Detection of the thermoacoustic combustion instabilities of a slot burner based on a diagonal-wise recurrence quantification

R. Hernandez-Rivera, G. Troiani, T. Pagliaroli, and A. Hernandez-Guerrero

AFFILIATIONS
1 Mechanical Engineering Department, University of Guanajuato, Salamanca, Mexico
2 Energy Technology Department, Sustainable Combustion Laboratory, ENEA C.R. Casaccia, Rome, Italy
3 Engineering Department, Università degli Studi Niccolò Cusano, Rome, Italy

ABSTRACT
This paper presents a nonlinear time-series analysis of the thermoacoustic instabilities of an experimental slot burner. The main objective was the calculation of indexes capable of detecting in advance the combustion instabilities by gradually increasing the flow Reynolds number of the pilot burner. A chaotic analysis based on diagonalwise measurements of the recurrence plots was performed on the basis of which the following indexes were calculated: the $\tau$-recurrence rate index $RR_\tau$, the $\tau$-determinism index $DET_\tau$, the $\tau$-average diagonal line length index $L_\tau$, and the $\tau$-entropy index $s_\tau$. A quantification carried out by means of the standard deviation $\sigma$ and mean values $\bar{\mu}$ of the diagonalwise measurements showed that the aforementioned indexes were successfully able to sort all cases under analysis mainly into two groups: the first three cases that correspond to the stable regime named “Combustion Noise” and the remaining cases that were associated with the unstable regime called “Combustion Instability.” Additionally, the particle image velocimetry optical method was applied in order to compute a new index based on the velocity fields. The results showed that the index $V_h$, based on the local heights of the velocity profiles of the central flame, was also capable of detecting the same two groups previously identified by the nonlinear analysis. Nevertheless, the most sensitive indexes were the indexes $RR_\tau$, $DET_\tau$, and $s_\tau$ since these indexes were able to detect the transition between the combustion noise and combustion instability regimes. Therefore, the present results proved that the proposed five indexes were effective precursors in order to detect in advance the combustion instabilities.

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I. INTRODUCTION

In engineering applications where combustion processes take place within confined chambers, an interaction between noise sources, primarily produced by flame dynamics and unsteady heat release rates, and sound pressure waves may occur. There are basically two types of coupled interactions: a weak interaction in which the combustion remains mainly stable and a strong interaction where the acoustic fluctuations are synchronized about a resonant frequency of the burner producing self-sustained high-amplitude oscillations. The procedure begins when a driving process produces perturbations in the upstream flow such as velocity fluctuations. These velocity oscillations eventually induce heat release fluctuations in the downstream flow which finally excite acoustic oscillations. Then, a feedback process, usually performed by the acoustic wave propagation, couples these perturbations to the driving process, generating new and synchronized velocity fluctuations that may create a resonant interaction and conduct to self-sustained oscillations. As a consequence, the heat release and pressure fluctuations may oscillate at the same frequency. Another important issue to consider is the Rayleigh criterion, which states that the combustion process will add energy to the acoustic oscillations only if the phase difference between the heat release and pressure oscillations is lower than 90°.
Thus, the combustion instabilities will occur only if the energy added to the acoustic mode, by the combustion process, is higher than the energy losses, e.g., radiation and convection of acoustic energy out of the combustor, viscous dissipation, and heat transfer.

Therefore, the combustion instabilities are an oscillatory motion of gases that detrimentally affect the performance, reducing the life time and operability ranges of combustors; producing structural vibrations; and increasing surface heat transfer rates, thrust oscillations, and eventually failure of the combustion systems. Such instabilities are represented by acoustic modes that grow from low-amplitude pressure oscillations, through a transient regime, to a saturation process where constant high-amplitude pressure oscillations are sustained, a condition known as limit-cycle. In most cases, the acoustic modes are mainly longitudinal and transverse, e.g., in the study of a swirl stabilized combustor with the aim to analyze the effects of hydrodynamic features of a swirling flow on the thermoacoustic instabilities; the results showed that the instabilities occurred via a lock-in mechanism between heat release fluctuations (induced by hydrodynamic features) and a mixed first tangential and quarter wave longitudinal mode of the combustor. However, modern annular gas turbines have also displayed azimuthal modes which are more complex and less studied. Based on the frequencies and oscillation amplitudes, combustion processes can be classified into two categories: (i) a stable combustion regime named combustion noise composed of low-amplitude aperiodic fluctuations without a dominant frequency and (ii) an unstable combustion regime called combustion instability represented by high-amplitude periodic oscillations with a clear dominant oscillation frequency. Moreover, the transition from the combustion noise regime to combustion instability regime was indicated to occur by means of intermittent high-amplitude bursts composed of periodic oscillations that emerge among low-amplitude aperiodic oscillations.

Hence, since nonlinear interactions dominate the dynamics of the system when the oscillations of the instabilities grow exponentially until reaching a constant limit-cycle amplitude, an insufficient understanding of the nonlinear aspects of the thermoacoustic instabilities may incorrectly predict the amplitude of the oscillations and cause an inadequate control of the combustion systems. Therefore, nonlinear methods must be applied. For example, and with regard to studying the connection between the combustion instabilities and the flame blowout of a lean premixed laminar conical flame, nonlinear time-series analyses such as bifurcation plots and reconstruction of the phase space trajectories were employed. The flame was confined in a vertical glass duct where the flame location was vertically varied and pressure fluctuations were measured at different locations. The bifurcation analysis, which consists in plotting the highest local amplitudes of the pressure fluctuations, revealed several regions based on the vertical location of the flame: (i) limit-cycle oscillations represented by an isolated single loop attractor visualized in its phase space associated with periodic oscillations with a single dominant frequency, (ii) a dense toroidal attractor that is related to a quasiperiodic state because of the presence of a second frequency peak along the dominant one, (iii) burst oscillations composed of low-amplitude bursts (representing the steady state) and high-amplitude bursts (representing the unstable limit-cycle) where the attractor is composed of a small cloudy core (steady state) surrounded by large circular orbits (unstable limit-cycle), and (iv) the flame blowout that takes place. This study proved that the application of nonlinear analysis is an efficient tool in order to identify the system dynamics oscillations.

Moreover, a nonlinear time-series analysis was also employed to study the dynamic behavior of the combustion instabilities of a lean premixed gas-turbine combustor. Thus, pressure fluctuations were acquired experimentally for several increments of the equivalence ratio. In addition, the phase spaces of the pressure fluctuations were reconstructed by using Takens’ embedding theorem in which an adequate time lag was selected by employing the mutual information method, instead of using the autocorrelation function, because it captures more efficiently the determinism of the time-series. The results showed three different behaviors: (i) the pressure fluctuations are composed of irregular low-amplitude oscillations in which their attractor fills the core of the phase space without showing a deterministic behavior, (ii) intermittent large-amplitude pressure fluctuations emerge among low-amplitude fluctuations where their attractor in the phase space is basically composed of an inner small ring surrounded by an outer large ring, and (iii) the high-amplitudes of the pressure oscillations become regular and their phase space displays only the trajectories of the outer large ring where the periodicity is highly remarkable. Therefore, this study showed that the dynamics of the combustion instabilities occurs via a transition from stochastic to periodic oscillations among low-dimensional chaotic oscillations.

