Chaos, synchronization, and desynchronization in a liquid-fueled diffusion-flame combustor with an intrinsic hydrodynamic mode

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ABSTRACT
We experimentally investigate the nonlinear dynamics of a thermoacoustically self-excited aero-engine combustion system featuring a turbulent swirling liquid-fueled diffusion flame in a variable-length combustor. We focus on the steady-state dynamics via simultaneous measurements of the acoustic pressure in the combustor and the heat release rate (HRR) from the flame. When the combustor length is increased following the onset of thermoacoustic instability, we find that the pressure signal transitions from a period-1 limit cycle to chaos, whereas the HRR signal remains chaotic owing to the presence of an intrinsic hydrodynamic mode in the flame. When the hydrodynamic mode is filtered out of the data, we find that the pressure and HRR signals are in generalized synchronization. However, when the hydrodynamic mode is retained in the data, we find that the pressure and HRR signals are either weakly phase synchronized or desynchronized. This study has two main contributions: (i) it shows that a liquid-fueled diffusion-flame combustor can exhibit dynamics as complex as those of its gaseous-fueled premixed-flame counterparts and (ii) it highlights the need to be exceptionally careful when selecting a diagnostic signal from which to calculate nonlinear measures of self-excited thermoacoustic oscillations, because our experiments show that the pressure and HRR signals can be desynchronized by the presence of a hydrodynamic mode in the flame at a frequency different from that of the dominant thermoacoustic mode.

I. INTRODUCTION
Thermoacoustic instabilities are a recurring problem in various combustion devices, such as industrial furnaces, aircraft engines, and...
land-based gas turbines. Such instabilities arise from positive coupling between the heat-release-rate (HRR) oscillations of an unsteady flame and the pressure oscillations of its surrounding acoustic field. Such instabilities manifest as self-excited flow oscillations in the combustor. If their amplitudes are sufficiently high, they can bring about operational problems such as flame flash-back and blow-out as well as increased pollutant emissions, reducing the reliability and efficiency of the overall combustion system. This is becoming an increasingly serious problem as the need to meet new environmental standards has necessitated that gas turbines be operated in the very regime where they are the most susceptible to thermoacoustic instabilities.

It is helpful to define at the outset what is meant by the term “thermoacoustic instability.” In this paper, we define thermoacoustic instability as a stable state of combustor operation in which at least one discrete periodic mode is self-excited to an amplitude above the background noise level by positive coupling between the HRR oscillations of an unsteady flame and the pressure oscillations of the combustor. It is important to recognize that this definition does not preclude the existence of additional periodic or broadband modes, which could interact with the original self-excited periodic mode to produce quasiperiodic or chaotic oscillations, as has been demonstrated in previous experimental and numerical studies. Although this state is stable, it is customary to refer to it as being “thermoacoustically unstable,” if only to distinguish it from a “thermoacoustically stable” state, which is also stable but does not contain any self-excited modes caused by feedback between HRR and pressure. In the literature, such a thermoacoustically stable state is often synonymous with combustion noise, which was traditionally assumed to be purely stochastic but has recently been shown to be chaotic.  

A. Thermoacoustic instabilities in liquid-fueled combustors

Thermoacoustic instabilities can occur in both gaseous-fueled and liquid-fueled combustors, but the driving mechanisms are thought to be more complex in the latter. In liquid-fueled combustors, the interactions between spray processes (e.g., primary and secondary atomization, droplet and ligament transport and coalescence, and liquid evaporation and mixing) and other physical and chemical processes (e.g., oscillatory hydrodynamic and acoustic fields, and chemical reactions) can produce additional HRR oscillations, providing another means of exciting thermoacoustic instabilities. For instance, when an oscillatory acoustic field interacts with a turbulent spray flame, the former can modify the latter, including its droplet size distribution, its spatial distribution of droplet number density and velocity, and its evaporation and mixing characteristics. In turn, this can lead to spatiotemporal variations in the equivalence ratio, producing additional HRR oscillations.

The more sophisticated geometry of liquid-fuel injectors can give rise to additional instability modes in a combustor. For example, experiments on a liquid-fueled diffusion-flame combustor have revealed the existence of three different types of modes: hydrodynamic modes, Helmholtz modes, and longitudinal modes. The Helmholtz modes were found to be particularly sensitive to the geometry of the liquid-fuel injector. However, owing to the complex interactions among spray processes, hydrodynamics, acoustics and combustion, liquid-fueled combustors have been less well studied than gaseous-fueled combustors, particularly from the viewpoint of nonlinear dynamics, synchronization, and chaos.

B. Chaos in thermoacoustic systems

Self-excited thermoacoustic oscillations in combustion systems were initially assumed to be simply periodic, but recent experiments have shown that they can also be chaotic. For example, Kabiraj et al. found chaos in the self-excited thermoacoustic oscillations of a laboratory-scale combustor powered by a set of laminar conical premixed flames at an equivalence ratio close to the lean blow-out limit. In subsequent experiments, Gotoda et al. and Kabiraj et al. found similar chaotic thermoacoustic oscillations in model gas-turbine combustors powered by turbulent premixed flames at equivalence ratios far from the lean blow-out limit.

Besides the observation of chaos itself, two classical routes to chaos have also been identified in self-excited thermoacoustic systems. Using low-order G-equation simulations of a confined two-dimensional laminar premixed flame, Kashinath et al. found that chaos can emerge via the period-doubling route and the Ruelle–Takens–Newhouse (RTN) route. This supports earlier observations made (i) by Subramanian et al. on the period-doubling route to chaos in a reduced-order model of a horizontal Rijke tube, and (ii) by Kabiraj et al. of the RTN route to chaos in laboratory-scale experiments on a self-excited tube combustor powered by seven laminar conical premixed flames stabilized on a copper block.

These observations of chaos were all made on unforced thermoacoustic systems, but recent studies have shown that chaos can also emerge as a result of external forcing. For example, low-order simulations and experiments have shown that the external open-loop application of periodic acoustic forcing on a ducted laminar premixed Bunsen flame can give rise to chaos via the period-doubling route and the Afraimovich–Shilnikov route involving the breakdown of a phase-locked torus.

As the foregoing literature review has shown, chaotic oscillations can be found in both simple and complex thermoacoustic systems, with or without external forcing. However, it is worth mentioning that nearly all previous observations of chaos in thermoacoustic systems were made on gaseous-fueled premixed-flame combustors. A number of seminal studies—most notably from Pawar et al. and Kobayashi et al.—have investigated the nonlinear thermoacoustics of liquid-fueled combustors, but have yet to identify chaos in their dynamics. In the present study, we provide experimental evidence of chaos in a liquid-fueled diffusion-flame combustor.

