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Self-aggregation Phenomenon and Stable Flow Conditions in a Two-Phase Flow Through a Minichannel

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Abstract: By increasing a water flow rate of the two-phase (air–water) flow through a minichannel, both the partitioning of air slugs into air bubbles of different sizes and small air bubbles aggregation into larger air bubbles were identified. These phenomena were studied in detail by using the corresponding sequences of light transmission time series recorded with a laser-phototransistor sensor. To distinguish any instabilities in air slugs along with their break-ups and aggregations, the recurrence plots and recurrence quantification analysis were applied.

Keywords: Bubble Dynamics; Recurrence Plots; Recurrence Quantification Analysis; Two-Phase Flow.

1 Introduction

Two-phase flow is typical of technical systems where, in addition to a liquid phase, a gas-phase appears. This phenomenon occurs in large boilers producing steam for turbines or in pumps, which operate close to a vapour pressure of a fluid being pumped. Many studies are carried out to investigate the nature of two-phase flow in pipes and minichannels [1–4]. Gas–liquid two-phase flow patterns were visualised and several distinctive flow patterns were identified by Triplett et al. [5] and Serizawa et al. [6].

Wang et al. [7] showed the flow patterns of an oil–gas–water mixture, identifying the flow patterns of air in water by using non-linear analysis methods, including the Hurst and Lyapunov exponents, and correlation dimension. On the other hand, Jin et al. [8] demonstrated that the correlation dimension and Kolmogorov entropy are sufficient to identify the flow patterns. Further, the results of non-linear analysis with respect to temperature and pressure fluctuations in microchannels are discussed by Mosdorf et al. [9].

Methods denote the dynamics of oil–water flow, that is, recurrence plots (RP) and recurrence quantification analysis (RQA), are used for identifying the flow patterns and to assess their complexity. Recently, Faszczewski et al. [10] used the RP to analyse the flow patterns in a vertical minichannel. Timung and Mandal [11] performed the modelling and identification of the flow patterns of gas–liquid flow through a circular microchannel using probabilistic neural networks.

This study focuses on identification and description of flow patterns that occur during the flow of a liquid–gas (water–air) mixture in a horizontal minichannel. More specifically, the RP- and RQA-based approaches are adopted [4, 10] and applied in order to analyse the laser light transmission time series, recorded by a laser-phototransistor sensor during the experiment. In this study, the bifurcation conditions are described, namely a transition from slugs or larger elongated bubble flows to the flow of smaller bubbles and vice versa. From the studies of Triplett et al. [5] and Serizawa et al. [6], where the stationary conditions (for fixed volume flow rate) were used, here, quasistatic conditions were used (by slowly changing the water volume flow rate). The main objective of this study is to identify the flow patterns by RQA parameters.

2 Experimental

During the experiment, it was analysed that the data recorded for different flow patterns (water–air at 21 °C) in a 3-mm-diameter circular channel. Figure 1 shows the schematic design of the experimental stand. Owing to the size of the minichannel, it was necessary to use
a special generator of mini bubbles (Fig. 1a) to produce bubbly flow inside the channel. The proportional pressure regulator (metal work regtronic with an accuracy of 1 kPa) was used to maintain the constant overpressure in the supply tank (Fig. 1a) at 50 kPa. Flow patterns were recorded using the Phantom v610 digital camera at 5000 fps (1280 × 64 pixels). The amount of air flowing through the minichannel was measured by a laser-phototransistor sensor (Fig. 1a).

Data from the sensors were acquired by a acquisition system (Data translation 9804, an accuracy of 1 mV for voltages in the range of −10 V to 10 V), (Fig. 1a) at a sampling rate of 1 kHz. A schematic design of the laser-phototransistor sensor is shown in Figure 1b.

Figure 2 illustrates the evolution of the flow with change in the water volume rate flow $q$, ranging from 0.021 l/min to 0.211 l/min. One can observe different patterns of slugs and bubbles aggregations during the flow. From the top (a), long slugs (extremely elongated large air bubbles) are observed, and then, with increase in $q$, the smaller slugs are noted, and finally, in cases (e–f), the slugs become very small, but they still have a substantially large length compared to the diameter of the minichannel. At the same time, small bubbles can also be observed in case (f).

