Typical chaotic dynamics in squid giant axons

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Abstract

We discuss that the dynamics of squid giant axons is a typical example of chaotic dynamics. There are two reasons why the dynamics of squid giant axons is a typical example of chaotic dynamics. The first reason is that the maximal Lyapunov exponent has a positive value which coincides with the metric entropy. The second reason is that the analysis of recurrence plot shows that the underlying dynamics of squid giant axon is consistent with Devaney’s definition of deterministic chaos.

Keywords: deterministic chaos; squid giant axons; maximal Lyapunov exponent; metric entropy; recurrence plot

1. Introduction

Deterministic chaos has been attracting lots of attention for the last 50 years. Although many systems were claimed to be of deterministic chaos, there are only few examples of real systems for which the nature of deterministic chaos has been confirmed by time series analysis. Here, we focus on dynamics of squid giant axons and show that the underlying dynamics is of typical deterministic chaos form the two viewpoints: Pesin’s equality and recurrence plot. The remaining parts of the paper are organized as follows. In Section 2, we observe the dataset of squid giant axon. In Section 3, we analyze the dataset by the maximal Lyapunov exponent and the

FIG. 1. Time series of squid giant axon.
metric entropy. In Section 4, we further characterize the dataset using their recurrence plot. Lastly, we conclude this paper.

2. Dataset

The dataset of squid giant axon is shown in Fig. 1 (see [1] for how the measurements were taken). We stimulated the squid giant axon by periodic pulses. At the leading edge of each stimulating pulse, we measured the membrane potentials stroboscopically. We normalized the measurements of membrane potentials so that they vary between 0 and 1.

When we take its return plot, plotted points look very close to a one-dimensional map (Fig. 2). But, they do not form a one-dimensional map exactly since there are some points where more than one values on the vertical axis are associated with a value on the horizontal axis.

3. Pesin’s equality

The first viewpoint is that the maximal Lyapunov exponent and the metric entropy have the same positive value.

We estimated the maximal Lyapunov exponent by using the method of Kantz [2]. The method characterizes how exponentially fast two nearby orbits diverge along the time axis.

The result is presented in Fig. 3. We found that two nearby orbits tend to diverge exponentially fast. We can observe a relatively wide scaling region. The exponent is about 0.42 bits/observation.

Second, we try to estimate the metric entropy. To define the metric entropy, let us introduce a partition to generate, from a time series, a symbolic sequence. For a symbolic sequence, we can calculate entropy per symbol. Suppose that we maximize the entropy per symbol over all partitions. Then, the entropy per symbol is the metric entropy. If such a partition is finite, then it is called a generating partition. A generating partition is not trivial for a higher-dimensional dynamics.

We estimate the metric entropy by estimating a generating partition from the time series with the method of [3] (see Fig. 2 for the estimated generation partition), obtaining the symbolic sequence, and estimating the metric entropy using a context tree [4]. Find the details of the used methods in [3] and [4]. Our estimate for the metric entropy is 0.42 bits/observation. This estimate of the metric entropy agrees with the estimate of the maximal Lyapunov exponent. This relation is called the Pesin’s identity [5].

FIG. 2. Return plot of the dataset. In this figure, the colours show the estimated generating partition.

FIG. 3. Maximal Lyapunov exponent estimated by the method of Kantz [2].
Therefore, in the system of squid giant axon, the Pesin’s identity seems to hold, which is a typical characteristic of deterministic chaos.

4. Devaney’s chaos on recurrence plot

Next, we try to further characterize the time series by a recurrence plot [6, 7]. Recurrence plots are two-dimensional figures originally proposed for visually characterizing time series. Both axes of a recurrence plot correspond to the same time axis. For a pair of times, if the distance between the corresponding states is closer than a pre-defined threshold, then we plot a point at the corresponding place. Otherwise, we do not plot a point.

Recurrence plots can show various characteristics of time series. For example, a recurrence plot of random noise has points spread uniformly randomly. A recurrence plot of periodic wave shows periodic patterns. A recurrence plot of deterministic chaos such as the logistic map can have many diagonal line segments. Therefore, diagonal lines characterize deterministic systems.

There are some known properties of recurrence plots. We can estimate correlation dimension and correlation entropy from recurrence plots [8, 9]. Moreover, we can demonstrate that a recurrence plot can contain almost all information related to topology and distances except for spatial scales since we can reproduce the rough shape of original time series from a recurrence plot [10].