C. Synchronization between pressure and HRR oscillations after the onset of thermoacoustic instability

Experiments by Kabiraj et al. and simulations by Kashinath et al. have shown that, after the onset of thermoacoustic instability, the pressure and HRR signals from a combustor tend to oscillate similarly to each other, in so far as they show the same dynamical state when processed with tools from nonlinear time-series analysis. It is thus often assumed that, during thermoacoustically unstable operation, the dynamical state of a combustor—whether it be
periodic, quasiperiodic or chaotic — can be inferred from either the pressure or HRR signal, with little to choose between the two. Recently, this assumption of dynamical similarity between the pressure and HRR signals has been further supported by experiments conducted within the framework of mutual synchronization. For example, Pawar et al.\textsuperscript{33}, Mondal et al.\textsuperscript{34}, and Godavarthi et al.\textsuperscript{35} have shown that when a turbulent bluff-body-stabilized premixed-flame combustor transitions from a thermoacoustically stable state characterized by combustion noise to a thermoacoustically unstable state characterized by a periodic limit cycle, the pressure and HRR signals transition from being desynchronized and aperiodic to being synchronized and periodic. The transition between these two stable states was found to occur via a state of intermittent phase synchronization, in which the pressure and HRR signals are synchronized during periodic epochs but are desynchronized during aperiodic epochs. Furthermore, analysis of the limit-cycle state revealed that it was made up of two different substates: (i) a weakly correlated limit-cycle state featuring phase synchronization and (ii) a strongly correlated limit-cycle state featuring generalized synchronization. In the present study, we find similar, but not identical, synchronization dynamics under different operating conditions as well as a previously unreported state featuring neither phase nor generalized synchronization between the pressure and HRR signals, despite the system being thermoacoustically unstable. As shown in Sec. IV F, this desynchronization is believed to be due to the presence of an intrinsic hydrodynamic mode in the flame.

D. Contributions of the present study

In this experimental study, we explore the nonlinear dynamics of a thermoacoustically self-excited aero-engine combustion system: a turbulent swirling liquid-fueled diffusion flame in a variable-length combustor. On increasing the combustor length while keeping all other control parameters constant, we find that the pressure signal transitions from a period-1 limit cycle to chaos, whereas the HRR signal remains chaotic owing to the presence of a hydrodynamic mode in the flame. This study has two main contributions: (i) it shows that a liquid-fueled diffusion-flame combustor can exhibit dynamics as complex as those of gaseous-fueled premixed-flame combustors, and (ii) it underscores the need to be extra careful when choosing a diagnostic signal from which to calculate nonlinear measures of self-excited thermoacoustic oscillations, because our experiments show that the pressure and HRR signals can oscillate differently from each other during thermoacoustic instability, if a sufficiently robust hydrodynamic mode exists in the flame at a frequency different from that of the dominant thermoacoustic mode.

This paper is organized as follows. We describe the test rig and measurement diagnostics in Sec. II, discuss the nonlinear time-series tools used for data analysis in Sec. III, examine the transition from periodicity to chaos in a thermoacoustically self-excited combustor in Sec. IV, and conclude with the highlights of this study in Sec. V.

II. EXPERIMENTAL SETUP

Experiments are performed on a laboratory-scale aero-engine combustion rig burning liquid fuel (Jet A-1). The full details of this rig can be found in Ref. 24, so only an overview is given here. As shown in Fig. 1, the rig consists of an air-blast swirl injector, an inlet plenum (inner diameter: ID: 83 mm; length, L\textsubscript{p}: 350 mm), an optically accessible quartz tube (ID: 109 mm; length: 405 mm), and a double-walled steel combustor (ID: 78 mm), whose length can be adjusted to three different values: L\textsubscript{i} = 1000 mm (short), 2200 mm (medium), and 2500 mm (long). These three values are specifically chosen to generate a wide range of nonlinear dynamics in the combustor. The injector consists of three axial swirlers, as shown in Fig. 1(a). A counter-rotating stream of air is produced by two concentric pilot swirlers, which help to improve atomization and mixing. A sixteen-vane axial swirler (swirl angle of 50°) is installed in the main air passage. The air flow is metered with a thermal mass flow controller (Sierra Instruments FlatTrak 780S: ±0.5% FS), and the liquid-fuel flow (Jet A-1) is metered with a positive displacement flow meter (Max Machinery P214: ±0.2%). An electric motor (Dayton 5G5D58) is used to drive a gear pump (Parker Hannifin D05A2A) to pressurize the liquid-fuel supply upstream of the injector. Preheated dry air (T\textsubscript{i} = 200 °C) flows to the injector at a bulk velocity of u\textsubscript{i} = 29.6 m/s. The pilot equivalence ratio is varied from \( \phi_{\text{pilot}} = 2.5 \) to 7.0 (±0.8%), producing a global equivalence ratio of \( \phi_{\text{global}} = 0.375 \) to 1.050 (based on the main air flow rate). All the test conditions produce a diffusion flame, which is ignited at startup by a high-voltage spark igniter. Table 1 lists the experimental parameters and test conditions.

![FIG. 1. Cross-sectional views of (a) the air-blast swirl injector and (b) the overall experimental setup, which is representative of an aeronautical gas-turbine combustor. All dimensions shown are in millimeters. The full details can be found in Ahn et al.\textsuperscript{24}](image-url)

Chaos 29, 053124 (2019); doi: 10.1063/1.5088735

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TABLE I. Experimental parameters and test conditions.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet plenum length, $L_p$</td>
<td>350 mm</td>
</tr>
<tr>
<td>Total combustor length, $L_c$</td>
<td>1000, 2200, 2500 mm</td>
</tr>
<tr>
<td>Inlet air temperature, $T_i$</td>
<td>200 °C</td>
</tr>
<tr>
<td>Inlet air bulk velocity, $u_i$</td>
<td>29.6 m/s</td>
</tr>
<tr>
<td>Air flow rate, $m_{air}$</td>
<td>33.0 g/s</td>
</tr>
<tr>
<td>Pilot equivalence ratio, $\phi_{pilot}$</td>
<td>2.5–7.0</td>
</tr>
<tr>
<td>Global equivalence ratio, $\phi_{global}$</td>
<td>0.375–1.050</td>
</tr>
</tbody>
</table>

Eight piezoelectric pressure transducers (PCB 112A22: ±1%), each mounted in its own water-cooled jacket, are used to measure the acoustic pressure in the combustor. A photomultiplier tube (PMT, Hamamatsu H7732–10), equipped with a bandpass filter (309 ± 5 nm), is used to measure the global OH chemiluminescence from the flame, which is used as an indicator of the global HRR. The analog output from the PMT is amplified (PCB 482A16) and digitized at 3000 Hz for 8 s on a 16-bit data acquisition system (TEAC model LX–110). A high-speed camera (Photron FASTCAM SA-X2), equipped with a Sigma 24–70 mm F/2.8 lens, is used to capture time-resolved videos of the flame at 6000 frames/s.