At this stage, the pressure inside the pipe is high enough to destroy the slugs. Naturally, the new mechanism, combined with the different speeds of mixed size bubbles, leads to the new patterns. The longer bubbles, such as slugs, became more stable again [cases (j)–(o)]. They are separated by a larger volume of water and mediated by fairly small air bubbles. For larger $q$, the defragmentation of elongated bubbles to smaller ones and the pattern (a) that resembles the previous one (f) due to the presence of short slugs are observed. The differences in the new slugs formations are in a separation volume, which is covered by water with a large number of small bubbles.

To classify these changes, more systematic studies were carried out using the laser-phototransistor sensor. More specifically, the time series of laser light transmission was recorded during the particular flows. Six representative cases are given in Figure 3a–f (upper panels). It can be noted that the laser light transmission increases with increase in water volume rate. The recurrence statistical analysis was carried out after measurements, and the results were presented in the subsequent section. In these calculations, the Cross Recurrence Plot Toolbox for Matlab was used [12].

### 3 Recurrence Plots and Recurrence Quantification Analysis

The dynamics of the underlying phenomena can be investigated by means of recurrences that are calculated for each visited state of the reconstructed trajectory. This method was developed by Eckmann [13] and extended by Webber [14], Casdagli [15] later by Marwan et al. [16, 17] and others. This approach was used for both short deterministic and noise-affected experimental data [18, 19].

Two points on a trajectory are marked as neighbours if they are close enough to each other. This can be expressed by the distance matrix $R$ with its element $R_{ij}$ given by [13]:

$$R_{ij} = \Theta(\epsilon - |x_i - x_j|),$$

where $\epsilon$ is the threshold value and $\Theta(x)$ denotes the Heaviside function. The number of recurrence points depends on both the underlying dynamics and the cosineing the threshold value. A standard technique for approximations is that the threshold value should not be higher than a few percentage of the total number of points [17].

According to Takens [20] and the laser light transmission $x$ time series, the following vectors $x$ in the embedding space are defined:

$$x(t) = (x(t), x(t - \delta t \Delta t), x(t - 2\delta t \Delta t), \ldots, x(t - (m - 1)\delta t \Delta t),$$

where $\Delta t=0.1$ ms is a sampling period. The phase space reconstruction is performed by using the standard methods of the first minimum of average mutual information and
flow rate was constant

\[ q = 0.179; (r) 0.179; (s) 0.190; (t) 0.190; (u) 0.211, \] respectively. The air

\[ k = 0.147; (l) 0.137; (m) 0.158; (n) 0.158; (o) 0.158; (p) 0.169; (q) 0.063; (e) 0.084; (f) 0.1048; (g) 0.1053; (h) 0.116; (i) 0.126; (j) 0.126; \] water volume flow rate

\[ \text{l/min}: (a) 0.021; (b) 0.042; (c) 0.042; (d) \]

Photographs of the selected flow cases at a decreasing

Figure 2: Photographs of the selected flow cases at a decreasing water volume flow rate \( q/\text{l/min} \): (a) 0.021; (b) 0.042; (c) 0.042; (d) 0.063; (e) 0.084; (f) 0.1048; (g) 0.1053; (h) 0.116; (i) 0.126; (j) 0.126; (k) 0.147; (l) 0.137; (m) 0.158; (n) 0.158; (o) 0.158; (p) 0.169; (q) 0.063; (e) 0.084; (f) 0.1048; (g) 0.1053; (h) 0.116; (i) 0.126; (j) 0.126;

the nodal fraction of false neighbours [21–23]. The same

embedding methods are used for comparison with the

assumption of the smallest time delay \( \delta \) and then the

highest dimension \( m \). In this way, the system could be
described the dimension \( m = 7 \) and the delay \( \delta i = 7 \).

Figure 3a–f (in the lower panels) compare the recur-

ence plots (RP) of the threshold \( \epsilon = 3 \sigma \), where \( \sigma \) denotes the standard deviation of a particular time series depicted in the upper panels. It is noted that one criterion is used to all the cases in order to obtain 7 % recurrences in the minimal case. To estimate the distances, the Euclidean distance in the reconstructed phase space of the laser light transmission time series for six representative cases, obtained from the flows shown in Figure 2, was used. The horizontal and vertical axes represent the time instants \( i \) and \( j \) to which the distance formula (1) is applied.

Here, some interesting observations can be made. The clear checkerboard structure, which can be seen in most of figures, implies two states of intermittency [24, 25] in the light transmission time series; however, its type can be assessed with changing \( q \). Here, these intermittences are mostly related to switching between better and worse transparency depending on the presence of slugs and/or bubbles. It is worth noting that in the presence of very short slugs mediated by the small bubbles (Fig. 3f) the laser light is strongly scattered by moving surface of the slugs and bubbles resulting in fast oscillations of \( x \).