On the other hand, and with a view to develop an index acting as the earliest warning signal of an impending oscillatory combustion instability, linear and nonlinear analyses were applied to a liquid rapid mixer oil burner that was operated from stable to unstable conditions by increasing the fuel mass flow (liquid oil). Time histories of pressure and radiant energy fluctuations were acquired simultaneously by means of a microphone and a photodiode. The linear methods were performed by applying cross correlations and spectral analyses to the time-series. On the other hand, the nonlinear analysis was carried out by reconstructing the pressure and radiant energy phase spaces of their time-series by employing the time delay embedding method. Then, the interdependence index was computed for several windows along the time-series. This index measures the similarity between the original and mutual neighborhoods of corresponding attractor points, where the mutual nearest neighbors of a particular point are those points that fall within the same attractor but are evaluated at the occurrence times of the original neighbors (around a corresponding point) in the other attractor. The results showed that the linear analyses, performed here by cross correlations and spectral analyses, were not capable of detecting the combustion instabilities since high values of cross correlations were found in stable and unstable conditions as well as dominant frequencies in both conditions. Conversely, the results of the nonlinear analysis showed that the interdependence index was able to identify complex nonlinear correlations and synchronizations more accurately than the linear analyses and correctly detect the impending combustion instability.

Furthermore, a nonlinear time-series analysis based on a recurrence quantification applied to two experimental laboratory-scale combustors, a swirl-stabilized and a bluff-body-stabilized backward-facing-step, was used in order to compute three indexes (precursors) able to recognize an impending combustion...
instability. In the experiments, pressure fluctuations were acquired by using piezoelectric transducers and several cases were investigated by gradually increasing the Reynolds number of both combustors. The nonlinear time-series analysis consisted in reconstructing the phase spaces by means of the algorithms Average Mutual Information (AMI) and Averaged False Nearest Neighbor (AFNN) in order to estimate an optimum time lag and an appropriate embedding dimension, respectively. Afterward, the dynamics of several combustion regimes were studied by tracking the recurrences of the trajectories in their phase spaces, i.e., the recurrence plots (RPs) were obtained by identifying the time when the system returns nearly to the same area in the phase space. The three indexes based on this recurrence quantification were the recurrence rate index RR that measures the density of recurrence points within the recurrence plot (RP), the time index \( t_0 \) that quantifies the vertical lines of the RP, and the Shannon entropy index \( s \) that measures the diagonal lines of the RP. The results showed that the three indexes correctly identified the combustion regimes from combustion noise to combustion instability.

Therefore, nonlinear analyses based on the reconstruction of the phase space of measured pressure fluctuations have proved to conserve the nonlinear behavior of the combustion processes. In addition, the RP quantification showed to be an effective tool in order to calculate indexes based on the type of measurement: either the density of recurrence dots or vertical and diagonal lines. Nevertheless, ordinary RP quantification produces only one value per RP, i.e., each RP is entirely quantified and a single value is exclusively obtained. The distribution of recurrence points inside each RP was not considered and investigated in the aforementioned studies. On the other hand, another type of recurrence quantification, based on a diagonalwise measurement, quantifies separately each diagonal line (parallel to the main diagonal) of a RP. This technique provides a quantified distribution of the evolution of recurrence dots within each RP. Consequently, in this paper, the combustion instabilities were analyzed by means of a diagonalwise quantification in order to examine the distribution of recurrent points inside each RP that may give a new insight into the nature of instabilities, information that is not currently available in the literature.

Hence, in the present study, pressure fluctuations were acquired from an experimental slot burner by gradually increasing the flow Reynolds number of both pilots. A Kaplan-Glass test was applied with the aim to study the degree of determinism of the pressure time-series acquired during conditions of stable combustion in order to determine whether the combustion noise regime is effectively composed of chaotic oscillations. Moreover, the development of the recurrence points inside each RP was studied with an aim to calculate new indexes based on the distribution of the recurrence dots in order to identify the several regimes of the combustion processes and detect in advance a combustion instability. On the other hand, the optical method called Particle Image Velocimetry (PIV) was simultaneously applied along with the pressure fluctuations with the goal to obtain the velocity fields. The objective here was to visualize and study the velocity fields during stable and unstable conditions of combustion with a view to compute an index able to detect the combustion instabilities.

The manuscript is organized as follows: Sec. II gives an introduction of the theoretical background with regard to the methods that were employed in order to apply nonlinear analyses, Sec. III shows the setup of the experimental measurements, Sec. IV gives an examination of the results, and Sec. V presents the final discussions and conclusions.

II. THEORETICAL BACKGROUND

A. Phase space reconstruction

In this article, the combustion instabilities were analyzed by quantifying the RPs. Thus, in order to obtain the RPs, the pseudophase space was primarily reconstructed by employing the time delay embedding method\(^ {17-19} \) where the delay vectors are given by the following equation:

\[
y_i(d) = (x_i, x_{i+t}, \ldots, x_{i+(d-1)t}),
\]

where \( i = 1, 2, \ldots, N_0 = (d - 1)t, N_0 \) is the time-series length, \( x_i \) is a time-series, \( y_i \) is the \( i \)th reconstructed vector, \( d \) is the embedding dimension, and \( t \) is the time delay.

B. Average mutual information (AMI)

In the first place, an optimum time delay \( \tau_{opt} \) was determined by using the Average Mutual Information (AMI) algorithm by means of the following equation:

\[
I(X, Y) = \sum_{k=1}^{N_y} \sum_{l=1}^{N_x} p_{x_k, y_l} \ln \left( \frac{p_{x_k, y_l}}{p_{x_k} p_{y_l}} \right),
\]

where \( X = x_i \) represents the original time-series, \( Y = x_{i+t} \) corresponds to the delay time-series, \( p_{x_k} \) and \( p_{y_l} \) are the marginal probabilities, \( p_{x_k, y_l} \) is the joint probability, and \( N_x \) and \( N_y \) are the number of bins of the X and Y histograms, respectively. With the aim to compute the marginal probability \( p_{x_k} \), a histogram with \( N_x \) bins of the vector X was initially calculated. Then, the number of elements within each bin is divided by the total number of elements of the entire histogram. A similar procedure is followed to calculate \( p_{y_l} \) by using the vector Y and the \( N_y \) bins. Concerning the joint probability \( p_{x_k, y_l} \), a matrix is first created with \( N_x \) columns (representing the marginal probability \( p_{x_k} \)) and \( N_y \) rows (expressing the marginal probability \( p_{y_l} \)). Thus, in each column (a particular bin \( N_x \)), the elements of the vector X within this selected bin are searched along the vector Y and a new histogram with \( N_y \) bins is calculated (based on the found elements) and eventually divided by the total number of elements of the new histogram. The procedure is then repeated on each column until the matrix of size \( N_x \times N_y \) is filled. Finally, Eq. (2) is used to compute the values of AMI by gradually increasing the time delay \( \tau \) in the vector Y. Hence, AMI measures the shared information between two random vectors X and Y based on the transmitted information of one vector by giving information of the other, e.g., if the vectors X and Y are independent, then AMI is equal to zero. Concerning the time delay \( \tau \), a short delay will not yield new information since both vectors are nearly the same; on the contrary, a long delay will yield vectors that are statistically independent and will not be connected.\(^ {18} \) Therefore, an optimal delay \( \tau_{opt} \) was selected by gradually delaying the vector Y and choosing the first minimum value of AMI.

