($i = 1, \ldots, N$). Such a neighborhood is denoted as fixed amount of nearest neighbors
(FAN). Other commonly used neighborhood is that with a fixed radius, first used by
Zbilut.\textsuperscript{12}

In the carried out research, the results of which are discussed in the paper, the
FAN criterion was used. The smallest sufficient embedding dimension $m$ was esti-
mated with the use of the false nearest neighbors algorithm.\textsuperscript{13} Appropriate time
delay $\tau$ was determined by searching for the first minimum of the mutual information
function.\textsuperscript{14} Phase spaces of the considered systems were reconstructed by delay
embedding.\textsuperscript{1} All the necessary computations were performed with the use of func-
tions implemented in the CRP Toolbox for MATLAB.\textsuperscript{15}

3. Results of the Carried out Research
In this section, the results of the research into properties of selected mechanical
systems: damped mechanical system, undamaged and cracked aluminum plates
carried out by means of the RPs method are presented.\textsuperscript{16} The results of the RPs
method application to the rotational machine state model-based diagnostics are also
discussed.
3.1. Damped mechanical system

Time history [Fig. 1(a)] of the considered system is described by the following equation:

\[ y = A \cdot e^{-\alpha t} \cdot \sin \omega t, \quad \alpha = 1 \]  

System phase space is presented in the Fig. 1(b).

It was determined that optimal (in the sense of the mutual information criterion) value of time delay \( \tau = 34 \) while embedding dimensions, computed with the use of the false nearest neighbors method, \( m = 2 \). RP determined for these parameters on the basis of the considered system time history (1) is presented in the Fig. 2.

As a result of damping presence, diagonal lines visible on the estimated RP are rippled and fading away toward the upper left and lower right corner.

3.2. Aluminum plate system

Research was carried out for two aluminum plates of dimensions \( 400 \times 150 \times 2 \text{ mm} \), density \( 2780 \text{ kg/m}^3 \), Young’s modulus \( 72400 \text{ MPa} \) and Poisson’s ratio 0.33: undamaged plate and cracked plate (presented schematically in the Fig. 3). In the course of the experiment, the considered plates were freely suspended on elastic cords and excited to vibrations with the use of the electromagnetic shaker. In the individual tests, the plates were subjected to the harmonic exciting signals of frequencies corresponding to the frequencies of their 1st, 3rd and 6th mode shapes, respectively.

Selection of exciting signals of such frequencies is motivated by the fact that for these frequencies characteristic behavior of the cracked plate has been observed: crack opening in plane of plate (for the 1st natural mode shape), crack opening in
plane perpendicular to the plane of plate (for the 3rd natural mode shape) and friction of crack edges (for the 6th natural mode shape).\textsuperscript{17}

Analysis of the measured displacement time histories was carried out with the use of the RPs method and aimed at detection of changes in the determinism of undamaged and damaged system responses in the function of the excitation frequency.

In case of RPs estimated on the basis of the considered plates responses to the harmonic excitations of the frequencies $f_e$ corresponding to the frequencies of their first mode shapes $f_{\text{first mode shape}}$ [Figs. 4(a) and 5(a)], diagonals are long and continuous, which proves deterministic dynamical behavior of the considered plates.

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Fig. 2. RP ($m = 2, \tau = 34, \varepsilon = 0, 1$) determined for the considered damped system.

Fig. 3. Considered aluminum plate with a crack resulting from carried out fatigue tests and scheme of the test stand\textsuperscript{15}: computer and acquisition system (1), Polytec PSV-400 scanning vibrometer (2), tested plate (3), infrared camera (4), optical system (5), signal generator (6).
RPs determined from displacement time histories of undamaged and cracked aluminum plates excited with the frequencies corresponding to the 3rd vibration modes \( f_{\text{third mode shape}} \) are presented in the Figs. 4(b) and 5(b), respectively. Diagonal lines exhibited by both diagrams are more disturbed than in the case of the 1st vibration mode excitation, particularly for the undamaged plate. This pattern indicates the increase of irregularities of dynamic behavior of both plates resulting possibly from the increase in the frequency of excitation.

Fig. 4. RPs determined for undamaged plate excited to vibrations by harmonic signals of frequency (a) \( f_e = f_{\text{first mode shape}} \), (b) \( f_e = f_{\text{third mode shape}} \), (c) \( f_e = f_{\text{sixth mode shape}} \).