Figure 3c shows a large number of short slugs rarely mediated by air bubbles. Figure 3c,e, and f illustrate the structure smearing that could correspond to the larger complexity in bubbles and slugs sizes, and, consequently, to their speeds. On the other hand, more regular points distributions are visible in Figure 3a,b, and d. These cases correspond to the observations of longer air slugs and elongated bubbles.

Webber and Zbilut [14] and later Marwan et al. [16, 17, 26] developed the recurrence quantification analysis (RQA) for recurrence plots. The RQA analysis includes the recurrence rate variable, \( RR \), which expresses the system’s ability to return to the neighbourhood of previous state and has the following definition:

\[
RR = \frac{1}{N^2} \sum_{i, j} R^q(i, j).
\]

Furthermore, the RQA was used to identify diagonal lines through their maximal lengths, \( L_{\text{max}} \). The RQA provides the probability \( p(l) \) or \( p(v) \) of line distribution according to their lengths \( l \) or \( v \) (for diagonal and vertical lines). They are calculated as follows:

\[
p(z) = \frac{P(z)}{\sum_{z=z_{\text{max}}} P(z)},
\]

where \( z = l \) or \( v \) depend on diagonal or vertical structures in a specific recurrence plot. \( P(z) \) denotes the histogram of \( z \) lengths and a fixed value of \( \epsilon \), \( N_{z} \) is the number of diagonal or vertical lines (depending on the definition of \( z \)). Measures, such as determinism \( DET \), laminarity \( LAM \), trapping time \( TT \), and average length \( L \), are based on probabilities \( P(z) \)

\[
DET = \frac{\sum_{l=1}^{N} lP^l(l)}{\sum_{l=1}^{N} lP^l(l)},
\]

\[
LAM = \frac{\sum_{v=1}^{N} vP^v(v)}{\sum_{v=1}^{N} vP^v(v)},
\]

\[
L = \frac{\sum_{l=1}^{N} lP^l(l)}{\sum_{l=1}^{N} lP^l(l)}.
\]
Figure 3: Normalised time series of the laser light transmission $x'$ versus $i$ in sampling units ($x' = (x - x_{av})/\sigma(x)$), where $x$ is the original signal measured in volts, $x_{av}$ denotes its average value and $\sigma(x)$ its standard deviation, respectively. The corresponding recurrence plots for the cases 3a–f (with $q = 0.021, 0.042, 0.105, 0.137, 0.179, 0.211$ l/min) were taken from the following regions in Fig. 2a,b,g,l,q,u, respectively. The embedding parameter dimension was applied as $m = 7$, delay in the sampling units $\delta i = 7$, while RQA threshold $\epsilon = 3\sigma$, where $\sigma$ denotes the standard deviation of the corresponding $x$ series. The sampling period was $\Delta t = 0.1$ ms.
self-aggregation phenomenon and stable flow conditions (Figs. 2l and 3d for $q=0.137$ l/min).

This is related to the values of $q$ for which the elongated large bubbles are stable. They occur together with a low number of residual small bubbles. The recurrence rate, $RR$, (Fig. 1a) shows that the system returns to the same state most frequently at $q=0.06$ and 0.014, whereas between these value, the system behaves differently. It is concluded that for $q=0.06$ and 0.014, the fluctuations of laser transmissions appear between a small number of states, while for $q=0.10$, the number of states available significantly increases. This conclusion is supported by determinism, $DET$, and laminarity, $LAM$. At $q=0.06$, we observe the minimum, which indicates that the system becomes more random and unpredictable. The maxima $1/L_{max}$ correspond to the dynamical divergence that increases for three values of $q$. The maxima $1/L_{max}$

\[
TT = \frac{\sum_{v=v_{min}}^{N_v} vP^v(v)}{\sum_{v=v_{min}}^{N_v} P^v(v)},
\]

On the other hand, the transitivity, $TRAN$, is a network topological parameter based on the distance matrix elements $R_{ij}$:

\[
TRAN = \sum_{i=1}^{N} \frac{\sum_{j,k=1}^{N} R_{ij} R_{jk} R_{ki}}{\sum_{j,k=1}^{N} R_{ij} R_{jk}}.
\]

These parameters were studied systematically with change in water volume rate flow $q$. The results are shown in Figure 4. It is noted that these figures show a non-monotonic behaviour of the studied flow phenomenon, namely they confirm the effect of resonating self-organisation (Figs. 2l and 3d for $q=0.137$ l/min).

