Applications of Nonlinear Time-Series Analysis in Unstable Periodic Orbits Identification – Chaos Control in Buck Converter

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Abstract—Recently, there has been an extensive interest in research on the analysis and control of chaotic behavior in nonlinear systems. Various control problems can be defined for such systems, such as targeting the trajectories to a desired point, stabilizing unstable periodic orbits etc. The recurrence plot is a two dimensional representation technique that brings out distance correlations in a time series and make it instantly apparent whether a system is periodic or chaotic. In this paper, the proposed method is applied to control chaos in buck converter. In particular, the resonant parametric perturbation in the perturbing signal is studied.

I. INTRODUCTION

Recent studies show that chaotic behaviour of Switched mode DC–DC converters is due to the nonlinearity of the switching operation which is an essential characteristic of the power electronic circuits. Since, converters are widely used in industries, analysis of chaos in converters assumes practical significance, failure of which will lead to erratic operation of the system.

Data from multi-dimensional nonlinear system are difficult to analyze. Structure may be masked by inherent complexities of the system. Many data analysis techniques such as spectral analysis hinge upon an assumption of linearity and strive to average out fluctuations believed to be noise. A different technique is required in which averaging out fluctuation would mean throwing out information about the dynamics of the system [1]. Recurrence plot (RP) is such a technique used to determine whether a nonlinear system is noisy or chaotic. RPs can be regarded as a general autocorrelation function of the series, because it relates its values at different times. Unlike the autocorrelation function, it takes into consideration the relative and absolute time, allowing analysis of non-stationary data [2].

This method allows the identification of system properties that cannot be observed using other linear and nonlinear approaches and is especially useful for analysis of non-stationary systems with high dimensional and/or noisy dynamics.

Recurrence plot analysis method has been proposed for robust analysis of data obtained from nonlinear switching buck converter. In so doing, recurrence plot is observed to be an useful tool for providing a complete image of dynamic course at a glance without the reference of model equations and physical variables [3].

The research described in this paper focuses on the recurrence plot analysis of DC–DC Buck converter with chaotic dynamics. The time series data obtained from this nonlinear dynamical system is analyzed for the transition of system behaviour from periodic to chaotic regime with the change in the system parameters and claims that the RPs are very effective in analyzing the set of unstable “n” periodic orbits that lie within a chaotic attractor.

II. CHAOS CONTROL – IDENTIFYING THE UNSTABLE PERIODIC ORBITS FOR A CHAOTIC SYSTEM

In most technical environments chaos is an undesired state of the system which one would like to suppress. Chaos control can help to re-establish at least a regularly oscillating output at a higher rate, with judiciously applied minimal perturbations.

Chaotic systems have the outstanding property that the attractor possesses a skeleton of Unstable Periodic Orbits (UPO). In strongly chaotic systems they are even dense. If a trajectory approaches a periodic orbit by chance, it will remain close for a while, until it is repelled by the instability and disappears in the chaotic sea.

For a systematic approach of the control problem, the first step is to find the location of the unstable periodic orbits from the experimental time series. When we look at a typical chaotic time series, this seems to be a hopeless task. Nevertheless, a chaotic trajectory reveals information about the periodic skeleton.

A pulse width modulated DC-DC converter operating in continuous conduction mode can be viewed as a linear time-varying system which passes periodically in time.

The Buck converter is used to convert a DC voltage to a lower DC voltage. The control is applied through an integrated voltage control feedback. This power electronic converter is extensively used in power supplies in industries, where regulating the voltage is desired.

Fig. 1 shows the schematic diagram of a buck converter that uses a Pulse Width Modulation (PWM) voltage loop.
The dynamic model can be written as:

\[
\frac{di(t)}{dt} = -\frac{1}{L} v(t) + \frac{u(t)}{L} E
\]

\[
\frac{dv(t)}{dt} = \frac{1}{C} i(t) - \frac{1}{RC} v(t)
\]  

(1)

where \(i\) is the inductor current, \(v\) the capacitor voltage, \(E\) a constant input voltage and \(u(t)\) the modulated signal which is zero when \(v_{co}(t) \leq v_c(t)\) (i.e., the switch is OFF) and is one when \(v_{co}(t) > v_c(t)\) (the switch is ON) [4],[5],[6].