RPs determined from displacement time histories of undamaged and cracked aluminum plates excited with the frequencies corresponding to the 3rd vibration modes \( f_{\text{third mode shape}} \) are presented in the Figs. 4(b) and 5(b), respectively. Diagonal lines exhibited by both diagrams are more disturbed than in the case of the 1st vibration mode excitation, particularly for the undamaged plate. This pattern indicates the increase of irregularities of dynamic behavior of both plates resulting possibly from the increase in the frequency of excitation.
The last considered case related to plates excited with the frequencies corresponding to the 6th vibration modes ($f_{\text{sixth\_mode\_shape}}$). This time some of the diagonal lines of estimated RPs [Figs. 4(c) and 5(c)] are broken and the presence of large white patches can be observed. The white patches, indicating abrupt changes in the dynamics, are particularly pronounced when the plot for the cracked plate is analyzed.

Fig. 5. RPs determined for cracked plate excited to vibrations with the use of harmonic signal of frequency (a) $f_e = f_{\text{first\_mode\_shape}}$, (b) $f_e = f_{\text{third\_mode\_shape}}$, (c) $f_e = f_{\text{sixth\_mode\_shape}}$. 

The last considered case related to plates excited with the frequencies corresponding to the 6th vibration modes ($f_{\text{sixth\_mode\_shape}}$). This time some of the diagonal lines of estimated RPs [Figs. 4(c) and 5(c)] are broken and the presence of large white patches can be observed. The white patches, indicating abrupt changes in the dynamics, are particularly pronounced when the plot for the cracked plate is analyzed.
Obtained results proved that determinism generally changes with the excitation frequency. Considered plates excited with the frequency corresponding to the first vibration mode has exhibited very regular behavior. In contrast, small irregularities and strong perturbations could be observed in the RPs estimated for the undamaged and cracked plates excited with the frequency corresponding to the 3rd vibration mode and for the 6th vibration modes, respectively. The presence of large perturbations, indicating abrupt changes in the dynamics of the plate, was particularly visible for the cracked plate excited with the 6th vibration mode.

Fig. 6. RPs determined for (a) undamaged system \( (m = 7, \tau = 1, \varepsilon = 0.5) \), (b) bearing slackness at non-driven shaft end \( (m = 7, \tau = 1, \varepsilon = 0.5) \), (c) misalignment \( (m = 7, \tau = 1, \varepsilon = 0.5) \).
3.3. Rotational machine state diagnostics with the application of the RPs method

Finally, the RPs method was applied to analysis of vibration acceleration time histories measured in vertical direction on the bearing housing at the rotational machine nondriven shaft end.

RPs estimated for undamaged rotational machine as well as machine with misalignment and bearing slackness introduced are presented in Fig. 6.

Although the differences between RPs estimated for different technical states of the considered rotational machine are not so distinct as in case of RPs presented in Sec. 3.2 of this paper, carried out analysis proved that the RPs method can be successfully used for the purposes of the machine state model-based diagnostics.

4. Conclusion

The paper concerns issues related to the application of the RPs method to research into dynamic properties of mechanical systems, detection of fatigue defects in metallic materials and model based diagnostics of mechanical systems.

The RPs method, providing information concerning time instants when the phase space trajectory of the dynamical system visits roughly the same area in the phase space, can be used as an efficient method supporting research into dynamic properties of any considered mechanical system. In the example provided in the paper, damping presence is clearly visible on the estimated RP.

In order to present possibilities of method application to detection of fatigue defects in metallic materials, the results of the research carried out for undamaged and cracked aluminum plates are presented. In the course of the carried out identification research the sample with crack of considerable length was considered. The further research will consist in determining the RPs method efficiency (the minimal crack length that can be detected) and method application to different types of structures (e.g. composite structures). On the basis of the obtained results it can be stated that the RPs method can be used for detection of changes in the dynamic behavior (determinism) of mechanical systems related to changes in the frequency of excitation or failure appearance. However, unequivocal classification of factors responsible for detected changes in the system dynamic behavior can be difficult to carry out.

In the last example, the RPs method application to the model based diagnostics of rotational machine with various types of damage (misalignment and bearing slackness) introduced was considered. Differences between RPs estimated for different technical states are distinct enough to be used in the course of the machine state model-based diagnostics.

Interpretation of information provided by recurrence plots method can be supported by the application of the recurrence quantification analysis (RQA), the idea of which consists in analysis of spatial distribution and density of recurrence points visible on RPs. The process of model based diagnostics carried out on the basis of
information provided by RPs can be automated by tracking changes in the values of RQA measures resulting from changes in system state (such as e.g. crack propagation).

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References


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