C. Averaged false nearest neighbor (AFNN)

Next, an appropriate embedding dimension \( d_e \) was estimated by using the Averaged False Nearest Neighbor (AFNN)\(^ {17} \) method
that actually is an optimized version of the False Nearest Neighbor (FNN) algorithm. The AFNN method computes the average change in the distances between a given point and its closest neighbor; meanwhile, the dimension $d$ is being increased. When the attractor is unfolded, any two points that stay close in the dimension $d$ will still be close in the dimension $d + 1$ and are called real neighbors; otherwise, they are named false neighbors. Then, an adequate embedding dimension $d_o$ is chosen when false neighbors are excluded. Essentially, the computation of AFNN begins with the following equation:

$$a(i,d) = \frac{\| y_i(d+1) - y_{n(i,d)}(d+1) \|}{\| y_i(d) - y_{n(i,d)}(d) \|},$$

(3)

where $i = 1, 2, \ldots, N_0 - d\tau$; $\|\|$ represents the measurement of Euclidean distance; $y_i(d + 1)$ is the $i$th reconstructed vector of the phase space with embedding dimension $d + 1$; $n(i,d)$ is an integer; and $y_{n(i,d)}(d)$ is the nearest neighbor of $y_i(d)$ in the $d$ dimension of the reconstructed phase space. Afterward, the quantity $a(i,d)$ is averaged in the following manner:

$$E(d) = \frac{1}{N_0 - d\tau} \sum_{i=1}^{N_0-d\tau} a(i,d).$$

(4)

To investigate the variation from $d$ to $d + 1$, the quantity $E(1)(d)$ is described as

$$E1(d) = E(d + 1)/E(d).$$

(5)

The minimum embedding dimension $d_o$ was selected when the variable $E(1)(d)$ stopped changing; meanwhile, the dimension $d$ was still being increased; thus, $d_o = d + 1$, where $d$ is the dimension at which the quantity $E(1)(d)$ stopped changing.

D. Recurrence plots

Once the values $\tau_{opt}$ and $d_o$ were obtained, the pseudophase space was reconstructed by means of Eq. (1). Afterward, the recurrences of the trajectories in the pseudophase space were measured by computing the RP based on the following equation:

$$R_{ij} = \Theta(e - ||y_i - y_j||),$$

(6)

where $i, j = 1, 2, \ldots, N_1$; $N_1$ is defined as $N_0 - (d_o - 1)\tau_{opt}$; $\Theta$ is the binary recurrence function; $e$ is the threshold distance; and $|| ||$ is a norm represented here as the Euclidean distance. During a recurrence, the distance between two points does not exceed the threshold $e$ and the Heaviside function yields a value $\Theta(0) = 1$; otherwise, there is a nonrecurrence and $\Theta(0) = 0$. In the recurrence matrix, a recurrence is denoted as $R_{ij} = 1$ and plotted as a black dot and a nonrecurrence is denoted as $R_{ij} = 0$ and plotted as a white dot.

E. Recurrence quantification analysis

The quantification of the RPs was carried out by means of a diagonalwise measurement. In this technique, each diagonal line parallel to the main diagonal of a RP is separately quantified, where the main diagonal of a square recurrence matrix is defined as the line that crosses the matrix from the lower left corner to the upper right corner. The distance between the diagonal line under analysis and the main diagonal is called "r," where $r = 0$ is the main diagonal, $r > 0$ are the diagonal lines above, and $r < 0$ are the diagonal lines below. Thus, the quantification of the RPs was initially performed by calculating the number of black diagonal lines $P_r(l)$ of length $l$ on each diagonal line by means of Eq. (7). Based on this calculation, some indexes were determined.

$$P_r(l) = \sum_{ij} (1 - R_{i-1j-1})(1 - R_{i+1j+1}) \prod_{k=0}^{l-1} R_{i+kj+k}.$$  

(7)

1. The $\tau$-recurrence rate index $RR_\tau$

The $RR_\tau$ index basically measures the density of black dots per diagonal line and is defined by the following equation:

$$RR_\tau = \frac{1}{N_1} \sum_{l=1}^{N_1-\tau} IP_r(l).$$

(8)

2. The $\tau$-determinism index $DET_\tau$

This index measures the ratio of black dots that form diagonal lines (of length greater than or equal to $l_{min}$) to all black dots per each diagonal line under analysis and is defined by the following equation:

$$DET_\tau = \frac{\sum_{l=l_{min}}^{N_1-\tau} lP_r(l)}{\sum_{l=1}^{l_{min}} lP_r(l)}.$$  

(9)

3. The $\tau$-average diagonal line length index $L_\tau$

The $L_\tau$ index measures the mean length of the black diagonal structures found on the diagonal line under analysis and is defined by the following equation:

$$L_\tau = \frac{\sum_{l=l_{min}}^{N_1-\tau} lP_r(l)}{\sum_{l=1}^{l_{min}} P_r(l)}.$$  

(10)

4. The $\tau$-entropy index $s_\tau$

The $s_\tau$ index measures the Shannon entropy of each diagonal line under analysis and is defined by the following equation:

$$s_\tau = -\sum_{l=l_{min}}^{N_1-\tau} P_r(l) \ln P_r(l),$$  

(11)

where the probability $p(l)$ of finding a diagonal line of length $l$ is given by $p(l) = P_r(l)/N_1$ and $N_1$ is the sum of all black diagonals of length $l$ found on the diagonal line under investigation.

F. The Kaplan-Glass test

On the other hand, in this research, the Kaplan-Glass test was applied with an aim to determine the degree of determinism of a time-series. Initially, the phase space of pressure fluctuations is reconstructed by means of Eq. (1). Then, the entire phase space is coarse grained into a grid, in which nonoverlapping hypercubes (boxes) of dimension $d$ and length $\varepsilon$ are employed. The main idea of this method is to evaluate points that are close in the pseudophase space but separated in time. Thus, the change in state from time $t_{ij}$...
to $t_{j,k} + \delta$ for each of the $n_j$ points within box $j$, where $k = 1, \ldots, n_j$ is given by
\[
\Delta \bar{x}_{j,k} = \sin \left( \frac{2\pi}{\lambda_c} \left[ t_{j,k} + (d - 1) \tau_{opt} + \delta \right] \right) - \sin \left( \frac{2\pi}{\lambda_c} \left[ t_{j,k} + (d - 1) \tau_{opt} \right] \right),
\]
(12)
where $\lambda_c$ is the characteristic length defined as twice the standard deviation of the time-series under analysis and $\delta$ is the time delay. Thereafter, the $n_j$ vector displacements $\Delta \bar{x}_{j,k}$ are normalized to unit length and subsequently added. By dividing this summed vector by $n_j$, it yields the resultant vector $\bar{V}_j$ given as
\[
\bar{V}_j = \frac{1}{n_j} \sum_{k=1}^{n_j} \frac{\Delta \bar{x}_{j,k}}{\| \Delta \bar{x}_{j,k} \|}.
\]
(13)

In the case of deterministic time-series, all $n_j$ vectors of box $j$ point precisely in the same direction and the magnitude of the resultant vector $\| \bar{V}_j \|$ is equal to one. On the contrary, stochastic time-series distribute the $n_j$ vectors randomly and isotropically, and as a consequence, the magnitude of the resultant vector $\| \bar{V}_j \|$ is lower than one with expectation $c_d \sqrt{n_j}$, where $c_d$ is a constant that depends on the embedding dimension $d$. Next, the magnitude of the resultant vector $\| \bar{V}_j \|$ is averaged separately for all boxes in which $n_j = n$, for $n \geq 2$. It produces the vector $\bar{T}_n$ defined as
\[
\bar{T}_n = \langle \| \bar{V}_j \| \rangle.
\]
(14)

Finally, these values are averaged in the following manner:
\[
\bar{\lambda} = \frac{(\bar{T}_n)^2 - c_d^2/n}{1 - c_d^2/n}.
\]
(15)

Deterministic time-series yield values of $\bar{\lambda}$ about one and stochastic time-series around zero.