III. NONLINEAR TIME-SERIES ANALYSIS

This section gives an overview of the time-series tools used to analyze the nonlinear dynamics of the combustor. Many of these tools have been used before to understand and model the nonlinear dynamics of nonreacting and reacting flow systems.

A. Phase space reconstruction

To examine the nonlinear dynamics of the combustor, we reconstruct its phase space using the time-delay embedding theorem proposed by Takens, with input data provided by two scalar time-series measurements: (i) the acoustic pressure fluctuations in the combustor $p_i(t)$, as measured with the pressure transducer mounted at location $p_i$ in Fig. 1(b), and (ii) the HRR fluctuations from the flame $q_i(t)$, as measured with the PMT described in Sec. II. The original attractor is unfolded into a $d$-dimensional Euclidean vector of time-delayed elements constructed from either $p_i(t)$ or $q_i(t)$. For $p_i(t)$, this yields $P_{i}(d) = [p_{i-1}, p_{i-2}, \ldots, p_{i-(d-1)}]$, where the index $i = (d-1)\tau + 1, \ldots, N$ denotes the $i$th reconstruction of the state vector. For the time-delayed vectors to capture adequate new information without being overly dependent of each other, it is necessary to choose an appropriate value of the time delay ($\tau$). In this study, the optimal value of $\tau$ is found by computing the first local minimum of the average mutual information function, which has been shown to be a reliable metric in previous studies of nonlinear thermoacoustic oscillations. The embedding dimension ($d$) is the dimension of the hyperspace onto which the original phase space is projected. Its value must be sufficiently large for the reconstructed attractor to be a one-to-one projection of the original attractor. In this study, we determine the minimum value of $d$ with the method proposed by Cao.

B. Correlation dimension

To explore the self-similarity of the reconstructed attractors, we use a topological metric known as the correlation dimension. This metric quantifies the number of active degrees of freedom in a dynamical system. Its value approaches one for a limit cycle (closed periodic orbit), two for a $\mathbb{T}^2$ quasiperiodic attractor (ergodic torus), a noninteger for a strange attractor (self-similar fractal), and infinity for a purely random signal (stochastic fluctuations). In this study, we calculate the correlation dimension with the method proposed by Grassberger and Procaccia. This involves first calculating the correlation sum $C_N(d, R)$, quantified as the probability that two points on an $d$-embedded time series $I_{i}(d) = [P_i(d) - P_j(d)]$ sit within a Euclidean distance of $R$ from each other

$$C_N(d, R) = \frac{2 \sum_{i=1}^{N} \sum_{j=i+1}^{N} \Theta(R - \|P_i(d) - P_j(d)\|)}{N(N-1)},$$

where $\Theta$ is the Heaviside step function, as defined by

$$\Theta = \begin{cases} 1 & \text{for } R - \|P_i(d) - P_j(d)\| > 0, \\ 0 & \text{for } R - \|P_i(d) - P_j(d)\| \leq 0. \end{cases}$$

In the self-similar range, $C_N(d, R)$ is expected to obey a power-law scaling with the following slope:

$$D_r(d, R) = \frac{\partial \log[C_N(d, R)]}{\partial \log(R)}.$$ Following Grassberger and Procaccia, we estimate the correlation dimension as the mean value of the local slope $[D_r(d, R)]$ within the self-similar scaling range.

C. Filtered horizontal visibility graph

A chaotic attractor can be partially identified by a positive maximum Lyapunov exponent. However, it is often difficult to estimate the Lyapunov exponent accurately because noise in turbulent combustion experiments can degrade the calculation, resulting in positive Lyapunov exponents even for periodic and quasiperiodic attractors. To overcome this problem, scientists have turned to alternative methods, one of which is the horizontal visibility graph (HVG). Introduced by Luque et al., the HVG offers a means of transforming a time series into a network structure using graph theory. Let $p_i, p_j, p_s$ be a time series of length $N$. The HVG algorithm assigns each stem of the time series to a node in the HVG. Thus, a series of $N$ data stems would map to an HVG with $N$ nodes. Two nodes $(i, j)$ in an HVG are considered to be linked if a straight line can be drawn between the tips of their stems ($p_i$ and $p_j$) in the time series, without intersecting any intermediate stems. In other words, nodes $i$ and $j$ are linked if the following geometrical criterion holds:

$$p_i \leq p_s \leq p_j,$$

for all $n$ such that $i < n < j$. This approach has been shown to be able to distinguish between noisy periodic, chaotic and stochastic signals. According to Nuñez et al., the mean degree of an HVG for an infinite periodic time series containing no repeated values within a period of $T$ is $K(T) = 4(1 - 1/T)$. For a noisy periodic signal, such as that which would be expected from turbulent combustion experiments, Nuñez et al. proposed a filtered HVG (f-HVG) for calculating $T$. The procedure involves mapping every
data stem $p_i$ to a node in the f-HVG, and then building a link between two nodes if $p_i > p_j$ for all $n$ such that $i < n < j$, where $f_{\text{HVG}}$ is the amplitude of a noise filter. In this study, we use the f-HVG and the histogram of $p(t)$ and $q(t)$ to explore the periodic and chaotic nature of the combustor dynamics.

IV. RESULTS AND DISCUSSION

In this section, we use the time-series tools described in Sec. II to analyze the nonlinear dynamics of the combustor described in Sec. II. As mentioned in Sec. I A, this combustor has been previously studied by Ahn et al. in the context of mode selection: it was found that three different types of modes—longitudinal modes, Helmholtz modes, and hydrodynamic modes—can arise in this combustor, depending on the specific values of the combustor length and the pilot equivalence ratio. As an extension of that previous study, the present study focuses on chaos and synchronization. Therefore, three different thermoacoustically self-excited states with chaotic features are examined, all of which occur after the onset of thermoacoustic instability. These three states correspond to the final attractors reached after a short transient lasting 5–7 s. The three states are reached by increasing the pilot equivalence ratio until it is sufficiently close to $\phi_{\text{plus}} = 7.0$ and by varying the combustor length across three discrete values (see Table I): $L = 1000, 2200$, and $2500$ mm. From here onwards, these three $L$ values will be referred to as the “short,” “medium,” and “long” combustors, respectively.