Actually, a recurrence plot can characterize deterministic chaos [11]. Devaney [12] proposed a mathematical definition of deterministic chaos. There are three conditions. The first condition is topological transitivity. The second condition is denseness of periodic points. The third condition is sensitive dependence on initial conditions. We look into the details of the three conditions. Topological transitivity means that any open set can have a non-empty intersection with any open set after applying the map several times. Denseness of periodic orbits means that periodic points are dense on the attractor. Namely, we can find at least a periodic point in any open set if periodic points are dense on the attractor. Sensitive dependence on initial conditions means that there exists $\delta > 0$ such that for any $x$ and for any neighborhood of $x$, there exist $y$ and integer $k$ such that after $k$ iterations, the distance between $x$ and $y$ becomes greater than $\delta$. Roughly speaking, in general, two neighboring points can be separated by a fixed amount of distance in the future.

Devaney’s definition of deterministic chaos is often used in mathematics. But, it is hard to apply to a time series of finite length since all the three conditions include the notion of any open sets. Therefore, we relax the three conditions by replacing any open sets with any open sets we use for defining a recurrence plot [11]. We call three relaxed conditions as r-topological transitivity, r-denseness of periodic points, and r-sensitive dependence on initial conditions.

For example, r-topological transitivity is the property that any neighborhood used to define the recurrence plot can have a non-empty intersection with any neighborhood used to define the recurrence plot after applying the underlying dynamics several times. It is easy to show that this condition is equivalent to the following condition: the minimum of the maximum plotted column is greater than the maximum of the minimum plotted column within a recurrence plot [11]. Therefore, this condition can be checked by the visual inspection of recurrence plot.

Similarly, r-denseness of periodic points is formulated as follows. First on each line which is separated from the central diagonal line by $k$, we check whether the number of switchings between plotted points and non-plotted points is few or not. If the number of switchings is few, it is likely that there are periodic points of period $k$. We gather time indexes of recurrence plot where periodic points of period $k$ are nearby. Let the set of time indexes as $P_k$. We take a union of $P_k$ over $k$. If this union is equal to a set of all time indexes for the recurrence plot, then we can find at least a periodic point for each time. Therefore, if the union of $P_k$ is equal to a set of all time indexes, then we say periodic points are r-dense.
The third relaxed condition, namely, r-sensitive dependence on initial conditions is much simpler. For each diagonal line which is apart from the central diagonal line by $k$, we have at least an interruption of plotted points.

Therefore, by replacing arbitrary open sets by open sets used for defining a recurrence plot, we can relax the three conditions of Devaney’s chaos. Since we relaxed the conditions, if one of the three conditions is denied, then the given time series is not consistent with Devaney’s chaos.

Let us see some examples (see Fig. 2 of [11]). In the case of random walk, r-topological transitivity is denied. Therefore, this time series is not consistent with Devaney’s chaos. In the case of autoregressive linear model, r-denseness of periodic points is denied. Therefore, this time series is not consistent with Devaney’s chaos. In the case of noisy periodic orbits, r-sensitive dependence on initial conditions is denied. In the case of Henon map, the recurrence plot shows that the underlying dynamics is r-topologically transitive, it has periodic points which are r-dense, and has r-sensitive dependence on initial conditions. Therefore, the time series of Henon map is consistent with Devaney’s chaos.

We applied this method to the dataset of squid giant axon (Fig. 4). Then, we find that the dataset of squid giant axon is with r-topological transitivity, r-denseness of periodic points, and r-sensitive dependence on initial conditions. Therefore, the dataset of squid giant axon is consistent with Devaney’s chaos.

5. Conclusions

In this paper, we have discussed the analysis of squid giant axon dataset. We first estimated the maximal Lyapunov exponent and the metric entropy. Both indexes show the same positive value of 0.42 bits/observation, implying that Pesin’s identity seems to hold. We also found using a recurrence plot that the dataset of squid giant axon is consistent with the mathematical definition of Devaney’s chaos. Therefore, the dynamics of squid giant axons seem to be a typical example of deterministic chaos.

Acknowledgements

This work was partially supported by a Grant in Aid for Scientific Research on Priority Areas, No. 17022012, and Scientific Research (A), No. 20246026, from the Japanese Ministry of Education, Culture, Sports, Science and Technology, and the Funding Program for World-Leading Innovative R&D on Science and Technology (FIRST Program) from the Japan Society for the Promotion of Science (JSPS). Y. H. was also partially supported by a Grant in Aid for Young Scientists (B), No. 21700249, from the Japanese Ministry of Education, Culture, Sports, Science and Technology.
References