III. EXPERIMENTAL SETUP

A. The slot burner

The experimental slot burner under analysis is shown schematically in Fig. 1. During the design stage of this burner, acoustic and aeroacoustic analyses under nonreactive conditions were previously performed.\textsuperscript{27}

The combustion chamber is composed of a transparent rectangular duct made of quartz which is open to the atmosphere at the top and has a wall thickness of 5 mm, a vertical length of 120 mm, and an inner cross-sectional area of $27 \times 10$ mm$^2$. The combustor is composed of a central exhaust nozzle slot and two pilots located on both sides; see Figs. 1(a) and 1(b). With regard to the central slot, it has an inner cross-sectional area of $3 \times 10$ mm$^2$, whereby a premixed mixture of methane and air is injected into the chamber. Moreover, each pilot has a cross-sectional area of $9 \times 10$ mm$^2$ and contains an array of $5 \times 4$ circular perforated holes with diameters of about 1.5 mm through which a premixed mixture of propane and air is ignited; see Fig. 1(c).

![FIG. 1. Experimental slot burner under analysis: (a) front view of the combustion chamber, (b) front view of the combustor (cross section), and (c) upper view of the combustor.](image-url)
B. The PIV and pressure data acquisition setup

In order to visualize the flow structures within the combustion chamber, the Particle Image Velocimetry (PIV) technique was performed. A double cavity Nd:YAG Quantel laser (Big Sky Laser 200 mJ at 532 nm) was used in order to produce during 8 ns a laser sheet with a thickness of about 1 mm along with an Imperx CCD camera B2041 with an image resolution of 2048 × 2048 px. A 105 mm lens equipped with a filter, which rejects all wavelengths except 532 nm, was used in conjunction with the camera, yielding a magnification factor of 27.17 px/mm. Figure 2 shows a schematic sketch of the experimental setup. A set of image pairs was acquired at a rate of 10 Hz in which the time delay between two continuous laser pulses was 10 μs. In the postprocessing processes, the interrogation windows were varied from 64 × 64 px to 16 × 16 px by employing a recursive technique. Aluminum dioxide with particle diameters of about 3 μm was used as the seeding material and introduced into the chamber through the central slot.

On the other hand, pressure fluctuations were acquired at a sampling frequency of 51.2 kHz by using a microphone located about 8 cm in front and 5 cm above the combustion chamber. The employed microphone was a Microtech Gefell 1/4” Electret-Measuring Microphone M 360 Class 1 with a frequency range from 20 Hz to 20 kHz and a high sensitivity of 12.5 mV/Pa. Throughout the experiments, the Reynolds number was computed by using the definition $Re = \frac{mA}{\mu A}$, where $m$ is the mass flow rate defined as $m = m_a + m_f$ (where the subscript “a” stands for air and “f” stands for fuel), $D_H$ is the hydraulic diameter defined as $D_H = \frac{4A}{P}$ (where $A$ is the cross-sectional area and P is the wet perimeter), and $\mu$ is the dynamic viscosity defined as $\mu = X_m \mu_a + X_f \mu_f$ (where $X_m$ is the molar fraction and the properties of air and fuel were considered constant at ambient conditions). In addition, the equivalence ratio was defined as $\phi = \frac{(m_f/m_a)_\text{actual}}{(m_f/m_a)_\text{stoich}}$. The experiments were conducted by holding the volumetric flow rates of methane and air of the central slot constant at 2717 and 25944 ml/min, respectively. The Reynolds number of the central slot was calculated by using an area $A$ and perimeter $P$ calculated from the rectangular exhaust section of the central slot. It yielded a constant $Re = 5000$ and equivalence ratio $\phi = 1$. In a similar manner, the propane volumetric flow rate of both pilots was kept constant along all experiments at 300 ml/min. On the contrary, the air volumetric flow rate of both pilots was varied from 5000 to 7000 ml/min in increments of 250 ml/min where the mass flow controllers have a measurement uncertainty less than 1%. The Reynolds number of both pilots was computed by considering the area $A$ and perimeter $P$ as 40 times the area and perimeter of a single perforated hole, respectively. In this study, $Re$ of both pilots was used as a control parameter and varied from 133 to 181 in increments of six, producing also variations in the equivalence ratio of the pilots from 1.37 to 0.98 (toward a leaner mixture). On the other hand, during the experiments, the burner was deactivated (the mass flow rates of air and fuel were not introduced into the combustor) and the background noise was recorded by the microphone. The frequency spectrum of the background noise showed to have amplitudes of about $1 \times 10^{-8}$. On the contrary, the cases under investigation displayed amplitudes around $1 \times 10^{-5}$. Due to the fact that the signal to noise level is very high, the background noise was not removed from the measurements.

IV. RESULTS

A. The time-series under investigation and linear analysis

Pressure fluctuations were acquired at a sampling frequency of 51.2 kHz. Nevertheless, and with the aim to reduce the computational cost, the data were undersampled to a frequency of 8.53 kHz where a bandpass filter with cutoff frequencies of 340 Hz and 1550 Hz was employed. Each case was analyzed by using a window of 1 s at a sampling frequency of 8.53 kHz, yielding 8530 samples per case study. Except for the case $Re = 151$, the time interval under analysis was always selected from 0 to 1 s. In the case $Re = 151$, the time interval was selected from 0.5 to 1.5 s because such selection represents more accurately its remaining time-series. Thus, the time-series of the pressure fluctuations under analysis are shown in Fig. 3. As is observed, the amplitudes of the oscillations concerning the cases from $Re = 133–145$ remain relatively small and constant; see Figs. 3(a)–3(c). Contrarily, a sudden growth in the amplitudes of the oscillations begins in the case $Re = 151$ and continues along the case $Re = 157$; see Figs. 3(d) and 3(e), respectively. Afterward, the amplitudes of the oscillations reach a relative high and constant amplitude, a phenomenon known in the literature as limit-cycle amplitude, see Figs. 3(f)–3(i). On the other hand, a linear study performed by a spectral analysis was applied to the time-series under investigation. Thus, the Power Spectral Density (PSD) was computed for each case study and the results are displayed in Fig. 4.

Figures 4(a)–4(c) depict a broadband noise spectrum without showing a dominant frequency typically observed in the spectra of the combustion noise regime. Concerning the case $Re = 151$, a dominant frequency of 750 Hz starts to emerge among the broadband noise; see Fig. 4(d). The remaining cases, from $Re = 157–181$, show a very remarkable dominant frequency (the fundamental one) at 750 Hz where the second harmonic at 1500 Hz is also observed. Higher harmonics are as well noticed; however, they are almost vanished due to the bandpass filter. A remarkable dominant
FIG. 3. Time-series under analysis for the cases (a) Re = 133, (b) Re = 139, (c) Re = 145, (d) Re = 151, (e) Re = 157, (f) Re = 163, (g) Re = 169, (h) Re = 175, and (i) Re = 181.