The figures show that the system returns to the same state most frequently at $q=0.06$ and 0.014, whereas between these values, the system behaves differently. It is concluded that for $q=0.06$ and 0.014, the fluctuations of laser transmissions appear between a small number of states, while for $q=0.10$, the number of states available significantly increases. This conclusion is supported by determinism, $DET$, and laminarity, $LAM$. At $q=0.06$, we observe the minimum, which indicates that the system becomes more random and unpredictable. The maxima $1/L_{max}$ correspond to the dynamical divergence that increases for three values of $q$. The maxima $1/L_{max}$

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Figure 4: RQA measures versus water volume flow rate $q$: (a) recurrence rate – $RR$; (b) determinism – $DET$; (c) laminarity – $LAM$; (d) divergence – $DIV=1/L_{max}$; (e) trapping time – $TT$; (f) transitivity (causality) – $TRAN$, respectively.
coincide with the minima in $RR$ conforming the instabilities by increasing available states. The occurrence of intermittency is more probable for reaching such maxima in the dynamical divergence ($1/L_{\text{max}}$). Particular attention should be payed on the trapping time parameter, $TT$. This measure signals the probability that the system will last in a single state. Once the clear increases in $TT$ are observed, the corresponding cases analysed by the RP (Fig. 3a and d) are represented by larger and clear checkerboard. Interestingly, this is the measure that directly indicates about the shape stability of air slugs or elongated bubbles. This stability strongly depends on the change of water flow rate $q$. Then, the bubbles merge into one large elongated slug upon increase in the water flow rate. Finally, transitivity is a topological measure of RP network itself. This can be linked to spatial and temporal properties of the system. Surprisingly, the $TRAN$ measure changes in a similar way to the trapping time, $TT$. Nonetheless, it should be noticed that a clear checkerboard formation leads to isolated square islands signalled by minimum in transitivity. Simultaneously, it is also possible to measure the cluster size of a square island. The smaller the size of square, the lower the value of transitivity. Consequently, for unstable slug and elongated bubble flows, the squares in the RP are irregular and of smaller sizes (Fig. 3c, e, and f). Then, the bubbles merge into one large elongated slug upon increase in the water flow rate $q$ (Fig. 4e). Then, the bubbles merge into one large elongated slug upon increase in the water flow rate $q$. The multiple peaks in patterns (Fig. 4) are due to aggregation of small bubbles into larger bubbles and slugs.

4 Conclusions

We observed multiple air slugs flow to bubbles flow bifurcations in the air–water flow patterns. The changes in pattern were caused by the instability of longer air slugs, which was increased for various values of water flow rate $q$. Using digital camera, the partitioning of air slugs to bubbles of different sizes with increasing $q$ was identified. These patterns were also recorded by a simultaneous laser light transmission. The RQA analysis was applied to the transmission light time series. It was confirmed that the bifurcations significantly increases in all examined measures ($RR$, $DET$, $TRAN$, $LAM$, $TT$, and $1/L_{\text{max}}$). This information was used to automate identification of air slug or elongated bubble occurrence, their average length and stability. Taking into account the fact that the air flow rate is stable, the main problem analysed in the study is the observation of merging of multiple small air bubbles into air slugs or elongated bubbles. This merging can be associated with a phenomenon of self-organisation that can occur at certain water flow rates.

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Nomenclature

$q$ Water volume flow rate
$q_0$ Air volume flow rate
$\mathbf{R}$ Recurrence plot
$\mathbf{RQA}$ Recurrence quantification analysis
$x$ Vector representation of $x$ in the reconstructed (embedding) phase space
$t$ Time
$\Delta t = 0.1 \text{ ms}$ Sampling time
$\delta i$ Time delay in the sampling units
$m$ Embedding dimension
$\epsilon$ Threshold value
$\theta$ Recurrence matrix
$\Theta$ Heaviside function
$R, R^c$ Recurrence rate
$l$ or $v$ Lengths for diagonal and vertical lines
$p(l)$ or $p(v)$ Probabilities of diagonal and vertical line distribution according to their lengths
$DET$ Determinism
$LAM$ Laminarity
$DIV = 1/L_{\text{max}}$ Divergence, inverse of the maximal diagonal line length
$TT$ Trapping time
$TRAN$ Transitivity

References