A. Phase space reconstruction

First, the phase space is reconstructed from the pressure signal, $p(t)$. For both the short and medium combustors ($L = 1000$ and 2200 mm), the phase portrait shows a closed periodic orbit [Figs. 2(a1) and 2(g1)], while the Poincaré map shows two clusters of trajectory intercepts [Figs. 2(b1) and 2(h1)]. These features indicate the presence of a period-1 attractor, which is characteristic of a limit cycle. The intercepts in the Poincaré map are somewhat scattered because the phase trajectories are randomly perturbed by background turbulence and noise. Similar perturbations have been reported before in laboratory experiments on an imperfectly premixed turbulent combustor. For the long combustor ($L = 2500$ mm), the phase portrait shows a strange geometrical object [Fig. 2(m1)], while the Poincaré map shows two broad cores with sparse trajectories around them [Fig. 2(n1)]. These features point to the presence of a chaotic attractor, which will be examined further below.

Next, the phase space is reconstructed from the HRR signal, $q(t)$. For the short and medium combustors ($L = 1000$ and 2200 mm), the phase portrait shows a strange geometrical object [Figs. 2(a2) and 2(g2)], while the Poincaré map shows a central blob of scattered intercepts [Figs. 2(b2) and 2(h2)]. These features, which, as a reminder, are found in the HRR signal, suggest the presence of a chaotic attractor—in stark contrast to the period-1 features found in the pressure signal [Figs. 2(a1), 2(b1), 2(g1), and 2(h1)]. For the long combustor ($L = 2500$ mm), the attractor can be seen to take on a strange geometrical structure [Figs. 2(m2) and 2(n2)] resembling that of the two suspected chaotic attractors found in the short and medium combustors [Figs. 2(a2), 2(b2), 2(g2), and 2(h2)]. The similarities shared by these three attractors suggest that the HRR of the flame remains similarly chaotic across all three combustor lengths, despite the pressure oscillating in a period-1 limit cycle in the short and medium combustors [Figs. 2(a1), 2(b1), 2(g1), and 2(h1)].

In summary, when the combustor length is increased following the onset of thermoacoustic instability, analysis of $p(t)$ indicates a transition from a period-1 limit cycle to chaos, whereas analysis of $q(t)$ indicates that the flame itself remains chaotic under the same test conditions. To investigate this difference, we compare the spectral content of $p(t)$ and $q(t)$ in Sec. IV B.

B. Spectral analysis

As in Sec. IV A, the pressure data are examined first. The pressure spectra show a dominant Helmholtz mode at $f_{H1}$ in the short combustor [Fig. 2(c1)] and a dominant longitudinal mode at $f_{L2}$ in the medium combustor [Fig. 2(i1)]. These pure-tone thermoacoustic modes are qualitatively consistent with the period-1 nature of the two attractors identified from phase space reconstruction [Figs. 2(a1), 2(b1), 2(g1), and 2(h1)]. Moreover, they quantitatively match the mode-selection criteria of Ahn et al. It is worth noting that in neither spectra [Figs. 2(c1) and 2(i1)] are there strong low-frequency components. For the long combustor [Fig. 2(i1)], the pressure spectrum shows multiple modes. Apart from a longitudinal mode at $f_{L2}$, a Helmholtz mode emerges at an incommensurate frequency of $f_{H1}$. The Helmholtz nature of this mode was confirmed by Ahn et al. The interaction between $f_{L2}$ and $f_{H1}$ gives rise to peaks at $f_{L2} \pm f_{H1}$. There is also a third mode at an incommensurate frequency, $f_3$. In our previous study, it was hypothesized that this mode ($f_3$) could be a linear combination of $f_{L2}$ and $f_{H1}$. However, if that were true, a T^3 torus attractor would appear in phase space, resulting in two clear rings in the Poincaré map. Recognizing that such rings are absent in Fig. 2(n1) and on reviewing the frequency scaling proposed by Ahn et al., we believe that the $f_3$ mode is an incommensurate longitudinal mode.

Next, the HRR data are examined. For the short and medium combustors [Figs. 2(c2) and 2(i2)], the HRR spectra show the same Helmholtz mode ($f_{H2}$) and longitudinal mode ($f_{L3}$) as the pressure spectra [Figs. 2(c1) and 2(i1)]. However, the HRR spectra [Figs. 2(c2) and 2(i2)] show markedly stronger components at low frequencies ($f \lesssim 70$ Hz; gray shading) than the pressure spectra [Figs. 2(c1) and 2(i1)]. This indicates that the flame is oscillating not only at $f_{H2}$ (short combustor) or $f_{L3}$ (medium combustor) but also across a range of low frequencies ($f \lesssim 70$ Hz) at which the acoustic pressure $p(t)$ is not oscillating. These low-frequency components can also be found in the HRR spectrum of the long combustor [Fig. 2(o2)]. The fact that these low-frequency components appear only in the HRR spectra, and not in the pressure spectra, suggests that they arise from an intrinsic hydrodynamic mode in the flame, rather than from an acoustic mode in the combustor. In Sec. IV F, the possibility of a hydrodynamic mode existing in the flame will be explored with the aid of high-speed chemiluminescence imaging.

C. Correlation dimension

As before, the pressure data are examined first. In Figs. 2(d1), 2(j1), and 2(p1), the local slope of the correlation sum...
FIG. 2. Pressure and HRR data sampled simultaneously for three different combustor lengths: (top two rows; red) $L_c = 1000$ mm or “short” combustor, (middle two rows; blue) $L_c = 2200$ mm or “medium” combustor, and (bottom two rows; green) $L_c = 2500$ mm or “long” combustor. For each row-pair at a fixed $L_c$, the upper row corresponds to the pressure data, and the lower row corresponds to the HRR data. From left to right, the columns show (a,g,m) the phase portrait, (b,h,n) the Poincaré map, (c,i,o) the frequency spectrum with its amplitudes normalized by their maximum value $\tilde{A} \equiv A / A_{\text{max}}$, (d,j,p) the local slope of the correlation sum as a function of the normalized hypersphere radius, with the self-similar scaling range shaded in gray, (e,k,q) the mean degree as a function of the noise filter amplitude, and (f,l,r) the histogram of oscillations normalized by their maximum values $\tilde{p}' \equiv p' / p'_{\text{max}}$ or $\tilde{q}' \equiv q' / q'_{\text{max}}$. Unless otherwise noted, phase space reconstruction is performed with $9000$ data points (time series of $3$ s) for an embedding dimension of $d = 3$ and an embedding time delay of $\tau = 1$ ms. The pilot equivalence ratio is fixed at $\phi_{\text{pilot}} = 7.0$. $(D_c)$ is plotted as a function of the normalized hypersphere radius $(R / R_{\text{max}})$ for three different embedding dimensions: $d = 8, 10,$ and $12$. These three values of $d$ are sufficiently large for $D_c$ to reach convergence in the self-similar scaling range (shaded in gray). At scales smaller than this range, however, $D_c$ fails to converge because stochastic noise dominates the signal, activating additional degrees of freedom and destroying any self-similarity in the attractor at small $R / R_{\text{max}}$. In the self-similar scaling range, the correlation dimension is $D_c \approx 1.1$ for both the short and medium combustors [Figs. 2(d1) and 2(j1)]. This value of $D_c$ is sufficiently close to 1 suggesting that the phase space topology is a one-dimensional orbit, consistent with our interpretation of this being a limit-cycle attractor. The relatively small deviation from a value of exactly 1 is believed to be due to the presence of inherent noise and turbulence—a factor also identified in