FIG. 4. Spectrum of frequencies where the vertical axis corresponds to the Power Spectral Density (PSD) for the cases (a) Re = 133, (b) Re = 139, (c) Re = 145, (d) Re = 151, (e) Re = 157, (f) Re = 163, (g) Re = 169, (h) Re = 175, and (i) Re = 181.
frequency is a characteristic feature of the combustion instability regime. Therefore, and based on the results of this linear analysis, it is concluded that the case \( Re = 151 \) indicates the beginning of the instabilities and, in addition, a possible transition between the cases without a dominant frequency (combustion noise regime) and with a remarkable frequency (combustion instability regime) because the amplitude of its remarkable frequency is relatively lower than the amplitudes of the dominant frequencies of the remaining cases.

1. The Kaplan-Glass test

The Kaplan-Glass test was performed in order to determine the degree of determinism of the time-series under analysis; see Sec. II F for a theoretical review. Based on the previous results of the spectral analysis, the first cases do not show a dominant frequency in their spectra, whereas the last cases display a very remarkable frequency. Thus, and with a view to analyze and highlight these conditions, the Kaplan-Glass test was applied only to the first and last cases, i.e., the cases of \( Re = 133 \) and 181, respectively. Hence, a grid composed of 32 boxes in 5 dimensions was employed by using an embedding dimension of \( d = 5 \). Figure 5 shows the results of the Kaplan-Glass test.

With regard to Fig. 5(a), it is observed that the case \( Re = 181 \) displays a higher determinism because its values \( \Lambda \) are closer to one. This is an expected result since this case shows a remarkable fundamental frequency in its spectrum, i.e., this time-series is deterministic. On the other hand, the case \( Re = 133 \) shows a lower determinism; however, its values \( \Lambda \) are still high along short time delays \( \delta \), and hence, this time-series may be considered as partially deterministic. In order to confirm the partial determinism of the case \( Re = 133 \), Fig. 5(b) shows the comparison between this case and its surrogate data, where the consecutive order of the vector components of its time-series was randomly changed in order to eliminate any determinism. As is noticed, the original time-series has higher values of \( \Lambda \) than its own surrogate data. The observed spikes of the surrogate data are produced when the time delay \( \delta \) is a multiple of the optimum time delay \( \tau_{opt} \) due to an overlap and must be ignored. Thus, due to the fact that each value of \( \Lambda \) is a multiple of the original time-series that is higher than its own surrogate data, and because such values are relatively high for short delays \( \delta \), it is concluded that the aperiodic oscillations of the time-series of the case \( Re = 133 \) can be considered as partially deterministic, which leads to the conclusion that the combustion noise regime is composed of deterministic chaos.

B. The chaotic nonlinear analysis

In order to calculate indexes capable of identifying in advance an impending oscillatory combustion instability, a nonlinear time-series analysis based on a chaotic study was primarily applied. As a first step, the pseudophase space was reconstructed by means of Eq. (1) by calculating an optimal time delay \( \tau_{opt} \) and an adequate embedding dimension \( d_{opt} \); see Secs. II B and II C, respectively. Table I shows the calculated values for all cases under analysis, where \( \lambda_A \) is the attractor’s size defined as the largest distance between two points within the phase space.

It is observed that high embedding dimensions are required in order to embed the attractors. Moreover, \( \lambda_A \) tends to be larger as \( Re \) is increased. The attractor’s sizes along the cases from \( Re = 133–145 \) and from \( Re = 163–181 \) nearly conserve the same size, respectively. However, a sudden growth of the attractor’s size is observed to begin in the case \( Re = 151 \) and moderately continue along the case \( Re = 157 \). It indicates that the case \( Re = 151 \) is the beginning of a transition. By taking into account the above computed values, Fig. 6 shows the reconstructed pseudophase spaces of all cases.

Figures 6(a)–6(c) show small attractors composed of a dense core in which the orbits are tangled, commonly observed in chaotic phenomena due to aperiodic oscillations. The attractor depicted in Fig. 6(d) shows a small dense core composed of tangled trajectories and some large circular orbits around it. This kind of attractor is commonly observed during a transition where the small dense core corresponds to a stable combustion and the large circular orbits correspond to an unstable combustion. Regarding the attractors from Figs. 6(e)–6(i), they are composed of a dense toroidal structure which is an indication of a quasiperiodic state because of a dominant frequency along with a second frequency peak. Hence, due to the type of oscillations of each time-series, two vast classifications were observed along the reconstructions of the phase space trajectories: (i) small attractors composed of tangled orbits related to the low-amplitude aperiodic oscillations regarding the time-series of the first three cases and (ii) attractors composed of large circular orbits related to the high-amplitude periodic oscillations concerning the time-series of the last five cases. Moreover, the fourth case displays an attractor formed by a combination of both characteristics, i.e., the attractor is composed of a small dense core that is surrounded by large circular orbits. This attractor gives evidence of a transition between both classifications.

![Fig. 5](image-url)
C. Quantification of recurrence plots

In order to obtain quantifiable results and with a view to calculate the indexes, a threshold \( \varepsilon \) has to be held constant throughout all cases and slightly larger than the smallest attractor’s size.\(^\text{13}\) Therefore, a threshold \( \varepsilon_0 \) was set as 103% the attractor’s size of the case \( Re = 133 \) and remained constant along all cases. Appendix A shows the development of the RPs by using the threshold \( \varepsilon_0 \). Figures 16(a)–16(c) basically display black square matrices because the threshold \( \varepsilon_0 \) is slightly larger than the attractor’s size of the first case and, in addition, because the first three cases essentially conserve the same attractor’s size. Nevertheless, the RPs for the cases of \( Re = 139 \) and 145 exhibit few isolated white dots because some orbits in their pseudophase space are occasionally larger than the threshold \( \varepsilon_0 \). Likewise, the RP depicted in Fig. 16(d) shows a pattern of black and white lines around the upper right corner and black patches close to the lower left corner. The pattern of black and white lines corresponds to the times when the large circular orbits of its attractor are larger than the threshold \( \varepsilon_0 \) and the black patches correspond to the times when the tangled trajectories of the small dense core are lower than the threshold. Conversely, the remaining cases, see Figs. 16(e)–16(i), display an arrangement composed entirely of diagonal lines because their attractors contain only circular orbits that are much larger than the threshold \( \varepsilon_0 \), and as a consequence, only periodic occurrences are observed (the RPs were enlarged; otherwise, the diagonal lines will not be visible). A complete arrangement of diagonal lines in the RP is an indication of periodic oscillations in which the horizontal (vertical) distances between two lines indicate the period.\(^\text{13}\) Hence, Fig. 16(i) shows that there are around 15 spaces within 0.02 s which yields a period of about 1.33 ms. This quantity matches the one previously calculated in the frequency spectrum of the same case, see Fig. 4(i), where a dominant frequency was detected at 750 Hz and produced a period of 1.33 ms.

On the other hand, in this paper, the development of the recurrence points (black dots) inside each RP was quantified. Such quantification may give new knowledge concerning how the combustion processes evolve within each RP. This approach is missed by using an ordinary quantification in which a single result is obtained in each RP. Therefore, the diagonalwise recurrence quantification was applied to the aforementioned sequence of RPs; see Sec. II E for a theoretical review. Since RPs are symmetric with respect to the main diagonal, the quantification above the main diagonal \((\tau > 0)\) is equal to the quantification below it \((\tau < 0)\), and thus, only the diagonals above the main diagonal were here analyzed. The results of the indexes \( RR_\tau, DET_\tau, L_\tau, \) and \( s_\tau \) are shown in Appendix B.