The system switches from one configuration to the other one whenever the control signal \(v_{co}\) is equal to the \(T\)-periodic carrier signal \(v_r\) which we assume to be the sawtooth signal (see figure 2):

\[
v_r(t) = a + \beta(t \text{ mod } T).
\]

(2)

In the schematic diagram of a Buck converter, the output voltage error with respect to the reference voltage is amplified to give a control voltage \(v_{co}\):

\[
v_{co}(t) = a \left( v(t) - V_{ref} \right).
\]

(3)

The chaotic behavior for buck converter \((L = 20mH; C = 47\mu F; R = 22\Omega; V_{ref} = 11V; a = 8.4; T = 400\mu S; V_r = 3.8V; V = 8.2V; E = 22 - 33V)\) observable in bifurcation diagram, figure 3, where \(E\) is bifurcation parameter.

This particular buck converter exhibits a complex bifurcation route, with the main bifurcation being period-doubling. When \(E\) exceeds about 32.27 V, the converter enters a chaotic region. Beyond 32.34 V, the chaotic attractor encounters crisis and expands to a larger chaotic attractor. Meanwhile, in some periodic windows along the main period-doubling bifurcation route, there are other periodic or chaotic attractors coexisting. For example, chaotic attractor coexists with the periodic attractor when \(E\) is about 24 V, and a period-doubling bifurcation route beginning at period-6 coexists with a period-2 attractor when \(E\) is about 30 V. The chaotic attractor, sawtooth and control voltage for buck converter are shown in figures 4 and 5, where \(E = 33V\).
In other words, in order to be able to compute a UPO, we have to retain from the RP just the lines parallel with the main diagonal whose lengths are higher than a specified value (threshold) \[9\].

Filtering the RP using a threshold, where \( E = 33\,\text{V} \) and \( E = 24\,\text{V} \), we can obtain the following one (see figure 8).

The above RP corresponds to a quasi-sinusoidal signal obtained from the buck converter using the next controlling procedure.

The method called resonant parametric perturbation \([4], [10]\) is now applied to stabilize the chaotic buck converter described in the foregoing for \( E = 32.27\,\text{V} \). We choose \( V_{\text{ref}} \) as the perturbation parameter. Essentially we replace \( V_{\text{ref}} \) by

\[ V_{\text{ref}} \rightarrow V_{\text{ref}} \left( 1 + \alpha \sin 2\pi f t \right) \]  \hspace{1cm} (4)

where \( \alpha \) is the perturbation amplitude, \( f \) is the perturbation frequency, and thus the term \((\alpha \sin 2\pi f t)\) is the resonant perturbation applied to \( V_{\text{ref}} \). The perturbation frequency \( f \) is set to the driving frequency, and the perturbation amplitude \( \alpha \) is varied from 0 to 0.25. We examine the steady-state behavior in terms of the sampled inductor current under the variation of \( \alpha \).

In particular, figure 8 shows the bifurcation diagram \( i(\alpha) \).

Fig. 9 reveals a deceptively lower “smallest effective perturbation amplitude” because of the afore-mentioned multiple switching effect.

Resonant parametric perturbation is applied with the perturbing signal perfectly synchronized to the switching frequency.
Notwithstanding this effect, we clearly observe the stabilization effect when resonant parametric perturbation is applied.

The attractor for buck converter is shown in figure 10, where $E = 33V$.

Experimental measurements have been taken to confirm the effectiveness of the method. The attractor is shown on the phase portrait of figure 11. As shown in this figure, the system is stabilized to a period-1 operation.

The evolution obtained for the buck converter (if $\alpha = 0.1$) can be represented as a recurrence plot (see figure 12).

III. CONCLUSIONS

In this paper we tried to show that the new nonlinear processing techniques such as RP can be used not only for visual inspection of real signals but also for analyzing and processing them. By using RP, we show that one can build nonlinear control techniques able to give better results when applying them to nonlinear REAL signals. The same tool can be used for determining signals behaviors and prediction, compressing signals, extracting characteristic parameters, synthesizing signals, comparing signals and as quality measuring tools, etc.

Resonant parametric perturbation is an effective nonfeedback approach for controlling chaos. Compared to the feedback chaos control methods, the nonfeedback chaos control methods are usually easier to apply and they require no prior knowledge of the system behavior. For switching converters, such nonfeedback chaos control allows easy incorporation with the existing control circuits, as demonstrated in the voltage-mode controlled buck converter studied in this paper.

REFERENCES