Chaos 29, 053124 (2019); doi: 10.1063/1.5088735
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previous laboratory experiments on turbulent combustors. For the long combuster [Fig. 2(p1)], the correlation dimension is \( D_2 \approx 3.5 \), which is a noninteger and thus indicative of a nonsmooth fractal. This is consistent with our interpretation of this being a strange chaotic attractor (see Secs. IV A and IV B).

Regarding the HRR data [Figs. 2(d2), 2(j2), and 2(p2)], we note that, as with the pressure data [Figs. 2(d1), 2(j1), and 2(p1)], \( D_1 \) achieves convergence in the self-similar scaling range for all three combustors (shaded in gray). The fact that convergence is achieved with a finite embedding dimension indicates that the signals are not dominated by stochastic noise at intermediate values of \( R/R_{max} \), although they are at smaller values of \( R/R_{max} \). This concurs with our earlier observation of sharp peaks in the frequency spectra (see Sec. IV B). The value of the correlation dimension is \( D_2 \approx 3.8, 3.8, \) and 4.2 for the short, medium, and long combustors, respectively. These values are all nonintegers, indicating the presence of strangeness. However, these values are also somewhat close to 4. In any experimental or numerical study, it is known that noise can contaminate the calculation of \( D_2 \), causing it to deviate from its true value. To explore the extent of this effect and verify the presence of chaos, we examine f-HVGs of \( p(t) \) and \( q(t) \) in Sec. IV D.

**D. Filtered horizontal visibility graph**

The f-HVG of the pressure signal \( (\tilde{p} = p/p_{amp}) \) is examined first. For the short and medium combustors [Figs. 2(e1) and 2(k1)], \( \tilde{K} \) converges to exactly 2 as \( f_{amp} \) increases. As mentioned in Sec. III C, the period \( T \) can be calculated from \( \tilde{K}(T) = 4(1 - 1/27) \). Therefore, \( \tilde{K} = 2 \) implies \( T = 1 \), which is consistent with our earlier assessment of these being period-1 attractors. For the long combustor [Fig. 2(q1)], \( \tilde{K} \) is yet to converge to 2, even when \( f_{amp} \) is at a value \( f_{amp} = 0.2 \) that has already produced \( \tilde{K} = 2 \) in the period-1 attractors of the short and medium combustors [Fig. 2(e1) and 2(k1)]. The absence of convergence in \( \tilde{K} \) indicates the absence of dominant periodic structures, which is consistent with a chaotic signal.

Besides the f-HVG, the histogram of \( \tilde{p}(t) \) is also examined. For a period-1 attractor unaffected by noise, the histogram would be bimodal with two distinct humps: one from the peaks of the input time series and one from the troughs. For a perfectly sinusoidal waveform, the height of the two humps would be identical. In the short and medium combustors [Figs. 2(f1) and 2(l1)], a bimodal distribution is indeed observed. However, in both cases, the hump at negative \( \tilde{p} \) values is higher and closer to \( \tilde{p} = 0 \) than the hump at positive \( \tilde{p} \) values. This asymmetry indicates that the time series is not perfectly sinusoidal, with sharp peaks and blunt troughs giving rise to a more pronounced hump at negative \( \tilde{p} \) values. In the long combustor [Fig. 2(r1)], the histogram shows a unimodal distribution. This is similar to that reported in previous experiments on laminar and turbulent combustors and indicates the presence of multiscale fluctuations, which are consistent with a chaotic signal. A multifractal analysis is presented in the Appendix to gain physical insight into the self-similar scaling of the system.

Next the f-HVG of the HRR signal \( (\tilde{q}(t)) \) is examined. In all three combustors [Figs. 2(e2), 2(k2), and 2(q2)], \( \tilde{K} \) is yet to converge to 2 within the \( f_{amp} \) range examined, indicating the absence of periodic structures in the HRR signal. Furthermore, all three histograms [Figs. 2(e2), 2(k2), and 2(q2)] show a unimodal distribution, indicating again the presence of multiscale fluctuations, consistent with chaotic motion.

In summary, by examining the phase portrait, Poincaré map, frequency spectrum, correlation dimension, f-HVG and histogram, we find evidence that \( p(t) \) and \( q(t) \) oscillate differently from each other, even during thermoacoustic instability. Specifically, on increasing the combustor length from \( L_c = 1000 \) to 2200 to 2500 mm for a pilot equivalence ratio of \( \phi_{pilot} = 7.0 \), we find that \( p(t) \) transitions from a period-1 limit cycle to chaos, whereas \( q(t) \) remains chaotic under the same test conditions. In Sec. IV E, we will explore the correlation between \( p(t) \) and \( q(t) \) with the aid of synchronization tools.

**E. Synchronization and desynchronization between pressure and HRR**

As the preceding sections have shown, following the onset of thermoacoustic instability, \( p(t) \) and \( q(t) \) need not necessarily oscillate similarly to each other. To investigate this further, we adopt the same synchronization metrics used in previous relevant studies. However, in addition to calculating these metrics for the raw \( p(t) \) and \( q(t) \) signals, we also calculate them for the same signals but after they have been bandpass filtered at the frequency of the dominant thermoacoustic mode \( f_{Hz} \) for the short combustor, and \( f_{Hz} \) for the medium and long combustors. The purpose of this filtering is to determine what causes \( p(t) \) and \( q(t) \) to oscillate differently from each other. The filtering is performed using a Butterworth bandpass filter with a bandwidth of \( \pm 20 \text{Hz} \), a roll-off of \(-48 \text{dB per octave, and the phase compensated by the forward-backward scheme of Gustafsson.} \) All quantities computed from the bandpass-filtered signals are denoted by the subscript \( f \).