Concerning the index \( RR_\tau \) that measures the probability when a state returns to its threshold after \( \tau \) time steps,\(^\text{13}\) its results are displayed in Fig. 17. As is shown, along the first three cases, the index \( RR_\tau \) is about one throughout all diagonals. These are expected results since the recurrence matrices of these cases are mainly black and contain full black diagonals on each diagonal under analysis. As a consequence, densities of black dots close to one are expected on each diagonal under consideration. On the contrary, the fourth case \( Re = 151 \), see Fig. 17(d), shows fluctuations throughout the diagonals. A closer examination of these oscillations is displayed in Fig. 7.

As Fig. 7(a) depicts, the first diagonals show cyclic fluctuations in which the amplitudes oscillate between 1 and 0.4 with a period of about 1.3 ms. This is an indication of a periodic pattern composed of complete and partial black diagonals. A partial black diagonal is a diagonal that contains about 50% black and 50% white lines, whereas a complete black diagonal is a diagonal that consists mainly of black lines. Likewise, an empty white diagonal is a diagonal that is essentially white, i.e., it almost does not contain black lines. Hence, there is a high probability followed by a middle one that the orbits recur to the threshold after \( \tau \) time steps. The middle probability is caused...
FIG. 7. Close-up of the oscillations of the index $RR_\tau$, for the case $Re = 151$: (a) first diagonals, (b) middle diagonals, and (c) last diagonals.

Conversely, the last diagonals shown in Fig. 7(c) display fluctuations in which their amplitudes periodically oscillate around 0 and 1 with a period of about 1.4 ms. This points out the presence of an oscillatory array of complete black and white diagonals which eventually evidence high and low probabilities for the system to recur to its threshold. In other words, this pattern reveals perfect periodic oscillations related to the larger circular orbits of its attractor. A similar behavior was observed along the last diagonals of the case $Re = 181$ in which a closer examination of its last diagonals, shown in Fig. 8(a), reveals a periodic distribution of near full black and empty white diagonals due to the oscillating values of the index $RR_\tau$ between one and zero, respectively, which eventually indicate perfect periodic oscillations of the system. Moreover, the spectral analysis of its cyclic oscillations throughout all diagonals shows a fundamental frequency of 750 Hz and several harmonics; see Fig. 8(b). This fundamental frequency coincides with the dominant frequency previously calculated in the spectral analysis of its time-series; see Fig. 4(i).

On the other hand, the indexes $DET_\tau$, $L_\tau$, and $s_\tau$ require the selection of a minimum length $l_{\text{min}}$ in order to reduce the tangential motion effect, an effect that produces thicker and longer diagonal lines in the RPs because a large threshold may also include consecutive points of the trajectory. The minimum length is usually set to 2 for most applications. Hence, in this study, the minimum length was considered as $l_{\text{min}} = 2$.

Based on the previous consideration of $l_{\text{min}}$, the results of the index $DET_\tau$ are shown in Fig. 18. The index $DET_\tau$ measures the proportion of black lines longer than $l_{\text{min}}$ with respect to all black due to the black patches displayed on the lower left side of its RP, see Fig. 16(d), which partially cover the periodic white diagonals that eventually become partial black diagonals. Then, instead of having a periodic pattern of black and white diagonals, due to the black patches, now this arrangement is composed of complete and partial black diagonals. On the other hand, Fig. 7(b) displays intermittent fluctuations of small amplitudes with a mean around 0.7 and pulsed distances between two peaks of about 1.4 ms. This result reveals an oscillating arrangement of consecutive pairs of partial black diagonals in which the first one is slightly fuller (darker) than the second one. The fuller partial black diagonals are produced by discontinued black diagonals and the emptier partial black diagonals due to the black patches that still cover the white diagonals.
lines on each diagonal under analysis. It can also be considered as the probability that a recurrence point belongs to a diagonal line. Thus, and because of the entire black RPs, the first three cases basically show values of $\text{DET}_\tau$ equal to one because all measured black lines were longer than $l_{\text{min}}$; see Figs. 18(a)–18(c). The following cases show variations of the index $\text{DET}_\tau$ based on their ratio of black lines longer than $l_{\text{min}}$. This index yields values equal to zero (on each diagonal under analysis) if black lines longer than $l_{\text{min}}$ were not found, but the number of total black lines is different from zero.

In order to determine the dominant frequencies of the distribution of the index $\text{DET}_\tau$, a spectral analysis was applied to the entire diagonals of the case $Re = 181$ and the results are presented in Fig. 9(b). As is observed, the spikes of the spectrum show a subharmonic at 375 Hz and several harmonics at 750 Hz, 1502 Hz, 2252 Hz, and so on. Therefore, and despite that Fig. 9(a) does not show a periodic behavior, the spectrum of the entire set of diagonals reveals the combustion instability frequency of 750 Hz previously calculated in its time-series.

Likewise, the results of the index $L_\tau$ are shown in Fig. 19. This index basically measures the average length of the black diagonals computed on each diagonal under investigation. As Fig. 19(a) depicts, along the case $Re = 133$, the average lengths of black diagonals decay linearly because its RP is entirely black and is only composed of complete black diagonals on each considered diagonal, e.g., for $\tau = 1$, the mean length is $N_1 - 1$, for $\tau = 2$ is $N_1 - 2$, and so on. In the cases $Re = 139$ and 145, the decay of the average lengths is still linear; however, some drops are observed because some considered diagonals no longer contain entire black diagonals.

The remaining cases simply display the average lengths of the black diagonals on each diagonal under investigation. As an example, Fig. 10(a) depicts a close-up of the average lengths along the case $Re = 181$ which reveals a periodic pattern composed of partial black and empty white diagonals. Moreover, a spectral analysis of the same case exposes harmonic frequencies at 752.1 Hz, 1500 Hz, 2992 Hz, and so on that coincide with the combustion instability of 750 Hz.

On the other hand, the results of the index $s_\tau$ are depicted in Fig. 20. This index essentially measures the disorder of the system by means of the Shannon entropy, e.g., systems that contain high disorders will produce high values of the Shannon entropy and vice versa. Hence, Fig. 20(a) shows the results of the case $Re = 133$ in which the index $s_\tau$ is equal to zero along all diagonals. Because the RP of this case is entirely black, each diagonal under analysis contains only one black line, the longest possible one, which crosses throughout the RP (from one side to the other). Therefore, since there is only one black line on each considered diagonal, there is a perfect order which eventually yields a value of the Shannon entropy equal to zero. Likewise, in the cases $Re = 139$ and 145, the values of the index $s_\tau$ begin to irregularly increase because their RPs now contain few white dots due to the occasional times when the largest orbits of their attractors were outside of the threshold. Such white dots produce several black lines of length $l$ on each diagonal under consideration that
eventually increase the disorder, and as a consequence, the values of the Shannon entropy.

It is important to mention that empty white diagonals will also produce values of entropy equal to zero due to the high order on such a diagonal. As an example, Fig. 11(a) shows a distribution of the first diagonals of the case $Re = 181$ in which the periodic high values of entropy correspond to the partial black diagonals and the cyclic zero values to the empty white diagonals. An analysis of the frequency spectrum of the same case reveals a well-defined dominant frequency at 747.9 Hz and several harmonics which once again matches the combustion instability frequency.