Figures 3(a), 3(f), and 3(k) show time traces of the pressure and HRR, both in raw form \( [p(t), q(t)] \) and in filtered form \( [p'(t), q'(t)] \), for the three different combustors. Also shown are the corresponding normalized frequency spectra [Figs. 3(b), 3(g), and 3(l)]. We divide each of the raw signals \( [p(t), q(t)] \) and each of the filtered signals \( [p'(t), q'(t)] \) into 10 equal segments. For each segment (0.3 s long), we calculate three synchronization metrics [62] [Figs. 3(c), 3(h), and 3(m)] the correlation between the probability of recurrence (CPR) of pressure and HRR, [Figs. 3(d), 3(i), and 3(n)] the probability of recurrence \( [P(r)] \) of pressure and HRR, and [Figs. 3(e), 3(j), and 3(o)] the instantaneous phase difference between pressure and HRR \( (\psi_{p'q'} \text{ or } \psi_{q'p'}) \), as computed with the Hilbert transform.

Although the Hilbert phase is physically defined only for narrowband signals, it can still provide valuable information when such signals are contaminated with weak broadband components, such as those arising from chaos. In our study, we take a conservative approach by using two other synchronization metrics \( \text{CPR and } P(r) \) to validate the results obtained via the Hilbert phase.

We begin by examining the raw (unfiltered) data. For the short and medium combustors [Figs. 3(a)–3(f)], \( p(t) \) and \( q(t) \) are only weakly correlated in time and even more weakly correlated in amplitude [Figs. 3(a1) and 3(f1)]. This view is supported by the fact that the mean CPR is well below one [Figs. 3(c) and 3(h)]. CPRmean = 0.48–0.55 and \( \psi_{p'q'} \) drifts unboundedly in time, with
intermittent epochs of phase locking [Figs. 3(c) and 3(i)]. However, the peaks in $P(\tau)$ for $p'(t)$ and $q'(t)$ are well correlated in time but not in amplitude [Figs. 3(d1) and 3(i1)]. Collectively, these observations indicate that, for the short and medium combustors, $p'(t)$ and $q'(t)$ are in a weak form of phase synchronization.\(^{2,3}\) For the long combustor [Figs. 3(k)–3(o)], the mean CPR is further reduced [Fig. 3(m); CPR\text{mean} = 0.29], the peaks in $P(\tau)$ for $p'(t)$ and $q'(t)$ are not correlated in time or amplitude [Fig. 3(n1)], and $\psi_{p',q'}$ drifts unboundedly in time, albeit with intermittent epochs of phase locking [Fig. 3(o)]. These observations indicate that, for the long combustor, $p'(t)$ and $q'(t)$ are desynchronized, even though the system is thermoacoustically self-excited.

To determine the cause of this desynchronization, we bandpass filter $p'(t)$ and $q'(t)$ according to the procedure described above. For all three combustors, we find that the time traces of $p'_f(t)$ and $q'_f(t)$ are well correlated in both time and amplitude [Figs. 3(a2), 3(f2), and 3(k2)]. In effect, the bandpass filtering removes the low-frequency components from the HRR spectra and the secondary modes from the pressure spectra [Figs. 3(b), 3(g), and 3(l)]. As a result, the mean CPR approaches one [Figs. 3(c), 3(h), and 3(m)]; CPR\text{mean} = 0.99], the peaks in $P(\tau)$ for $p'_f(t)$ and $q'_f(t)$ become well correlated in both time and amplitude [Figs. 3(d2), 3(i2), and 3(n2)], and $\psi_{p'_f,q'_f}$ becomes constant in time, with continuous phase locking [Figs. 3(e), 3(j), and 3(o)]. Taken together, these observations indicate that, when bandpass filtered at the frequency of the dominant thermoacoustic mode, $p'(t)$ and $q'(t)$ undergo generalized synchronization.\(^{2,3}\) Thus, the Rayleigh criterion is satisfied, implying that the flame is transferring sufficient energy to the acoustic field to balance the losses incurred through viscous and thermal damping as well as acoustic advection and radiation.\(^{2,3}\) These findings suggest that a possible cause of the unfiltered system’s departure from generalized synchronization could be the presence of low-frequency fluctuations in $q'(t)$. As mentioned in Sec. IV B, these low-frequency fluctuations are believed to arise from an intrinsic hydrodynamic mode in the flame, rather than from an acoustic mode in the combustor.\(^{2,3}\) In Sec. IV F, we will examine the mechanisms behind these low-frequency HRR fluctuations using high-speed chemiluminescence videos.

The desynchronization observed between the unfiltered $p'(t)$ and $q'(t)$ signals in the long combustor [Figs. 3(k)–3(o)] deserves
Further attention. In Fig. 4, we show the dynamics of this combustor as it transitions from [Fig. 4(c)] a thermoacoustically stable state characterized by low-amplitude chaos (this state is sometimes referred to as “combustion noise” in the literature), to [Fig. 4(d)] a thermoacoustically unstable state characterized by weak limit-cycle oscillations (see Ref. 24 for details of the line-dominated spectra), and finally to [Fig. 4(e)] a thermoacoustically unstable state characterized by high-amplitude chaos. These transitions are induced by increases in $\phi_{\text{pilot}}$ for a fixed combustor length ($L_c = 2500$ mm). In the thermoacoustically stable state ($\phi_{\text{pilot}} = 2.5$) [Fig. 4(c): purple], before the emergence of self-excited oscillations, $p'(t)$ and $q'(t)$ are desynchronized, as evidenced by the absence of temporal and amplitude correlations between the peaks in $P(\tau)$ for $p'(t)$ and $q'(t)$. This assessment is supported by analysis of the permutation spectrum (not shown for brevity), whose standard deviation contains forbidden patterns, indicating the presence of deterministic chaos. When $\phi_{\text{pilot}}$ increases to 3.8 [Fig. 4(d): orange], the system transitions to weak limit-cycle oscillations, as evidenced by the emergence of a closed-loop orbit in the phase portrait [Fig. 4(b): middle inset]. This transition is accompanied by increases in the mean CPR [Fig. 4(a)] and in the degree of temporal correlation between the peaks in $P(\tau)$ for $p'(t)$ and $q'(t)$, although the amplitudes of $P(\tau)$ are still not well correlated [Fig. 4(d)]. These observations suggest a state of weak phase synchronization, similar to that which was observed in the short and medium combustors of Fig. 3. When $\phi_{\text{pilot}}$ increases to 7.0 [Fig. 4(e): green], the system transitions to the chaotic state identified in Figs. 3(k)–3(o). This transition is accompanied by an approximate doubling of the root-mean-square amplitude of $p(t)$ [Fig. 4(b)] without a significant change in the mean CPR [Fig. 4(a)]. Nevertheless, an inspection of Fig. 4(e) reveals weakened temporal and amplitude correlations between the peaks in $P(\tau)$, indicating that $p'(t)$ and $q'(t)$ are no longer phase synchronized. In summary, when $\phi_{\text{pilot}}$ increases from 2.5 to 7.0 for a fixed combustor length ($L_c = 2500$ mm), the system first transitions from a low-amplitude chaotic state to a medium-amplitude limit-cycle state with weak phase synchronization between $p'(t)$ and $q'(t)$. It then transitions to a high-amplitude chaotic state with no synchronization between $p'(t)$ and $q'(t)$. However, it should be noted that if $p'(t)$ and $q'(t)$ were bandpass filtered at the frequency of the dominant thermacoustic mode (as per the procedure described above), then generalized synchronization would occur, which is consistent with the Rayleigh criterion and the results of Fig. 3.

F. Origin of chaos in the HRR of the flame

The $q'(t)$ signal analyzed in Secs. IV A–IV E was collected with a PMT, which means that it represents the global HRR of the flame. To explore the local HRR structures within the flame body, we analyze time-resolved videos of the chemiluminescence emission from the flame (see Sec. I). Specifically, we extract a time series of the chemiluminescence intensity from each pixel in a video (total of 6000 frames over 1 s) and apply a fast Fourier transform (FFT) to compute the dominant spectral amplitude and its corresponding temporal frequency. We then normalize the maximum spectral amplitude found at each pixel location by the maximum value found over the entire image domain: $A_l = A_l/A_{l,\text{max}}$. To delineate the flame boundary, we black out the pixel locations where $A_l < 0.1$. In Fig. 5, we show two representative test cases, both from the medium combustor ($L_c = 2200$ mm): (top row) a thermoacoustically stable state at $\phi_{\text{pilot}} = 2.5$ where both $p'(t)$ and $q'(t)$ indicate a stable fixed point, and (bottom row) a thermoacoustically unstable state at $\phi_{\text{pilot}} = 7.0$ where $p'(t)$ indicates a stable period-1 limit cycle but $q'(t)$ indicates chaos.
FIG. 5. Spatial distribution of HRR from the flame in the medium combustor ($L_c = 2200$ mm) at two different operating conditions: (top row) a thermoacoustically stable state at $\phi_{pilot} = 2.5$ where both $p'(t)$ and $q'(t)$ indicate a stable fixed point, and (bottom row) a thermoacoustically unstable state at $\phi_{pilot} = 7.0$ where $p'(t)$ indicates a stable period-1 limit cycle but $q'(t)$ indicates chaos. Shown are (a), (d) the normalized FFT amplitude at the dominant frequency, (b), (e) the dominant frequency, and (c), (f) the raw instantaneous flame image. In (b), (e), the frequency resolution is 2 Hz.

For the thermoacoustically stable state, there are coherent regions of high $\tilde{A}_I$ (red) near the injector outlet [Fig. 5(a)]. These regions, along with some of those around them, oscillate at a low frequency of around 62 Hz [Fig. 5(b)]. This frequency is well below the values expected of a Helmholtz mode. Furthermore, the fact that the regions of high $\tilde{A}_I$ are concentrated near the shear layers downstream of the injector outlet, rather than being more uniformly distributed, suggests that these oscillations are not due to a longitudinal mode either. Instead, recognizing that the flame structure resembles vortex breakdown [Fig. 5(c)], we speculate that the low-frequency HRR oscillations are due to hydrodynamic instabilities arising from vortex breakdown. Such instabilities are commonly found in swirling flames and can affect the HRR dynamics through a variety of shear layers, recirculation zones, and precessing vortex cores.

For the thermoacoustically unstable state, the regions of high $\tilde{A}_I$ (red) are seen to shift downstream, leaving the near-field region of the injector relatively undisturbed [Figs. 5(d) and 5(f)]. The resulting spatial distribution of $\tilde{A}_I$ is consistent with that of a longitudinal thermoacoustic mode. As Fig. 5(e) shows, most of the flame body is dominated by the frequency of the longitudinal mode ($f_{L2}$) and its second harmonic ($2f_{L2}$), which corroborates the discussion of Sec. IV B. Furthermore, two symmetric regions of low-frequency oscillations ($f_{low} \lesssim 70$ Hz) can be found in the same shear-layer regions, where $\tilde{A}_I$ was dominant in the thermoacoustically stable case [Fig. 5(a)]. These oscillations are immediately downstream of the swirl injector outlets and are therefore most likely to be hydrodynamic in origin.

Because hydrodynamic modes are known to project themselves more strongly onto the HRR field than onto the acoustic field, this interpretation would explain why the $q'(t)$ spectrum [Fig. 2(ii)] contains stronger low-frequency components than the $p'(t)$ spectrum [Fig. 2(i)].

In summary, when the system is thermoacoustically stable, the flame’s HRR is dominated by a hydrodynamic mode at a relatively low frequency ($f \approx 62$ Hz). However, when the system is thermoacoustically unstable, the flame’s HRR is dominated by two distinctly different modes: a low-frequency hydrodynamic mode ($f_{low} \lesssim 70$ Hz) and a high-frequency longitudinal mode ($f_{L2}$). The hydrodynamic mode projects itself more strongly onto the HRR field than onto the acoustic field, leading to stronger low-frequency components in the $q'(t)$ spectra than in the $p'(t)$ spectra (see Fig. 2). In this particular combustor, the presence of a hydrodynamic mode in the flame is what physically prevents $p'(t)$ and $q'(t)$ from undergoing generalized synchronization.
V. CONCLUSIONS