D. Indexes based on the diagonalwise recurrence quantification

In order to calculate indexes capable of identifying an impending combustion instability, a single value of each index must be quantified on each case. Hence, two approaches were proposed based on the standard deviation $\sigma$ and mean value $\mu$ of the distribution of the diagonalwise quantification.

Figure 12 shows the results with regard to the approach based on the standard deviation where the subscript $\sigma$ indicates this procedure. As is observed, the index $L_{\sigma}$ shows a distinct classification of all cases into two groups: (i) the first three cases that display high and constant values because of the linear decay of the mean lengths and (ii) the remaining cases that show low values about zero.

Moreover, the indexes $RR_{\sigma}$ and $DET_{\sigma}$ also sort the cases into two classes: (i) zero values along the first three cases and (ii) higher values for the remaining ones. However, the index $DET_{\sigma}$ shows an intermediate value in the fourth case ($Re = 151$) which indicates a possible transition. On the other hand, the index $s_{\sigma}$ classifies all cases into three groups: (i) the first three cases that display values of entropy near zero due to a low disorder inside the RPs, (ii) the fourth case displayed as a transition because of its intermediate value, and (iii) the remaining five cases with higher entropy due to a higher disorder of the system.

Regarding the approach based on the mean values, Fig. 13 depicts the results where the subscript $\mu$ denotes this method. Concerning the index $RR_{\mu}$, it distinctly classifies the cases into three classes: (i) the first three cases with values about one due to their black RPs that contain densities of black dots close to one, (ii) the fourth case that displays a transition due to the shown intermediate value, and (iii) the remaining five cases that display low values due to a pattern of periodic black and white diagonals in their RPs. Concerning the index $DET_{\mu}$, no classification was clearly observed. The first three cases show values equal to one; however, the remaining cases just display slightly lower values. On the contrary, the index $L_{\mu}$ clearly shows an assortment divided into two groups: (i) high values along the first three cases due to long black diagonals and (ii) about zero values for the rest of the cases due to partial black diagonals of smaller lengths. Likewise, the index $s_{\mu}$ sorts the cases into two classes: (i) the first three cases that depict values around zero due to their black RPs that contain a single long black line on each diagonal under analysis indicating a high order, i.e., low entropy, and (ii) the rest of the cases that display higher values of entropy because of the higher disorder that partial black diagonals introduce due to a mixture of black and white lines.
similar behavior was observed in the analysis of a trapped vortex clearly identified the stable and unstable regions of combustion. A index, an index based on a chaotic analysis (a nonlinear method), frequencies in the spectral analyses, were found along the regions of since high values of cross correlations, as well as dominant fre-

Example, in the analysis of a liquid rapid mixer oil burner, linear methods in order to detect correctly the instabilities rather than mentioning that chaotic analyses have proved to be more reliable fying transition values of the indexes. Additionally, it is important that the present stochastic observable and may be used as the deviations shown in margin of error of the present results, the values of the standard deviation demonstrated that only slight changes were found, almost negligible. This is an indication that the present nonlinear analysis is robust and slightly independent of the selected threshold. Concerning the margin of error of the present results, the values of the standard deviation shown in Fig. 12 can also be considered as the variations of the present stochastic observable and may be used as the uncertainties of the mean values displayed in Fig. 13.

Therefore, based on the above outcome of the indexes and the results previously discussed in Sec. IV B, it was concluded that the four cases (Re = 133–145) represent a stable combustion concerning the combustion noise regime, the fourth case (Re = 151) indicates a transition toward instabilities, and the last five cases (Re = 157–181) imply an unstable combustion for the combustion instability regime. Due to the fact that transition values were identified, the present analysis also showed its capacity to detect the combustion instabilities prior to the actual manifestation by identifying transition values of the indexes. Additionally, it is important to mention that chaotic analyses have proved to be more reliable methods in order to detect correctly the instabilities rather than linear analyses such as cross correlation and spectral analysis. For example, in the analysis of a liquid rapid mixer oil burner, linear analyses were not capable of detecting the combustion instabilities since high values of cross correlations, as well as dominant frequencies in the spectral analyses, were found along the regions of stable and unstable combustion. However, the interdependence index, an index based on a chaotic analysis (a nonlinear method), clearly identified the stable and unstable regions of combustion. A similar behavior was observed in the analysis of a trapped vortex combustor where the spectral analyses showed dominant frequencies along stable and unstable regions of combustion. Nevertheless, a chaotic analysis based on a recurrence quantification correctly identified the stable and unstable regions of combustion. Moreover, in the present study, the chaotic analysis identified the case Re = 151 as a transition point; however, the spectral analysis was unable to distinguish this case from further cases (starting with Re = 157 to Re = 181) because all these cases also showed frequency peaks, but they were totally unstable; see Fig. 4. Furthermore, in the study of an experimental nonpremixed high pressure laboratory combustor in which the lengths of the combustor ducts were varied in order to trigger the combustion instabilities, the Rayleigh criterion was used as an index by calculating the correlation between pressure and combustion heat release oscillations. As the results showed, the Rayleigh index was unable to identify stable and unstable conditions that were observed numerically and experimentally. It was assumed that other driving and damping mechanisms were operating along with the Rayleigh process. Hence, the present chaotic analysis, a nonlinear analysis based on a diagonalwise recurrence quantification, offers a more reliable method in order to correctly detect the thermoacoustic instabilities.

On the other hand, an acoustic Finite Element Method (FEM) simulation applied to the burner under analysis, by including the inlet ducts and without modeling porous media in the location of the pilots (due to the perforated holes), revealed that the first four longitudinal modes of resonance occur at frequencies of 200, 605, 781, and 965 Hz, respectively. Thus, the dominant frequency at 750 Hz, visualized in the spectral analysis from Figs. 4(e)–4(f), may be related to the third longitudinal mode of resonance at 781 Hz. Due to the fact that the instability frequency is lower than the third natural acoustic mode of the combustor, it is assumed that the coupling process between the heat release oscillations and the pressure fluctuations might be associated with a coupled convective-acoustic mode 35; however, this analysis is beyond the scope of this paper.

E. The velocity fields

The optical method Particle Image Velocimetry (PIV) was applied simultaneously along with the acquisition of the pressure fluctuations. The main objectives were to obtain a flow visualization of the flame under stable and unstable conditions and to establish a new index based on the velocity fields. Hence, 1000 image pairs (time instants) were acquired at a rate of 10 Hz in which the field of view was reduced in order to study only the flow within the combustion chamber. Thus, for each image pair, the horizontal and vertical velocity components were calculated by finding the vector displacements by means of a cross correlation and the time delay between two laser pulses of 10 μs. Consequently, the velocity magnitude |V| was calculated by using both velocity components. On the other hand, and with regard to the cross correlation algorithm, as a general rule each interrogation window requires at least three seeding particles in order to correctly calculate the vector displacement of this interrogation window. When the PIV method is applied to combustion processes, the density of tracing particles tends to greatly decrease in the combustion product zone of the flame due to a thermal expansion, and as a consequence, the cross correlation algorithm is not able to correctly evaluate the
vector displacements in such regions. Thus, instead of observing an increment in the velocity around the combustion product zone due to the thermal expansion, an unphysical drop of velocity (noise) is usually visualized in the velocity fields. In order to obtain reliable velocity profiles, isocontours were employed in which the velocity magnitudes higher than a threshold velocity \( V_o \) delimit the velocity profiles by setting all velocities within the combustion product zone as zero (regions in which the velocities are lower than \( V_o \)). Thus, in the present study, the loss of the velocity signal due to the thermal expansion in the combustion product zone (around the central flame) was delimited to the velocities of the reactants by setting a value \( V_o = 20 \) m/s. Figure 14 shows a sequence of isocontour images concerning the cases \( Re = 133 \) (representing a stable combustion) and \( Re = 181 \) (representing an unstable combustion).