In this study, we have experimentally investigated the nonlinear dynamics of a thermoacoustically self-excited aero-engine combustion system—a turbulent swirling liquid-fueled diffusion flame in a variable-length combustor. We focused on the steady-state dynamics via simultaneous measurements of the acoustic pressure in the combustor and the HRR from the flame. We analyzed the pressure and HRR signals using a combination of linear and nonlinear tools, including the phase portrait, Poincaré map, frequency spectrum, correlation dimension, f-HVG, histogram of oscillations, CPR, P(τ), and ψ̅,ψ′. When the combustor length is increased following the onset of self-excited thermoacoustic oscillations, we find that the pressure signal transitions from a period-1 limit cycle to chaos, whereas the HRR signal remains chaotic as a result of the presence of an intrinsic hydrodynamic mode in the flame. Analysis of high-speed chemiluminescence videos, along with pressure and HRR spectra, shows that the hydrodynamic mode projects itself more strongly onto the HRR field than onto the acoustic field, thus preventing the HRR and pressure signals from undergoing generalized synchronization, even during thermoacoustic instability. When the hydrodynamic mode is filtered out of the data, we find that the pressure and HRR signals undergo generalized synchronization, which is consistent with the findings of previous studies on self-excited combustors without an independent hydrodynamic mode, i.e., after the onset of flow-acoustic lock-in. However, when the hydrodynamic mode is retained in the data, we find that the pressure and HRR signals are either weakly phase synchronized or desynchronized. There are two main contributions of this study: (i) we show that a liquid-fueled diffusion-flame combustor can exhibit complex nonlinear dynamics, including chaos and varying degrees of synchronization, which is reminiscent of the behavior of gaseous-fueled premixed-flame combustors and (ii) we show that one should be particularly careful when selecting a diagnostic signal from which to calculate nonlinear measures of self-excited thermoacoustic oscillations, because our experiments show that the pressure and HRR signals can be desynchronized by the presence of a hydrodynamic mode in the flame at a frequency different from that of the dominant thermoacoustic mode. Knowledge of this information could aid the development of both passive and active techniques for controlling self-excited thermoacoustic oscillations in combustion devices such as gas turbines.

APPENDIX: MULTIFRACTAL ANALYSIS

Given the broad spectra observed at low frequencies, particularly for the HRR signal [Figs. 2(c2), 2(f2), and 2(o2)], it is natural to ask whether or not the system dynamics is multifractal. This question is important because its answer could provide useful insight into the self-similar scaling of the system. The scaling of a multifractal signal cannot be described by just a single fractal dimension but involves interwoven subsets of different fractal dimensions. Previous experiments by Gotoda et al. and Nair and Sujith have shown that multifractality can arise in turbulent combustion systems, both before and after the onset of thermoacoustic instability. This information has been used, for example, to develop early warning indicators of thermoacoustic instability.

In this study, we use multifractal detrended fluctuation analysis (MFDFA) to explore the multifractal scaling properties of p′(t) and q′(t). The procedure, as applied to p′(t), is as follows:

(i) The cumulative deviate series is calculated as

\[ p' = \sum_{i=1}^{N} (p'_i - \bar{p}) \]  \hspace{1cm} (A1)

where \( \bar{p} \) is the mean of the entire time series of length N. The series \( p' \) is then divided into n nonoverlapping segments \( \{p'_k, k = 1, 2, \ldots, n\} \), each with length s.

![FIG. 6. Multifractal spectra of (top row) p′(t) and (bottom row) q′(t) for the three combustors. The spectra are estimated with −2 ≤ q ≤ 2. The pilot equivalence ratio is fixed at φpilot = 7.0.](image-url)
(ii) The variance of each segment is determined as

\[ \overline{F_{i,k}} = \frac{1}{s} \sum_{j=1}^{s} \left( p_i(j) - \overline{p_i} \right)^2, \]

where \( \overline{p_i} \) is a linear fit for segment \( p_i \).

(iii) A generalized \( q \)-th order fluctuation function \( (F_{i,k}^q) \) is defined by averaging over all segments

\[ F_{i,k}^q = \left( \frac{1}{N} \sum_{k=1}^{N} (F_{i,k}^q)^{\frac{1}{q}} \right)^{\frac{1}{q}}. \]

For \( q = 0 \), the structure function is defined

\[ F_{i,k}^0 = \exp \left( \frac{1}{2n} \sum_{k=1}^{n} \log (F_{i,k}) \right). \]

The structure function obeys a power law as a function of scale \( s_i = \ldots s, (n-1)s + 1, \ldots ns \) with an exponent of \( H(q) \) (i.e., \( F_{i,k}^q \propto n^{H(q)} \)). The \( q \)-th order Hurst exponent \( (H(q)) \) is related to the \( q \)-th order mass exponent \( \tau_q = qH(q) - 1 \).

A monofractal time series and white noise would have a mass exponent with a nonlinear dynamics of a self-excited thermoacoustic system subjected to acoustic forcing.

Both a monofractal time series and white noise would have a mass exponent with a nonlinear dynamics of a self-excited thermoacoustic system subjected to acoustic forcing.

\[ q = 0, \tau_q = qH(q) - 1. \]

Both a monofractal time series and white noise would have a mass exponent with a nonlinear dynamics of a self-excited thermoacoustic system subjected to acoustic forcing.

The multifractal spectrum \( f(\alpha) \) and \( \alpha \) are found via a Legendre transform

\[ \alpha = \frac{\partial q}{\partial q}, \]

\[ f(\alpha) = qa - \tau_q. \]

The multifractal spectrum \( f(\alpha) \) and \( \alpha \) is useful for characterizing the scale-invariant structure of a time series containing both small-scale and large-scale fluctuations.

Figure 6 shows the multifractal spectra of (top row) \( p'(t) \) and (bottom row) \( q'(t) \) for the three combustors. For the short and medium combustors \( (Figs. 6(a) and 6(b)) \), the multifractal spectra of \( p'(t) \) cluster around a singularity strength of \( \alpha = 0 \), indicating no scale invariance and no fractal structures. This is consistent with the fact that period-1 oscillations occur at just a single time scale: \( 1/f_{12} \) for the short combustor or \( 1/f_{12} \) for the medium combustor. The corresponding multifractal spectra of \( q'(t) \) are broad curves (Figs. 6(d) and 6(e)), indicating the presence of multifractality, i.e., that fluctuations of different amplitudes follow different scaling laws. Physically, this implies that energy is transferred between different time scales via multiplicative processes.

For the long combustor \( (Figs. 6(c) and 6(f)) \), the multifractal spectra of \( p'(t) \) and \( q'(t) \) also show broad curves, indicating again the presence of multifractality and multiplicative processes.

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ACKNOWLEDGMENTS

This work was funded by the Research Grants Council of Hong Kong (Project Nos. 162357/16 and 262028/15) and the Ministry of Trade, Industry & Energy (MOTIE) of the Republic of Korea (Grant No. 10067074).

Chaos 29, 053124-12 (2019); doi: 10.1063/1.5088735
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