As is observed, in the stable condition represented by a red line, the central flame slightly oscillates and basically remains steady. On the contrary, the flame of the unstable condition marked with a black line shows strong oscillations in the upper part of the combustion chamber. In general, it is noticed that during unstable conditions the vertical height of the central flame tends to be higher in comparison with the stable conditions. A physical interpretation of this effect may be explained by means of the acoustic Finite Element Method (FEM) simulation applied to the burner, where a pressure antinode was observed to be located at the top of the combustion chamber, where it is open to the atmosphere. Pressure nodes are defined as the regions along the standing waves where the pressure remains always equal to zero; on the contrary, pressure antinodes are those regions where the pressure oscillates from high to low amplitudes. Hence, and since the combustion instabilities are characterized by higher amplitudes of the pressure fluctuations in comparison with a stable combustion, the strong sinusoidal oscillations at the top of the combustor would tend to vertically lift the central flame during negative pressure oscillations (vacuum pressure) and tilt it leftward or rightward during positive pressure oscillations.

1. **Index based on the velocity fields**

By taking into account the previous results in which the central flame tends to be higher in the unstable condition, a new index based on the height of the central flame was proposed in order to identify the combustion instabilities. Let us recall that the height of the central flame was delimited by the velocity magnitudes higher than \( V_o = 20 \) m/s, i.e., the overall shape of the central flame was defined by the high speed of the reactants. Hence, the heights of the central flame were calculated in each time instant throughout all cases. In order to accomplish this step, the height of the central flame was established as the vertical point at which all velocity magnitudes within the central flame dropped below \( V_o = 20 \) m/s. Afterward, the average height of the central flame along the stable case \( Re = 133 \) was calculated and used as a fixed threshold \( \zeta_o = 0.035 \) m (with respect to the y-axis of Fig. 14). Then, and by holding the threshold \( \zeta_o \) constant throughout all cases, the local height of the central flame of each time instant was compared against the threshold \( \zeta_o \), e.g., if the local height was higher than the threshold \( \zeta_o \), it was marked as one; otherwise, it was denoted as zero. Finally, the number of ones and zeros was summed up and divided by the total time instants under analysis. Based on this procedure, the results of the index \( V_h \) are shown in Fig. 15 where the subscript \( h \) stands for “height,” i.e., an index based on the height of the central flame.

The index \( V_h \) basically sorts the cases into two groups: (i) the first three cases that show intermediate values because the local heights of the flame were partially higher than the threshold \( \zeta_o \) and (ii) the remaining cases that display values about one because mainly all local heights of the flame were higher than the threshold \( \zeta_o \). In order to increase the distinction between these two groups, only the first 100 time instants were used for the calculation of the index \( V_h \). A further selection of time instants will tend to decrease the differences but still classify the cases into two groups, i.e., the results are qualitatively equal. Therefore, the results of the index \( V_h \), based on the height of the central flame, are in agreement with the previous conclusions stated in Sec. IV D. In other words, the index \( V_h \) also relates the first three cases to the combustion noise regime due to a relative stable flame and the last six cases to the combustion instability regime due to the stronger oscillations of the flame. However, this index was unable to detect the transition previously identified by other indexes in the fourth case. A possible reason for that is perhaps that the intermittent bursts visualized along the fourth case already produce strong oscillations over long periods of time which...
are equivalent to the observed strong oscillations along the unstable cases.

V. CONCLUSIONS

In this paper, an experimental slot burner was analyzed by means of a nonlinear time-series analysis applied to measured pressure fluctuations. The slot burner was composed of a central exhaust nozzle slot (where a premixed mixture of methane and air was injected into the chamber) and two pilots located on both sides (where a premixed mixture of propane and air was ignited). As a control parameter, the Reynolds number of both pilots was used and varied from $Re = 133–181$ in increments of six in which the burner approached instabilities as $Re$ was increased, i.e., toward leaner mixtures. The main objective was the calculation of indexes capable of detecting in advance an impending combustion instability. Concerning the nonlinear analysis, diagonalwise measurements were applied to the recurrence plots through which four indexes were proposed: the $\tau$-recurrence rate index $RR_\tau$, the $\tau$-determinism index $DET_\tau$, the $\tau$-average diagonal line length index $L_\tau$, and the $\tau$-entropy index $s_\tau$. A unique value of each index per case was obtained by quantifying the diagonalwise measurements by means of two approaches based on their standard deviations $\sigma$ and mean values $\mu$. The results showed that all indexes based on both approaches were able to sort all cases under analysis into two groups: the first three cases that correspond to the "Combustion Noise Regime" and the remaining cases that were associated with the "Combustion Instability Regime." Additionally, by performing the optical method Particle Image Velocimetry (PIV), a flow visualization of the unstable condition revealed strong oscillations in the upper part of the combustion chamber and a relative higher average height of the velocity profile of the central flame in comparison with the stable condition. Thus, a new index, named here as the index $V_h$, based on the local heights of the velocity profiles of the central flame was calculated. The results showed that the index $V_h$ was also capable of identifying the same two groups previously computed by the nonlinear analysis. Nevertheless, the best indexes were $RR_\mu$, $DET_\sigma$, and $s_\sigma$, where the subscripts $\mu$ and $\sigma$ stand for the mean and standard deviation approach, respectively, since these indexes were able to detect the transition between the combustion noise and combustion instability regimes, i.e., between the stable and unstable conditions. Therefore, the results presented in this research proved that the five indexes were effective precursors in order to detect in advance the combustion instabilities.

APPENDIX A: RECURRENCE PLOTS

![Figure 16](image_url)
APPENDIX B: DIAGONALWISE MEASUREMENTS OF THE RECURRENCE PLOTS (RPS)

FIG. 17. Sequence of the index $RR$, based on the RPs of Appendix A for the cases (a) $Re = 133$, (b) $Re = 139$, (c) $Re = 145$, (d) $Re = 151$, (e) $Re = 157$, (f) $Re = 163$, (g) $Re = 169$, (h) $Re = 175$, and (i) $Re = 181$.

FIG. 18. Sequence of the index $DET$, based on the RPs of Appendix A for the cases (a) $Re = 133$, (b) $Re = 139$, (c) $Re = 145$, (d) $Re = 151$, (e) $Re = 157$, (f) $Re = 163$, (g) $Re = 169$, (h) $Re = 175$, and (i) $Re = 181$. 
FIG. 19. Sequence of the index $L_\tau$ based on the RPs of Appendix A for the cases (a) $Re = 133$, (b) $Re = 139$, (c) $Re = 145$, (d) $Re = 151$, (e) $Re = 157$, (f) $Re = 163$, (g) $Re = 169$, (h) $Re = 175$, and (i) $Re = 181$.

FIG. 20. Sequence of the index $s_\tau$ based on the RPs of Appendix A for the cases (a) $Re = 133$, (b) $Re = 139$, (c) $Re = 145$, (d) $Re = 151$, (e) $Re = 157$, (f) $Re = 163$, (g) $Re = 169$, (h) $Re = 175$, and (i) $Re = 181$. 
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